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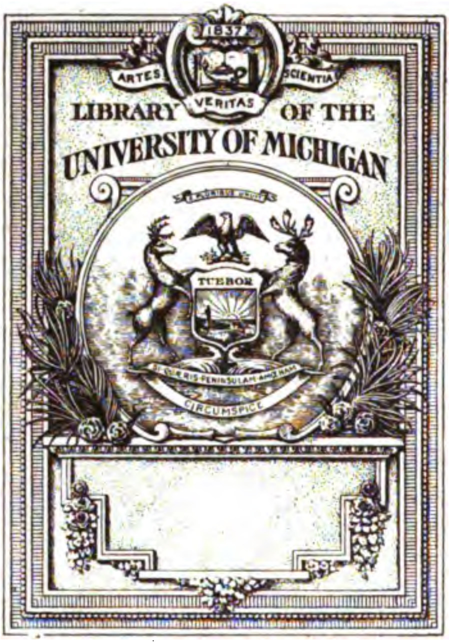

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DYNAMOS AND MOTORS

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DYNAMOS AND MOTORS

A TEXT BOOK FOR
COLLEGES AND TECHNICAL SCHOOLS

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DIRECT-CURRENT AND
ALTERNATING-CURRENT MACHINES

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PREFACE.

There is a demand for a simple physical treatise on Dynamo Electric Machinery apart from such subjects as Electric Distribution and Wiring, and Electric Lighting, and this book has been arranged to meet this demand.

South Bethlehem, Pa.,
June 22, 1909.

339659

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DYNAMOS AND MOTORS.

CHAPTER I.

THE ELECTRIC CURRENT. CHEMICAL EFFECT.

1. The electric current. — When a wire is connected to the terminals of an ordinary battery, certain phenomena are produced and an *electric current* is said to flow through the wire. A wire in which an electric current is flowing is sometimes called an *electric wire* for brevity. The production of an electric current always requires a *generator* such as a battery or a dynamo. The path of the current is usually a wire and it is termed the *electric circuit.* If the path is complete, leading out from the generator and returning to it without break or interruption, the circuit is said to be *closed;* otherwise the circuit is said to be *open.* A steady electric current always flows in a closed circuit, that is, in a circuit which goes out from the generator and returns to it, and the current ceases to flow when the circuit is broken (opened).

Certain substances such as metals and salt solutions may form portions of an electric circuit. Such substances are called *electrical conductors.* Other substances, such as glass, hard rubber, air, and dry wood, cannot form a portion of an electric circuit, that is, the electric current cannot flow through them to any appreciable extent. Such substances are called *insulators.*

Energy must be supplied to an electric generator (chemical energy in the case of a battery, mechanical energy in the case of a dynamo), and this energy reappears in various parts of the electric circuit through which the current flows. Thus, energy reappears as heat in an electric lamp and as mechanical work in an electric motor.

The chemical effect. — When a solution of a chemical compound forms a portion of an electric circuit, the compound is, in general.

decomposed by the current. This chemical effect of the electric current is exemplified in the practical operation of electroplating.

In the following brief outline of the theory of the electric current the chemical effect is discussed first, because it forms the most intelligible basis for the measurement of current, although the fundamental method of current measurement is based upon the magnetic effect.

The magnetic effect. — A wire through which an electric current flows is called an electric wire for brevity. The region surrounding an electric wire is a magnetic field, and it is to the action of this field that many of the most important phenomena of the electric current are due.

The heating effect. — A wire, or any substance which forms a portion of an electric circuit, has heat generated in it by the current. This heating effect of the electric current is exemplified in the ordinary glow lamp, the incandescent carbon or metal filament of which forms a portion of an electric circuit.

The hydraulic analogue of the electric current. — The electric current in a wire may be thought of as something flowing through the wire very much as water flows through a pipe. Thus a battery or a dynamo producing an electric current through a circuit of wire is analogous to a pump forcing water through a pipe, the pipe leading out from the pump and returning to it, so that the water is pumped through the pipe again and again. Energy must be supplied to the pump as to an electric generator, and this energy reappears in the various parts of the circuit through which the water flows. Thus heat is generated because of the frictional resistance of the pipe, and if a water motor is included in the water circuit, a portion of the energy expended at the pump reappears as mechanical work in the motor.

Water flowing through a pipe does not produce any disturbance in the region surrounding the pipe corresponding to the magnetic field surrounding an electric wire. Therefore the hydraulic analogue of the electric current is of no help in giving one a conception of the magnetic effect of the electric current; and, in the study of those phenomena of the electric current which depend upon its magnetic effect, the hydraulic analogue must be used with caution.

2. The chemical effect of the electric current. — When a solution of a chemical compound forms a portion of an electric circuit, the compound is, in general, decomposed by the current, as stated above. Thus, melted salts, and acids and salts in solution are decomposed by the electric current. This chemical decomposition is called *electrolysis*, and the liquid in which electrolysis takes place is called an *electrolyte*. Electrolysis is usually carried out in a vessel provided with two flat plates of metal or carbon which serve to lead the current into and out of the electrolyte. Such an arrangement is called an *electrolytic cell* and the plates of metal or carbon are called the *electrodes*. The electrode upon which the metallic constituent of a solution is deposited is called the *cathode* and the other is called the *anode*. It is customary to speak of the current as flowing into an electrolytic cell at the anode and out of the cell at the cathode, that is, *the electric current is considered to flow in the direction in which the metallic constituent of the solution is carried in an electrolytic cell.*

Electro-chemical equivalent of a metal. The international standard ampere. — The amount of metal deposited by electrolysis is proportional to the strength of the current and to the time that the current continues to flow, and the amount of metal in grams deposited in one second by a unit of current is called the electro-chemical equivalent of the metal.

The fundamental definition of the ampere is based upon a consideration of the magnetic effect of the electric current, but the most intelligible definition of the ampere is that which was adopted as a legal standard by the International Electrical Congress of 1893. *The ampere is a current of such strength as to deposit 0.001118 gram of metallic silver per second from an aqueous solution of pure silver nitrate.* The c.g.s. unit of current is equal to 10 amperes and it is called the *abampere*.

3. The voltaic cell.* — The chemical action that is caused by the flow of current through an electrolytic cell is confined wholly to the immediate neighborhood of the electrodes, and this chemical action is usually *forced*, that is, work has to be done to bring it about, or in other words, an external electric generator has to be used to push the current through the cell. In some cases however, the chemical action which takes place in an electrolytic cell is a source of energy. Such an electrolytic cell produces a current in any circuit to which it is connected, and it is called a *voltaic cell.*

Example: When a strip of clean zinc and a strip of copper or carbon are dipped into dilute sulphuric acid, no chemical action takes place. When the plates are connected together by a wire, however, a current immediately starts to flow through the circuit, causing SO_4 to be liberated at the zinc plate and H_2 to be liberated at the copper or carbon plate. The hydrogen (H_2) passes off as a gas, and the SO_4 combines with the zinc forming zinc sulphate which goes into solution. This reaction, as a whole, is a source of energy and the energy developed by it goes to maintain the electric current in the circuit.

The Heating Effect of the Electric Current.

4. Electrical resistance. — When an electric generator produces an electric current in a circuit, part of the work expended in driving the generator reappears as heat in the various parts of the circuit. The electric current seems to be opposed by a kind of resistance somewhat analogous to the resistance which a pipe offers to the flow of water, and a portion of a circuit is said to have more or less resistance according as more or less heat is generated in it by a given current. Thus the filament of a glow lamp has a much greater resistance than the copper connecting wires inasmuch as a great deal more heat is generated in the filament than is generated in the wire.

---

* See *Primary Batteries*, by H. S. Carhart, Allyn and Bacon, Boston, 1891, for detailed information concerning voltaic cells; see *The Theory of the Lead Accumulator* by Friedrich Dolezalek (English translation by C. L. von Ende, published by John Wiley & Sons), for detailed information concerning the action of the lead storage cell; and see *Storage Battery Engineering*, by Lamar Lyndon, McGraw & Co., N. Y., for practical information concerning storage cells.

Joule's law. — The rate at which heat is generated in a given wire is proportional to the square of the current flowing in the wire, or, the amount of heat generated in a given wire is proportional to the square of the current and to the elapsed time, that is

$$H = RI^2t \tag{1}$$

in which H is the amount of heat generated by the current I in t seconds, and R is a constant for the given wire. The constant R [**units-of-heat-per-(unit current)^2-per-second**] is adopted as the numerical measure of the resistance of the wire, and it is called the resistance of the wire.

The ohm. — When in equation (1), H is expressed in joules,* I in amperes, and t in seconds, then R is expressed in ohms. That is, a wire has a resistance of one ohm when one joule of heat is generated in it per second by an ampere of current.

The abohm. — When in equation (1), H is expressed in ergs, I in abamperes and t in seconds, then R is expressed in terms of a unit called the c.g.s. unit of resistance, or the abohm. One ohm is equal to 10^9 abohms.

Resistivity. — The resistance of a wire is proportional to the length of the wire and inversely proportional to its sectional area. That is

$$R = \rho \cdot \frac{l}{s} \tag{2}$$

in which R is the resistance of a wire of given material, l is the length of the wire, s is the sectional area of the wire, and ρ is a constant which is called the resistivity, or specific resistance, of the material of which the wire is made. The resistivity of a material is evidently equal to the resistance of a wire of unit length and of which the sectional area is unity. Electrical engineers ordinarily express length of wire in feet and sectional area in circular mils. In this case ρ is the resistance of a wire one mil in diameter and one foot long; the "mil-foot" of wire as it is sometimes called. The resistance of a "mil-foot" of ordinary commercial copper wire at ordinary room temperature, say, 20° C., is about 10.4 ohms.

---

* Heat is best expressed in terms of its energy value. Thus one erg of heat is the quantity of heat which is the equivalent of one erg of work ; one joule of heat is the quantity of heat which is the equivalent of one joule of work. There are 4.2 joules in one calorie.

Influence of temperature on resistance. — The resistance of a wire depends not only upon length, size, and material, but also upon temperature. Most metallic wires increase in resistance with rise of temperature.

The increase of resistance of a given wire due to a rise of temperature is proportional to the initial resistance and approximately proportional to the rise of temperature; that is, if R_0 is the resistance of a wire at some standard temperature, say 0° C., then the increase of resistance when the wire is heated to $t°$ C. is equal to $\beta R_0 t$, where β is a proportionality factor. Therefore the total resistance R_t of the wire at t°C. is $R_t = R_0 + \beta R_0 t$, or

$$R_t = R_0(1 + \beta t) \tag{3}$$

The quantity β is called the temperature coefficient of resistance of the given material. The value of β for high grade commercial copper is about 0.004 per centigrade degree.

Solutions of acids and salts, graphitic carbon, and nearly all so-called insulating materials such as glass and rubber, decrease in resistance with rise of temperature; in other words their temperature coefficient is negative.

TABLE.—RESISTIVITIES AND TEMPERATURE COEFFICIENTS.

	a	*b*	*c*
Aluminum wire (annealed) at 20° C.	27.4×10^{-7}	16.5	+0.0039
Copper wire (annealed) at 20° C.	17.24×10^{-7}	10.4	+0.0040
Iron wire (pure annealed) at 20° C.	95×10^{-7}	58.0	+0.0045
Steel telegraph wire at 20° C.	150×10^{-7}	91‡	+0.0043‡
Steel rails at 20° C.	120×10^{-7}	72‡	+0.0035‡
Mercury at 0° C.	943.4×10^{-7}	—	+0.00088
Platinum wire at 0° C.	89.8×10^{-7}	54.0	+0.00354
German-silver wire at 20° C.	212×10^{-7}	127‡	+0.00025‡
Manganin wire (Cu 84, Ni 12, Mn 4) at 20° C.	475×10^{-7}	286	—*
"Ia Ia" metal wire, hard (copper-nickel alloy) at 20° C	500×10^{-7}	300‡	—0.00001‡
"Climax" or "Superior" metal (nickel-steel alloy) at 20° C.	800×10^{-7}	480‡	+0.00067‡
Arc-lamp carbon at ordinary room temperature	0.005		—0.0003‡
Sulphuric acid, 5 per cent. solution at 18° C.	4.8 ohms		—0.0120†
Ordinary glass at 0° C. (density 2.54)	10^{15} ohms‡		
Ordinary glass at 60° C.	10^{12} ohms‡		
Ordinary glass at 200° C.	10^{8} ohms‡		

a = resistance in ohms of a bar 1 centimeter long and 1 square centimeter sectional area.

b = resistance in ohms of a wire 1 foot long and 0.001 inch in diameter.

c = temperature coefficient of resistance per degree centigrade (mean value between 0° C. and 100° C.).

5. Power required to maintain a current in a circuit expressed in terms of resistance and current. — When all of the work which is spent in maintaining a current appears in a circuit as heat, then the rate at which heat appears in the circuit is equal to the rate at which work is done in the maintenance of the current. Therefore, the power P required to maintain a current I through a wire having a resistance R is equal to RI^2 if no portion of the power is used for producing mechanical or chemical effects. That is:

$$P = RI^2 \tag{4}$$

in which P is the power in watts (joules per second) required to maintain a current of I amperes through a wire having R ohms of resistance. Of course equation (4) holds true when all quantities are expressed in c.g.s units. (P in ergs per second, I in abamperes, and R in abohms.)

---

* See temperature-resistance curve on page 33 of Franklin and MacNutt's *Elements of Electricity and Magnetism.*

† Between 18° C. and 19° C.

‡ These values differ greatly with different samples.

Electromotive Force and Ohm's Law.

6. Electromotive force. — It may seem from equation (4) that the power delivered by a generator should be proportional to the square of the current, but *the current delivered by a given generator under fixed conditions can be increased only by reducing the resistance of the circuit to which the generator delivers current.* In fact, the power delivered by a given generator under fixed conditions of running is very nearly proportional to the current so that the ratio P/I is nearly constant. This ratio P/I, whether it is constant or not, is called the *electromotive force* of a generator. That is:

$$P = EI \tag{5}$$

in which P is the power delivered by an electric generator, I is the current, and E is the electromotive force between the generator terminals. The electromotive force between the terminals of a generator is analogous to the pressure-difference between inlet and outlet of a fan blower. An increase of current delivered by a generator * usually causes a decrease of electromotive force between the terminals of the generator very much as an increased delivery of air by a fan, by opening the outlet for example, causes a decrease of pressure difference between inlet and outlet of the fan.

Unsatisfactory character of fundamental definition of electromotive force. — The definition of a physical quantity consists in every case of a concise statement of the fundamental method of measuring that quantity, and when this fundamental method of measuring a quantity involves operations which are not feasible under the ordinary conditions of practical work, the definition always seems more or less unsatisfactory. Thus, the above definition of the electromotive force, as **units-of-work-per-second-per-ampere,** assumes that the rate of doing work in pushing a current through a circuit is to be measured directly in mechanical units. This can indeed be done, but it is an operation of great difficulty and one that is seldom performed.

---

* Such as a separately excited dynamo, or a shunt dynamo.

Definition of electromotive force based on Ohm's law. — When all the work which is delivered to an electrical circuit is used to heat the circuit in accordance with Joule's law, a simple relation, called Ohm's law, exists between current, resistance, and electromotive force, and the simplest method of measuring electromotive force (and therefore the simplest definition of electromotive force) is based on Ohm's law.

7. Ohm's law. — The current produced by a voltaic cell, or, in general, by any electric generator, is inversely proportional to the resistance of the circuit.* This relation was discovered by G. S. Ohm in 1827, and it is called Ohm's law. A complete statement of Ohm's law together with a clear specification of the conditions under which the law applies may be derived as follows: The power output of an electric generator is equal to EI according to equation (5). **If the whole of this power is used to heat the circuit in accordance with Joule's law,** then we must have

$$EI = RI^2$$

according to equation (4). Therefore we have

$$E = RI \tag{6a}$$

or

$$I = \frac{E}{R} \tag{6b}$$

The volt. — When R in equations (6) is expressed in ohms and I in amperes, then E is expressed in terms of a unit called the volt. That is, one volt is an electromotive force sufficient to maintain one ampere of current through a wire having a resistance of one ohm. When, in equation (5), electromotive force is expressed in volts, and current in amperes, then the power P which is delivered to a circuit is expressed in watts.

---

* This statement and the statement given in the previous article to the effect that the power output of a generator is proportional to the current, are not exactly true, because of the fact that the electromotive force of a generator usually falls off in value to some extent, when the generator is called upon to give an increased current.

When, in equations (6), current is expressed in abamperes, and resistance in abohms, then electromotive force is expressed in c. g. s. units. The c. g. s. unit of electromotive force is called the *abvolt.* When, in equation (5), electromotive force is expressed in abvolts and current in abamperes, then the power P is expressed in ergs per second.

It is to be particularly noted that equations (4), (5), (6*a*), and (6*b*) apply to the whole or to any portion of an electrical circuit. Thus if R is the resistance of a transmission line delivering a current I to a receiving circuit, such as an electric motor, then RI^2 is the power lost in the line, and RI is the electromotive force lost in the line. This electromotive force lost in the transmission line on account of line resistance is called *line drop of electromotive force or pressure*, or simply *line drop.* Similarly the electromotive force RI used to overcome the resistance of the armature of an electric generator, is called *armature drop.*

8. Series and parallel connections. — When two portions of an electric circuit are so connected that the entire current in the circuit passes through both portions, the portions are said to be *connected in series.* When two portions of an electric circuit are so connected that the current in the circuit divides and a part of it flows through each portion, the portions are said to be *connected in parallel.*

The ordinary arc lamps which are used to light city streets are connected in series, and the entire current delivered by the lighting generator flows through each lamp. On the other hand, if the electromotive force of the generator is, say, 4,000 volts, and if there are 80 similar lamps in series, the electromotive force between the terminals of each lamp will be 50 volts. *The elec-*

tromotive force of a generator is subdivided among a number of receiving devices connected in series.

The ordinary glow lamps which are used for house lighting are connected in parallel between copper mains which lead out from the terminals of the generator, and, except for a slight loss of electromotive force in the mains, the full electromotive force of the generator acts upon each lamp. On the other hand, if the generator delivers, say 1,000 amperes, and if there are 2,000 similar lamps connected between the mains, the current in each lamp will be ½ ampere. *The current delivered by a generator is subdivided among a number of receiving devices connected in parallel.*

Electric generators may be connected in series. When this is done the electromotive force available for the maintenance of current is equal to the sum of the electromotive forces of the indidividual generators.

Electric generators may be connected in parallel.* When this is done the total current delivered by the set is equal to the sum of the currents delivered by the individual generators.

To determine the current in each of two branches of a circuit when the total current and the resistance of each branch are given.—The solution of this problem depends upon two principles, which were first pointed out by Kirchhoff and which are called Kirchhoff's laws, as follows:

(*a*) The current in the undivided part of a circuit is equal to the sum of the currents in the respective branches into which the circuit divides.

(*b*) Let A and B be the two points at which two or more branches of a circuit unite. The product of the resistance R_1 of any one of the branches into the current I_1 flowing in that branch is equal to the electromotive force between the branch points A and B. Therefore the product RI has the same value for every branch terminating in the points A and B.

---

* When the electromotive forces of two generators are not equal, then if the generators are connected in parallel, a very large current will in general be produced in the circuit formed by the two machines.

In the case of two branches we have from (*a*)

$$I = I_1 + I_2$$

and from (*b*) we have

$$R_1 I_1 = R_2 I_2$$

in which I is the total current, and the subscripts refer to the respective branches. Solving these two equations for I_1 and I_2, we have:

$$I_1 = \frac{R_2}{R_1 + R_2} \cdot I \qquad (7a)$$

$$I_2 = \frac{R_1}{R_1 + R_2} \cdot I \qquad (7b)$$

To determine the combined resistance of a number of coterminous branches of a circuit. — Let I be the total current flowing in the circuit, E the electromotive force between the branch points, and R_1, R_2, R_3, ... the resistances of the respective branches. Then E/R_1, E/R_2, E/R_3, etc., are the currents in the respective branches, and from Kirchhoff's law we have

$$I = \frac{E}{R_1} + \frac{E}{R_2} + \frac{E}{R_3} \cdots$$

Now, the combined resistance, R, of all the branches is defined as that resistance through which the electromotive force E between the branch points could maintain the total current I; so that $RI = E$, or $I = E/R$. Substituting this value of I in the above equation, we have

$$\frac{E}{R} = \frac{E}{R_1} + \frac{E}{R_2} + \frac{E}{R_3} + \cdots .$$

whence

$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \cdots} \qquad (8)$$

MAGNETIC EFFECT OF THE ELECTRIC CURRENT.

9. The magnetic effect of the electric current. — Before describing the magnetic action of the electric current, it is necessary to consider some of the properties of ordinary magnets of steel. When iron filings are sifted upon a pane of glass under which a steel magnet is placed, the filings arrange themselves in lines. This arrangement of the filings is facilitated by gently tapping the glass. Thus Fig. 1 shows the arrangement of iron filings between two unlike poles of two magnets, and Fig. 2 shows the arrangement of iron filings in the neighborhood of a piece of soft iron placed between two unlike poles of two magnets. The appearance of these figures suggests that something emanates from the north pole of a magnet and converges upon the south pole of the magnet, or vice versa. In fact, the whole region in the neighborhood of the poles of a magnet is permeated by a certain magnetic influence, and such a region is called a *magnetic field*. The filaments of iron filings in Figs. 1 and 2 represent what are called the *lines of force* of the magnetic field.

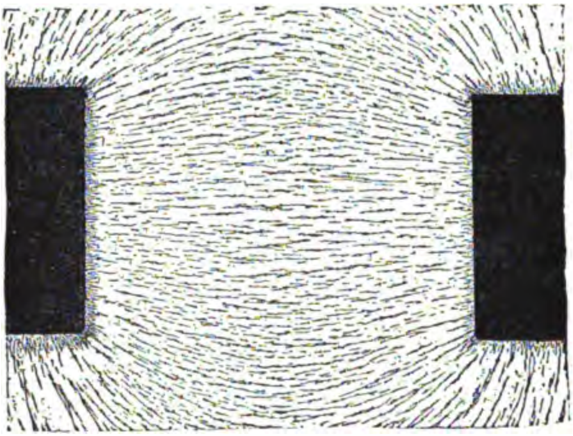

Fig. 1.

Figure 3 shows the arrangement of iron filings in the neighborhood of an electric wire; the black circle at the center represents a sectional view of the wire, and the wire is perpendicular to the plane of the figure. The lines of force of the magnetic field encircle the wire.

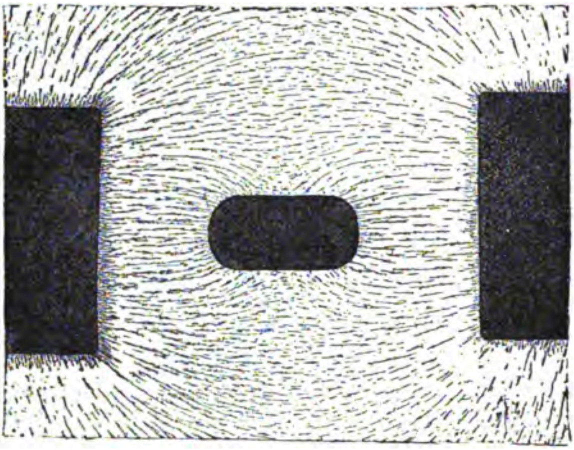

Fig. 2.

Figure 1 shows the approximately uniform magnetic field between two opposite magnet poles, and Fig. 4 represents the composite magnetic field produced between the two magnet poles of Fig. 1 when an electric wire is placed between them; the small

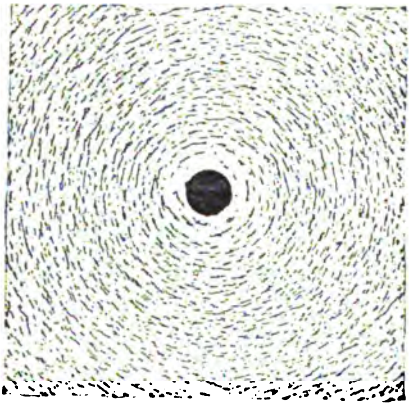

Fig. 3.

black circle in Fig. 4 represents the section of the wire, and the wire is perpendicular to the plane of the figure. The figures, 1 to 4, are from photographs.

Side force on an electric wire in a magnetic field. — The two unlike magnet poles in Fig. 1 attract each other. This attraction suggests the idea that the lines of force of the magnetic field are in a state of tension like stretched elastic filaments. The electric wire in Fig. 4 is pushed sidewise by the two magnet poles. This side force also may be attributed to a tension of the lines of force which bend around the wire in Fig. 4. The side force is proportional to the strength of current in the wire, and *the side force acting on one centimeter of length of a wire carrying one abampere of current is adopted as the measure of the strength of the magnetic field across which the wire is stretched.** That is, the strength of a magnetic field is **side-force-per-unit-length-of-wire-per-unit-current** flowing in the wire. Therefore, if we represent the intensity of a magnetic field by the letter $\mathcal{H}$, and the strength of current

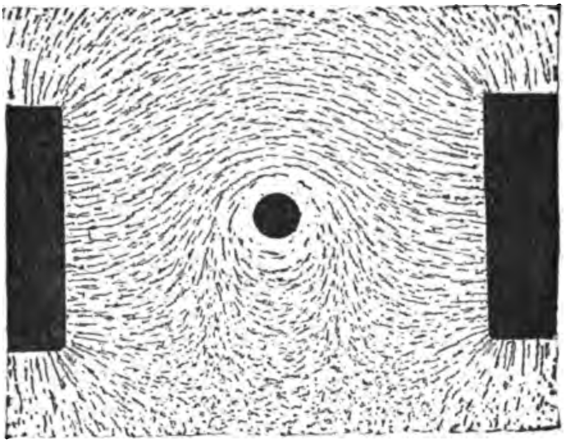

Fig. 4.

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*The idea of strength of a magnetic field is usually developed independently of the idea of current strength in a wire. The present outline of the principles of electricity and magnetism, however, is based upon the International Standard (Legal) definition of the ampere, for the sake of simplicity.

in a wire in abamperes by I, then $I\mathcal{H}$ will be the side force pushing on one centimeter of the wire, and the side force F acting on l centimeters of the wire will be

$$F = I\mathcal{H}l \tag{9}$$

in which F is the force in dynes pushing sidewise on l centimeters of wire stretched across a magnetic field of intensity $\mathcal{H}$ and carrying a current of I abamperes.

The Gauss. — When, in equation (9), I is expressed in abamperes, l in centimeters, and F in dynes, then $\mathcal{H}$ is expressed in terms of a unit which is called the gauss. That is, a magnetic field has an intensity of one gauss when it exerts a side force of one dyne upon one centimeter of an electric wire in which one abampere of current is flowing, the wire being at right angles to the lines of force of the magnetic field.

Direction of current. — The idea of direction of current which is given in Art. 2, namely, that the current flows in the direction in which the metallic constituent of an electrolyte is carried, is sometimes less convenient than the idea of direction of current which is based upon the magnetic effect of the current. The idea of direction which is based upon the magnetic effect of the current is as follows: The lines of force of the magnetic field surrounding an electric wire are circles as shown in Fig. 3, and a north magnet-pole tends to move around an electric wire in one direction whereas a south magnet-pole tends to move around the electric wire in the opposite direction, following the lines of force in each case. *The current is considered to flow in that direction along the wire in which a right-handed screw would travel if turned in the direction in which a north-magnet pole tends to move around the wire.*

10. Magnetic flux. — Consider a plane surface, s square centimeters in area, stretched across and at right angles to the lines of force of a uniform magnetic field of intensity $\mathcal{H}$. The product $s\mathcal{H}$ is called the *magnetic flux* across the surface. That is:

$$\Phi = s\mathcal{H} \tag{10}$$

in which Φ is the magnetic flux across a plane surface, s square centimeters in area, at right angles to a uniform magnetic field of intensity $\mathcal{H}$.

The unit of magnetic flux. — When, in equation (10), $\mathcal{H}$ is expressed in gausses and s in square centimeters, then the magnetic flux Φ is expressed in terms of a unit which is called the *maxwell.* That is, one maxwell is the amount of magnetic flux which crosses one square centimeter of area, when the area is at right angles to the lines of force of a magnetic field of which the intensity is one gauss. The unit of magnetic flux (the maxwell) is almost universally called *the line*, that is, a magnetic flux is nearly always expressed as so many lines.

INDUCED ELECTROMOTIVE FORCE.

11. Faraday's discovery. Induced electromotive force and induced current. — In 1831 Faraday discovered that a momentary electric current is produced in a coil of wire when a magnet is pushed into or withdrawn from the opening of the coil, or when an iron rod around which the coil is wound is magnetized or demagnetized. The motion of the magnet in the first case, or the changing magnetism of the iron rod in the second case, produces a momentary electromotive force in the coil, and this electromotive force produces a momentary electric current in the coil if the coil forms a portion of a closed circuit. Electromotive force and electric current produced in this way are called *induced electromotive force* and *induced current.*

Lenz's Law. — An induced current always opposes the action which produces it, and the work done in overcoming this opposition is the work which is expended in producing the induced current.

Examples. — (*a*) When a magnet is pushed into a coil the induced current is in such a direction as to tend to push the magnet out of the coil; more work must, therefore, be done in pushing the magnet into the coil than would be done if the circuit of the coil were open so that no induced current could be produced;

this extra work is the work which is expended in producing the induced current.

(*b*) When an iron rod wound with wire is magnetized, the current induced in the winding of wire opposes the magnetization, and more work must be done to magnetize the rod than would be done if the induced current did not exist. This additional work is the work which is expended in producing the induced current.

A more general aspect of Lenz's law is given in Art. 28, where the relation between the electric generator and the electric motor is discussed.

12. Electromotive force induced in a straight wire moving sidewise across a uniform magnetic field. — With the help of Lenz's law the fundamental law of induced electromotive force may be derived. For this purpose it is sufficient to consider the simplest case, namely, the motion of a straight wire across a uniform magnetic field.

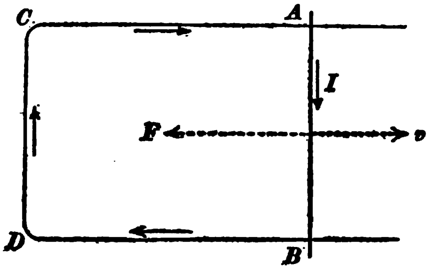

Fig. 5.

A straight wire AB, Fig. 5, slides sidewise at a velocity of v centimeters per second along two straight wires or rails CA and DB distant l centimeters from each other. The rails are connected at CD so that $ABCD$ is a closed circuit. The whole arrangement is placed in a uniform magnetic field of intensity $\mathcal{H}$, the direction of the field being perpendicular to the plane $ABCD$ and towards the reader.

The motion of the wire AB induces in it an electromotive force; this electromotive force produces a current in the circuit $ABCD$, and because of this current the wire AB is pushed sidewise by the magnetic field with the force F, of which the value in dynes is given by equation (9), namely,

$$F = I\mathcal{H}l \tag{1}$$

in which I is the strength of the induced current in abamperes.

Now Fv is the rate in ergs per second at which work is done in moving the wire AB against the opposing force F, and, from Lenz's law, this must be equal to the rate EI at which work is done by the induced electromotive force in maintaining the induced current I. That is:

$$Fv = EI \tag{ii}$$

in which E is the value of the induced electromotive force in abvolts.

Substituting the value of F from equation (i) in equation (ii) and solving for E, we have:

$$E = l\mathcal{H}v \tag{11a}$$

That is, the electromotive force E, in c. g. s. units (abvolts), induced in a straight wire l centimeters long moving sidewise at a velocity of v centimeters per second across a uniform magnetic field of intensity $\mathcal{H}$, is equal to the product $l\mathcal{H}v$. This result is expressed in abvolts and it is to be divided by 10^8 to reduce to volts, that is

$$E = \frac{l\mathcal{H}v}{10^8} \text{ volts} \tag{11b}$$

Expression of induced electromotive force in terms of number of lines of force cut per second.—During t seconds the sliding-wire AB, Fig. 5, moves over a distance vt and sweeps over lvt square centimeters of area. Multiplying this area by $\mathcal{H}$ gives the magnetic flux $\Phi(= lvt\mathcal{H})$ which passes through the area, and this flux Φ is the number of lines of force cut by AB during the t seconds, so that $\Phi/t(= l\mathcal{H}v = E)$ is the number of lines of force cut per second by AB. That is, *the electromotive force in abvolts induced in a wire is equal to the number of lines of force cut by the wire per second.* This proposition is here derived for a straight wire moving sidewise across a uniform magnetic field, but it is true for any wire, straight or curved, moving in any manner in any magnetic field, uniform or non-uniform.

Expression of induced electromotive force in terms of the rate of change of the magnetic flux enclosed by or passing through a circuit. — As the wire AB, Fig. 5, moves to the right, the enclosed area $ABCD$ increases, the magnetic flux through $ABCD$ increases, and the increase of this magnetic flux during a given time is equal to the number of lines of flux swept over by AB during that time; or in other words, the rate of increase of the magnetic flux through $ABCD$ is equal to the number of lines of force cut by AB per second. Therefore *the induced electromotive force in a circuit is equal to the rate at which the magnetic flux through the circuit is changing*, electromotive force being expressed in abvolts. This result is here derived for the very special case represented in Fig. 5, but it is true for any shape of circuit in any magnetic field uniform or non-uniform, and it is true whether the change of flux through the circuit be due to motion or to changing magnetism or to both.

Let the magnetic flux through a circuit be represented by Φ, then its rate of change is represented by $d\Phi/dt$, and we have

$$E = \frac{d\Phi}{dt} \qquad (12a)^*$$

in which E is the induced electromotive force in a circuit expressed in abvolts.

If the circuit has N turns of wire, then the electromotive force induced in each turn is $d\Phi/dt$, and the total induced electromotive force, expressed in abvolts, is:

$$E = N\frac{d\Phi}{dt} \qquad (12b)^*$$

In any given case an induced electromotive force may be

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* Equations (12a) and (12b) are more properly written with the negative sign, thus:

$$E = -\frac{d\Phi}{dt}, \text{ and } E = -N\frac{d\Phi}{dt},$$

inasmuch as an increasing flux produces an induced electromotive force which is in the direction in which a *left-handed screw* would have to be turned to cause the screw to travel in the direction of the magnetic field upon which the flux depends.

thought of as being produced by either (*a*) the cutting of lines of force at a definite rate by a wire, or (*b*) the change at a definite rate of the magnetic flux enclosed by a loop or coil of wire. These two actions are fundamentally identical, and the one or the other may be assumed as the basis of the discussion of any given case, according to convenience.

13. Eddy currents. Lamination. — When a piece of iron is magnetized or demagnetized, the changing flux through the central portions of the iron induces electromotive forces around the outer portions, and these electromotive forces produce what are called *eddy currents*. Eddy currents are also produced in a mass of metal which is near a moving magnet, or which moves in the neighborhood of a stationary magnet. Those parts of electrical machinery which are subject to rapid changes of magnetization are built up of thin sheets of iron or of iron wire so as to leave the iron *continuous* in the direction of the magnetization and *discontinuous* in the direction in which the eddy currents tend to flow. Such a mass of iron is said to be *laminated*. The iron parts of dynamo armatures and of transformers are always laminated.

Examples of eddy currents. — A solid iron rod soon becomes hot when it is placed in a coil of wire through which an alternating current flows, whereas a bundle of iron wires of the same size is but little heated when placed in such a coil; eddy currents are produced in the solid rod, whereas eddy currents are not produced in the bundle of iron wires.

A suspended magnet set swinging is quickly brought to rest if it is surrounded by a massive ring of copper, because the eddy currents induced in the copper by the moving magnet act upon the magnet with a force which is at each instant opposed to the motion.

A sheet of copper suddenly thrust between the poles of a strong magnet behaves as if it were moving in a viscous liquid. Eddy currents are induced in the copper; because of these currents the magnet exerts a force on the copper, and this force is always opposed to the motion.

INDUCTANCE.

14. The spark at break. — When an electric circuit is broken, the current continues to flow across the break for a short time, producing an electric arc or spark. This action of the electric current is suggestive of momentum, and, consequently, we shall speak of the *momentum of the electric current.* The intensity of the spark is a rough indication of the amount of this momentum.

The amount of momentum associated with a given current in a circuit made of a given length and size of wire depends upon the shape of the circuit and upon the presence of iron near the circuit. Thus, a current in a circuit *a*, Fig. 6, has but little momentum; the same current in circuit *b* has greater momentum, and the same current in circuit *c* has very much greater momentum.

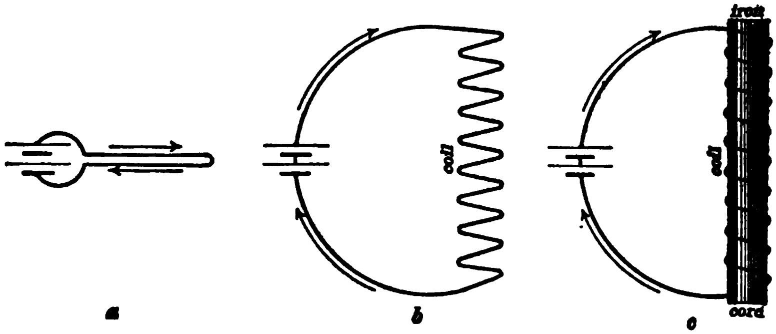

Fig. 6.

When the circuit of an ordinary incandescent lamp is broken, a very slight spark is produced; the same amount of current flowing through a coil of wire produces a much more intense spark when the circuit is broken; and a spark several inches in length may be produced if the coil of wire surrounds a core consisting of a bundle of iron wires.

The magnetic field which is produced by an electric current seems to be a state of motion of the ether, and the momentum of the electric current depends upon this magnetic field. Thus a current in the circuit *a*, Fig. 6, produces a very weak magnetic field, except in the small region between the wires, and the momentum of the current is small; the same current in circuit *b* produces an intense magnetic field inside of the coil, and the momentum of the current is correspondingly great; and the magnetism of the iron core in circuit *c* accounts for the very great momentum of the current in that circuit. In the following discussion the notion of *kinetic energy* is used in preference to the notion of *momentum* for the reason, mainly, that kinetic energy is expressed in mechanical units even when it is electrical kinetic energy.

15. The kinetic energy associated with a current in a circuit. Definition of inductance. — The ether motion which constitutes the magnetic field in the neighborhood of an electric circuit, represents kinetic energy, and the kinetic energy which is associated with an electric current is the total energy residing in the magnetic field produced by the current. We shall for convenience call this the kinetic energy of the current.

The amount of energy residing in a portion of a magnetic field is proportional to the square of the intensity of that portion of the field. This is analogous to the fact that the energy of a portion of a moving liquid is proportional to the square of the velocity of that portion. If the current in a circuit is doubled the field intensity is everywhere doubled, so that the energy of each portion of the field is quadrupled. Therefore the total energy of the field is quadrupled when the current is doubled, or in other words, the kinetic energy of a current in a given circuit is proportional to the square of the current, that is

$$W = \tfrac{1}{2}Li^2 \qquad (13)$$

in which W is the kinetic energy of a current, i, in the given circuit, and $(\frac{1}{2}L)$ is the proportionality factor. The quantity L is called the *inductance* of the given circuit.

Moment of inertia of a wheel. Analogue of inductance. — The kinetic energy of a rotating wheel resides in the various moving particles of the wheel. If the angular velocity ω of the wheel is doubled, the linear velocity of every particle of the wheel is doubled, so that the kinetic energy of every particle is quadrupled. Therefore, the total kinetic energy of a wheel is quadrupled when the angular velocity is doubled, so that the kinetic energy of a given wheel is proportional to the square of the angular velocity, that is

$$W = \tfrac{1}{2} K \omega^2$$

in which W is the kinetic energy of a rotating wheel, ω is the angular velocity of the wheel, and $(\frac{1}{2}K)$ is the proportionality factor. The quantity K is called the moment of inertia of the wheel.

Units of inductance. — If W in equation (13) is expressed in joules and i in amperes, then L is expressed in terms of a unit of inductance which is called a *henry*. That is to say, a circuit has an inductance of one henry when the kinetic energy of one ampere of current in the circuit is one half of a joule.

If W in equation (13) is expressed in ergs, and i in abamperes, then L is expressed in terms of the c.g.s. unit of inductance. This c.g.s. unit of inductance may be called the *abhenry*.

Inductance of a coil. — Strictly, one cannot speak of the inductance of anything but an entire circuit, inasmuch as every portion of a circuit contributes its share to the magnetic field at each and every point; it is, however, allowable to speak of the inductance of a coil when the terminals of the coil are near together, and when the remainder of the electric circuit does not produce any perceptible magnetic field in the region occupied by the coil.

Non-inductive circuits. — A circuit is said to be non-inductive under given conditions when the inductance of the circuit is negligible, that is, when the electromotive force $L \times di/dt$* is negligibly small, as compared with the electromotive force Ri which overcomes the resistance of the circuit. Thus, a given circuit might be considered to be non-inductive under conditions involving slow changes of current, whereas the same circuit could not be considered to be non-inductive under conditions involving very rapid changes of current. When a circuit consists of outgoing and returning wires side by side, its inductance is small and it may usually be considered to be non-inductive. The wires used in resistance boxes are usually arranged non-inductively. This may be done by doubling the wire back on itself, and winding the doubled wire on a spool. In this case the electromotive force between adjacent wires may be great, and the resistance coil may have considerable electrostatic capacity. In order to make a non-inductive resistance coil without this defect, the wire may be wound in one layer on a thin paper cylinder so as to bring the terminals as far apart as possible, and this cylindrical coil may then be flattened so as to reduce the region (inside) in which the magnetic field is intense.

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* See the next article.

Measurement of inductance. — The most accurate method for determining the inductance of a coil is by calculation from measured dimensions. This calculation can be carried out only when the coil is simple in shape, and even then the calculation is in most cases quite complicated. The simplest case is given in Art. 20. The inductance of an irregularly shaped coil may be determined by various electrical methods.*

16. Electromotive force required to cause a current to increase or decrease. — To keep a rotating wheel in uniform motion, a torque sufficient to balance the drag of friction must act upon the wheel. If the torque which acts upon the wheel is greater than the drag of friction, the wheel gains angular velocity, and if the torque which acts upon the wheel is less than the drag due to friction, the angular velocity of the wheel decreases.

To maintain a constant current in a circuit, an electromotive force equal to Ri must act upon the circuit to overcome the resistance of the circuit. If the electromotive force which acts upon the circuit is greater than Ri, the current increases in value, and if the electromotive force which acts upon the circuit is less than Ri, the current decreases in value. Let the electromotive force which acts upon a circuit exceed Ri by the amount e, then we have:

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* See *Absolute Measurements*, by A. Gray, Vol. 2, Part 2, pages 438 to 509.

See also a series of articles in the *Bulletin of the United States Bureau of Standards*, by E. B. Rosa, Vol. 1, page 125, and page 291; Vol. 2, page 87, page 161, and page 359; Vol. 3, page 1.

$$e = L\frac{di}{dt} \tag{14}$$

in which L is the inductance of the circuit, and di/dt is the rate at which current increases. When e is negative (electromotive force less than Ri) then di/dt is negative, that is, the current decreases.

Mechanical analogue of equation (14). — Whenever an unbalanced force e acts upon a body of which the mass is L, the body gains velocity at a rate di/dt such that

$$e = L\frac{di}{dt} \text{ (force equals mass times acceleration)}$$

Starting from the fact that force equals mass times acceleration, it can be shown that the kinetic energy of a moving body is equal to one half its mass times its velocity squared. The same argument reversed would show that force must be equal to mass times acceleration if kinetic energy is equal to one half mass times velocity squared; and an exactly similar argument would establish equation (14) on the basis of equation (13).

17. Self-induced electromotive force. Reaction of a changing current. — When one pushes on a wheel, causing its speed to increase, the wheel reacts and pushes back on the hand. This reacting torque is equal and opposite to the acting torque which is causing the increase of speed. When the speed of a wheel is increasing the reaction of the wheel is a torque opposed to its motion; when the speed of a wheel is decreasing the reaction is a torque in the direction of its motion. Similarly when an electromotive force acts upon a circuit and causes a current to increase or decrease, the changing current reacts; the reacting electromotive force is equal and opposite to the acting electromotive force, $L \times di/dt$, which is causing the current to change; when the current is increasing, the reaction of the current is an electromotive force opposed to the current; and when the current is decreasing, the reaction is an electromotive force in the direction of the current. The reaction of a changing current is called *self-induced electromotive force.*

18. Differential equations of growing and decaying currents. — A constant electromotive force $\mathcal{E}$ is at a given instant connected to a circuit of which the resistance is R and the inductance is L; after the current becomes fully established it is equal to $\mathcal{E}/R$ according to Ohm's law, but during the time that the current is being established a portion of $\mathcal{E}$ is used to overcome the resistance of the circuit and a portion of $\mathcal{E}$ is causing the current to increase. The portion used to overcome resistance is equal to Ri, and the portion used to cause the current to increase is equal to $L\,di/dt$, and therefore we have

$$\mathcal{E} = Ri + L\frac{di}{dt} \tag{15}$$

in which i is the value of the growing current at a given instant, and di/dt is its rate of increase.

When a circuit in which a given current is flowing is short-circuited and left to itself without any electromotive force acting to maintain the current, the current dies away; in this case equation (15) becomes

$I' = \frac{[illegible]}{R}$

$$0 = Ri + L\frac{di}{dt} \tag{16}$$

19. Definition of flux-turns. Flux-turns due to a current in a coil. — The induced electromotive force in a coil of wire is equal to $Z \times d\Phi/dt$, where Z is the number of turns of wire in the coil, and Φ is the flux through the coil (strictly, the flux through the mean turn of the coil). But Z times the rate of change of Φ is equal to the rate of change of the product $Z\Phi$. Therefore $Z\Phi$ is a quantity whose rate of change is equal to the electromotive force induced in a coil. This quantity $Z\Phi$ is called the *flux-turns.*

When a current flows through a coil the region surrounding the coil and inside of the coil becomes a magnetic field, and the lines of force of this magnetic field pass through the coil. That is, the current in the coil produces magnetic flux through the opening of the coil. Let Φ be the flux through the opening of a coil due to a given current i in the coil, and let Z be the number of turns of wire in the coil, and L the inductance of the coil (everything being expressed in c.g.s. units). Then

$$Z\Phi = Li \tag{17}$$

This is evident when we consider that the self-induced electromotive force in a coil is equal to $L \times di/dt$, according to Art. 17, and that $L \times di/dt$ is equal to the rate of change of Li; but the self-induced electromotive force is also equal to the rate of change of $Z\Phi$, so that $Z\Phi$ must be equal to Li.*

20. Calculation of inductance in terms of flux-turns per unit current. — According to equation (17) the inductance of a coil is equal to the quotient $Z\Phi/i$,

* In this argument algebraic signs are ignored, and Φ is assumed to be equal to zero when i is equal to zero.

where $Z\Phi$ is the flux-turns through the coil due to current i in the coil. Everything in this article is expressed in c.g.s. units except where the contrary is definitely stated. There are important cases in which the flux through the coil due to a given current may be easily calculated so that the inductance of such a coil may be easily determined.

Case I. Long solenoid. — Consider a coil of wire wound in a thin layer on a long cylindrical rod of non-magnetic material, wood, for example. Let r be the mean radius of the windings, l the length of the coil, and Z the number of turns of wire. The field intensity in the coil is $\mathcal{H} = 4\pi Zi/l$,* and the area of the opening of the coil is πr^2, so that the flux through the opening is $4\pi^2 r^2 Zi/l$, and for the flux turns, we have the value $4\pi^2 r^2 Z^2 i/l$, which, divided by i, gives the value of L as above explained. Therefore

$$L = \frac{4\pi^2 r^2 Z^2}{l} \tag{18}$$

in which the inductance L is expressed in abhenrys. To reduce to henrys, the right-hand member of equation (18) must be divided by 10^9.

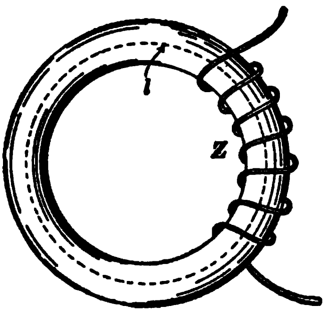

Fig. 7.

Case II. Coil wound on an iron core. — A coil of Z turns of wire is wound on an iron ring l centimeters in circumference (mean), and s square centimeters in sectional area, as shown in Fig. 7. The coil produces through the ring a magnetic flux which is equal to the magnetomotive force of the winding divided by the magnetic reluctance of the ring.† The magnetomotive force of the winding is $4\pi Zi$, and the magnetic reluctance of the ring is $l/\mu s$, i being a given value of current in the coil, and μ being the permeability of the iron. Therefore

$$\Phi = \frac{4\pi Zi}{\frac{l}{\mu s}}, \quad \text{or} \quad Z\Phi = \frac{4\pi\mu s Z^2 i}{l} = Li$$

so that

$$L = \frac{4\pi\mu s Z^2}{l} \tag{19}$$

It is to be particularly noted that the inductance of a coil of wire wound on an iron core varies with degree of saturation of the iron, inasmuch as the permeability of the iron is variable.

Case III. — The inductance of the two wires of a transmission line is given by the equation

$$L = 0.0017 \times \log_{10}\frac{d}{R} \times \text{length} \tag{20}$$

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* A proof of this equation may be found in Franklin and MacNutt's *Elements of Electricity and Magnetism*, page 102.

† See Appendix A of the first volume of Franklin and Esty's *Elements of Electrical Engineering*.

in which L is the inductance of the line in henrys, l is the length of the line (actual length of row of poles) in miles, d is the distance apart of the two wires center to center, and R is the radius of the wires, d and R being both expressed in the same units.

CAPACITY.

21. Electric charge. — A current of water through a pipe is a transfer of water along the pipe, and if q be the amount of water which during t seconds flows past a given point in the pipe, the quotient q/t is the rate of flow of water through the pipe, and this rate of flow may be spoken of as the *strength*, i, of the water current. Suppose the strength i of the water current to be given (rate of flow of water) then the amount of water flowing past a given point of the pipe in t seconds is given by the equation

$$q = it$$

Similarly, an electric current in a wire may be looked upon as a transfer of electricity along the wire, and the quantity q of electricity which flows past a point on the wire during t seconds may be defined as the product of the strength of the current and the time. That is

$$q = it \tag{21}$$

If the strength of the electric current is variable, equation (21) must be written in the form

$$\Delta q = i \cdot \Delta t \tag{22}$$

in which Δq is the small quantity of electricity which flows past a given point on the wire during the short interval of time Δt.

Quantity of electricity is usually spoken of as *electric charge* or simply as *charge*.

Quantity of water is the fundamental and easily measured thing in hydraulics, and *water current* is most conveniently defined as quantity of water per second. On the other hand, in the case of electricity, the fundamental and easily measured thing is electric current and *quantity of electricity* is most conveniently defined as the product of electric current and time.

Units of electric charge. — The quantity of electricity transferred in one second by one ampere is called a *coulomb;* q in equation (21) is expressed in coulombs when i is expressed in amperes and t in seconds.

22. Electrically charged bodies. — Consider two metal bodies, A and B, Fig. 8, which at a given instant are connected, as shown, to the terminals of a battery, or to any source of electromotive force. When the wire is connected a momentary pulse of current flows through the wire out of one body and into the other, and the bodies A and B are said to become *charged with electricity.* The body into which the momentary current flows is said to become *positively charged,* and the body out of which the momentary current flows is said to become *negatively charged,* that is the charge on one body is $+ q$ and the charge on the other body is $- q$. Electrically charged bodies always occur thus in pairs, the positive charge on one body being always associated with an equal negative charge on some other body or bodies.

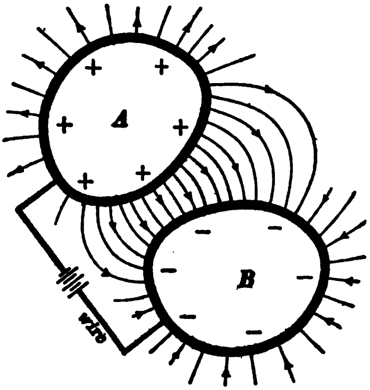

Fig. 8.

The electric field. The dielectric. — The region between the two bodies A and B, Fig. 8, is of course filled with some electrical insulator such as air, or oil, or glass. An insulator between two charged bodies is called a *dielectric.* This dielectric is the seat of a peculiar stress called the *electric field* which is similar in many respects to the magnetic field. The lines of force of this electric field trend somewhat as shown in the figure, touching the surfaces of A and B at right angles. These lines of force are thought of as going out from the positively charged body and coming in towards the negatively charged body.

Electrostatic attraction. — The charged bodies A and B, Fig. 8, attract each other. This attraction, which is called electrostatic attraction, shows that the lines of force of an electric field are in a state of tension and have a tendency to shorten. This tension of the lines of force pulls outwards on the surface of A and of B at each point. This outward pull on the surface of a charged body is very strikingly shown by pouring a viscid liquid over the sharp lip of a charged metal ladle. The liquid is pulled into fine jets by the lines of force which emanate from the liquid as it passes over the lip. When melted rosin is used in this way, the jets congeal into very fine fibers which float about in the air.

If a small charged body such as a pith ball be suspended anywhere in the region between A and B, it will be found to be acted upon by a force in the direction of the electric field, that is in the direction of the lines of force at the point where the pith ball is placed.

Need of large electromotive forces and good insulation. — The phenomena described above and, in fact, most of the phenomena of electrostatics are easily perceptible only when the bodies are charged by electromotive forces of many thousands of volts. The most convenient means for producing these large electromotive forces is the Holtz or Wimshurst electrical machine. When such a machine is used, however, the bodies A and B must be well insulated, because such electrical machines cannot supply charge at a rapid rate, that is, such a machine can deliver only a very small current.

Dielectric strength. — When the electromotive force E acting to charge two bodies A and B, Fig. 8, is increased, a value is eventually reached which breaks down, or ruptures, the dielectric and allows the charge on the bodies to pass in the form of an electric spark. When the bodies A and B are flat, as shown in Fig. 9, the electromotive force required to

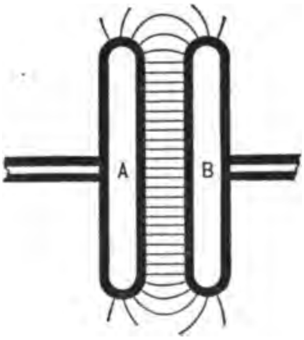

Fig. 9.

break down the intervening dielectric, is for a given dielectric, proportional to the distance x between the plates, and the quotient E/x is called the *electric strength* of the dielectric. The following table gives the strengths of various dielectrics.

TABLE.

DIELECTRIC STRENGTHS.

Substance.	Strengths in Volts per Centimeter.	Substance.	Strengths in Volts per Centimeter.
Oil of turpentine	94,000	Paraffined paper	360,000
Paraffine oil	87,000	Beeswaxed paper	540,000
Olive oil	82,000	Air (thickness 5 cm.)	23,800
Paraffine (melted)	56,000	CO_2 " "	22,700
Kerosene oil	50,000	O " "	22,200
Paraffine (solid)	130,000	H " "	15,100

23. The condenser.* Electrostatic capacity. — The amount of charge q which flows out of B into A, Fig. 8, when the battery is connected, is proportional to the electromotive force of the battery. Therefore we may write

$$q = CE \tag{23}$$

in which q is the charge that is drawn out of B and forced into A in Fig. 8 by a battery of which the electromotive force is E, and C is a constant depending upon the size and shape of A and B, and upon the intervening dielectric. This quantity C is called the *electrostatic capacity* or simply the *capacity* of the pair of bodies A and B. If the bodies A and B are in the form of metal plates separated by a thin layer of dielectric, their electrostatic capacity is large. Such an arrangement is called a *condenser*. Condensers of large capacity are made by sheets of tin foil separated by sheets of waxed paper or mica. The *Leyden jar* is a condenser made by coating the inside and outside of a glass jar with tin foil.

Units of capacity. — A condenser is said to have a capacity of one *farad* when one coulomb of charge is drawn out of one plate and forced into the other plate by an electromotive force of one volt; C in equation (23) is expressed in farads, when q is expressed in coulombs and E in volts. The farad is an enormously large capacity as compared with capacities ordinarily met with in practice, and the *microfarad* (one-millionth of a farad) is frequently used as a unit, for convenience.

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* See Art. 40 for a description of the Hydraulic Analogue of the Condenser.

24. Inductivity of dielectric. — The capacity of a condenser with plates of given size and at a given distance apart depends upon the dielectric. The quotient: *capacity of condenser with given dielectric divided by the capacity of the same condenser with air between its plates* is called the *inductivity* of the dielectric. For example, the inductivity of petroleum is about 2.04, that is, the capacity of a given condenser is about 2.04 times as great when the dielectric is petroleum as it is when the dielectric is air. A condenser is called an *air condenser*, a *mica condenser*, or a *paraffin condenser*, according to the dielectric used between the plates. The following table gives the inductivities of various dielectrics:

TABLE.

INDUCTIVITIES OF VARIOUS SUBSTANCES.

Glass	3 to 10	Shellac	2.95 to 3.60
Sulphur	2.24 to 3.84	Mica	4 to 8
Vulcanite	2.50	Quartz	4.5
Paraffin	1.68 to 2.30	Turpentine	2.15 to 2.43
Rosin	1.77	Petroleum	2.04 to 2.42
Wax	1.86	Water	73 to 90

25. Dependence of the capacity of a condenser upon the size and distance apart of its plates. — The capacity C of an air condenser is proportional to the area a of its plates, and inversely proportional to the distance x between its plates, that is C is proportional to a/x, or the capacity of a condenser with a given dielectric is proportional to ka/x, where k is the inductivity of the dielectric. Indeed the capacity of a condenser may be calculated by the equation

$$C = 884 \times 10^{-16} \times \frac{ka}{x} \tag{24}$$

in which k is the inductivity of the dielectric, x is the thickness of the dielectric in centimeters, and a is the area in square centimeters of the dielectric layer between the plates. This equation gives the value of C in farads.

The capacity of the two wires of a transmission line, the two wires being considered as the two "plates" of a condenser, is given by the formula

$$C = \frac{1.943 \times l}{10^8 \times \log_{10}\frac{d}{R}} \qquad (25)$$

in which C is expressed in farads, and l, d and R have the same significance as in equation (20), that is l is the length of the line (of poles) in miles.

26. Mechanical and electrical analogies. — The analogy between moment of inertia and inductance as pointed out in the discussion of inductance is but a small part of an extended analogy between pure mechanics and electricity. This extended analogy is here briefly outlined.

$x = vt \qquad (1)$ in which x is the distance traveled in t seconds by a body moving at velocity v.	$\phi = \omega t \qquad (2)$ in which ϕ is the angle turned in t seconds by a body turning at angular velocity ω.	$q = it \qquad (3)$ in which q is the electric charge which in t seconds flows through a circuit carrying a current i.
$W = Fx \qquad (4)$ in which W is the work done by a force F in pulling a body through the distance x.	$W = T\phi \qquad (5)$ in which W is the work done by a torque T in turning a body through the angle ϕ.	$W = Eq \qquad (6)$ in which W is the work done by an electromotive force E in pushing a charge q through a circuit.
$P = Fv \qquad (7)$ in which P is the power developed by a force F acting upon a body moving at velocity v.	$P = T\omega \qquad (8)$ in which P is the power developed by a torque T acting on a body turning at angular velocity ω.	$P = Ei \qquad (9)$ in which P is the power developed by an electromotive force E in pushing a current i through a circuit.
$W = \frac{1}{2}mv^2 \qquad (10)$ in which W is the kinetic energy of a mass m moving at velocity v.	$W = \frac{1}{2}K\omega^2 \qquad (11)$ in which W is the kinetic energy of a wheel of moment of inertia K turning at angular velocity ω.	$W = \frac{1}{2}Li^2 \qquad (12)$ in which W is the kinetic energy of a coil of inductance L carrying a current i.
$F = m\frac{dv}{dt} \qquad (13)$ in which F is the force required to cause the velocity of a body of mass m to increase at the rate $\frac{dv}{dt}$.	$T = K\frac{d\omega}{dt} \qquad (14)$ in which T is the torque required to cause the angular velocity of a wheel of moment of inertia K to increase at the rate $\frac{d\omega}{dt}$.	$E = L\frac{di}{dt} \qquad (15)$ in which E is the electromotive force required to cause a current in a coil of inductance L to increase at the rate $\frac{di}{dt}$.
$x = aF \qquad (16)$	$\phi = bT \qquad (17)$	$q = CE \qquad (18)$
$\frac{4\pi^2 m}{\tau^2} = \frac{1}{a} \qquad (19)$	$\frac{4\pi^2 K}{\tau^2} = \frac{1}{b} \qquad (20)$	$\frac{4\pi^2 L}{\tau^2} = \frac{1}{C} \qquad (21)$

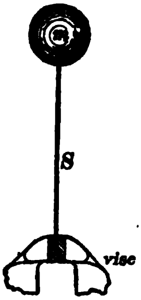

Fig. a.

A body of mass m is supported by a flat spring S, clamped in a vise as shown in Fig. *a*. A force F pushing sidewise on m moves it a distance x, which is proportional to F, according to equation (16). When started the body m will continue to vibrate back and forth and the period τ of its vibrations is determined by equation (19).

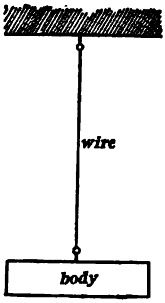

Fig. b.

A body of moment of inertia K is hung by a wire as shown in Fig. *b*. A torque T acting on the body will turn the body and twist the wire through an angle ϕ, which is proportional to T, according to equation (17). When started, the body will vibrate about the wire as an axis and the period τ of its vibrations is determined by equation (20).

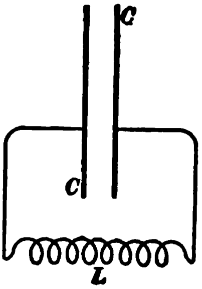

Fig. c.

A condenser C is connected to the terminals of a coil of inductance L as shown in Fig. *c*. An electromotive force E acting anywhere in the circuit pushes into the condenser a charge q, which is proportional to E, according to equation (18). When started the electric charge will surge back and forth through the coil, constituting what is called an oscillatory current and the period of one oscillation is determined by equation (21).

ELECTROMAGNETS.

27. The electromagnet * consists of an iron rod wound with insulated wire through which an electric current flows. The iron rod is called the *core*, the wire is called the *winding*, and the current is called the *exciting current*. The core of an electromagnet usually forms a complete, or nearly complete, iron circuit and a certain amount of magnetic flux is produced in this circuit, by the magnetizing action of the exciting current.

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* The theory of the electromagnet is discussed at length in Appendix A of Volume I of Franklin and Esty's *Elements of Electrical Engineering*. See also Ewing's *Magnetism in Iron and other Metals*, and DuBois' *The Magnetic Circuit in Theory and Practice*, translated by Atkinson.

A magnetic circuit is always complete, but the path of the magnetic flux may not be wholly in iron. Thus, the magnetic circuit of the dynamo contains the air gaps between the field poles and the iron core of the armature as shown in Fig. 11. The magnetic circuit of the transformer, on the other hand, is wholly of iron as shown in Fig. 12.

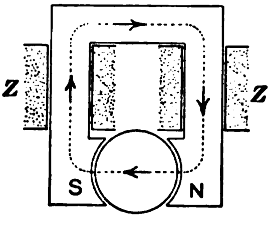

Fig. 11.

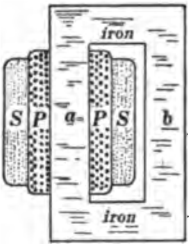

Fig. 12.

The magnetizing action of the exciting current of an electromagnet is called *magnetomotive force*, and it is proportional to the product of the current and the number of turns of wire in the winding. The unit of magnetomotive force which is used in this text is the *ampere-turn*, thus, the magnetomotive force of a

TABLE OF UNITS.*

ELECTRIC.

Name of Quantity.	Symbol.	Name of Practical Unit.	Name of c.g.s. Unit.	Practical Unit ÷ c.g.s. Unit.
Current	I, i	ampere.	(abampere.)	10^{-1}
Resistance.	R, r	ohm.	(abohm.)	10^{9}
Resistivity.	ρ	ohms of a centimeter cube.	abohms of a centimeter cube.	10^{9}
Conductance	$1/R$	(mho.)	——	10^{-9}
Conductivity	$1/\rho$	——	——	10^{-9}
Electromotive force	E, e	volt.	(abvolt.)	10^{8}
Inductance	L	henry	(abhenry or centimeter.)	10^{9}
Capacity	C	farad.	(abfarad)	10^{-9}
Inductivity	k	——	——	——
Dielectric strength		volts per centimeter.	abvolts per centimeter.	10^{8}

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* Names in parentheses have not been adopted internationally.

MAGNETIC.

Pole strength	m	—	—	—
Field intensity	$\mathcal{H}$	—	gauss.	—
Flux	Φ, ϕ	—	maxwell.	—
Flux density	$\mathcal{B}$	—	gauss.	—
Intensity of magnetization	$\mathcal{I}$	—	—	—
Magnetomotive force	$\mathcal{F}$	ampere-turn.	(gilbert.)	$4\pi \mid 10$
Reluctance	$\mathcal{R}$	—	(oersted)	—
Reluctivity	$1/\mu$	—	—	—
Permeance	$1/\mathcal{R}$	—	—	—
Permeability	μ	—	—	—

current of 2 amperes flowing through 3,000 turns of wire is 2 amperes × 3,000 turns of wire which is equal to 6,000 ampere-turns

CHAPTER II.

THE DYNAMO AND THE TRANSFORMER.

28. The dynamo.—The *dynamo electric machine*, or, simply, the *dynamo* is a machine for the production and maintenance of an electric current when the machine is supplied with mechanical power, or, conversely, for the development of mechanical power when the machine is supplied with electric current. When used for the former purpose the dynamo is called an *electric generator*, and when used for the latter purpose the dynamo is called an *electric motor*.

There are two distinct types of dynamo electric machines, namely (*a*) alternating-current machines, and (*b*) direct-current machines. The alternating-current generator delivers what is called an alternating current, that is, a current which is subject to rapid periodic reversals of direction. The direct-current generator, on the other hand, delivers a current which is not reversed in direction and which is more or less steady in value. All direct-current machines, except the so-called homopolar* dynamos, are essentially alternating-current machines with the addition of a commutator instead of collecting rings.

The action of the dynamo as a generator † is essentially as follows: A wire, see Fig. 5, is forced by an external source of mechanical power to move sidewise across a magnetic field. This motion induces an electromotive force in the wire. This electromotive force produces a current in the wire if the wire is a portion of a closed circuit. Because of this current the wire is pushed sidewise by the magnetic field with a force F which *opposes* the motion of the wire. Work is done by the external agent in moving the wire against the opposing force F, and the work so done goes to maintain the current in the wire.

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* The homopolar dynamo is described on page 71, Vol. I, Franklin and Esty's *Elements of Electrical Engineering*. See paper by H. E. Heath in the *Electrical World and Engineer*, Vol. XXXV, page 210, February 10, 1900. See also a paper by J. E. Noeggerath, *Transactions, American Institute of Electrical Engineers*, Vol. XXIV, pages 1-27, January 27, 1905.

† This statement and the following statement of the action of the dynamo as a motor constitute a complete statement of Lenz's Law. (See Art. 11.)

The action of the dynamo as a motor is essentially as follows: A current is forced (by an external generator) through a wire which is stretched across a magnetic field. This current causes the field to push sidewise on the wire. This force moves the wire sidewise and develops mechanical power. The motion of the wire induces in it an electromotive force E which *opposes* the flow of current in the wire. Work is done by the external generator in forcing the current against the electromotive force E, and the work so done goes to maintain the motion of the wire.

29. The alternating-current dynamo is the simplest form of dynamo electric machine. The essential features of the alternating-current dynamo are shown in Fig. 13. A wire W (perpendicular to the plane of the figure) is moved sidewise along the dotted circle so as to cut the magnetic lines of force which emanate from the inwardly projecting poles of a field magnet. While the wire is sweeping across a north pole an electromotive force is induced in it in one direction, and while the wire is sweeping

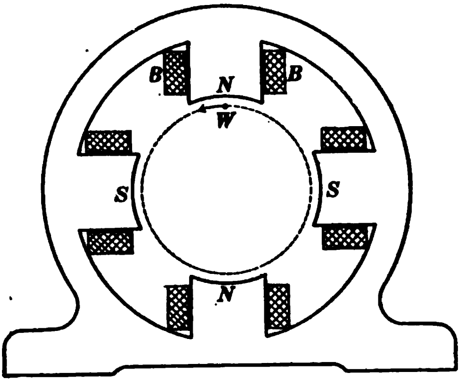

Fig. 13.

across a south pole an electromotive force is induced in it in the opposite direction. This repeatedly reversed electromotive force is called an *alternating electromotive force*, and it produces *alternating current* in the wire and in the outside circuit to which the ends of the wire are connected.

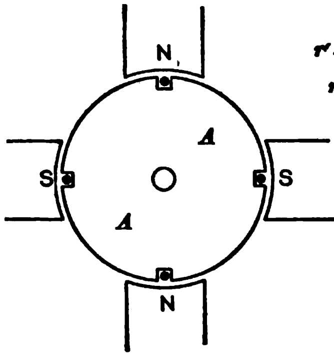

Fig. 14a.

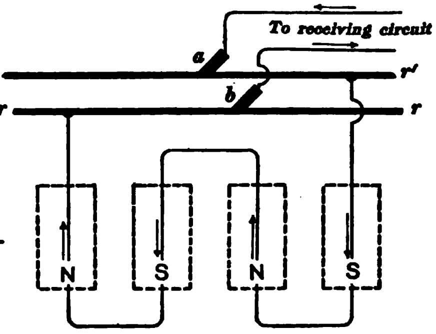

Fig. 14b.

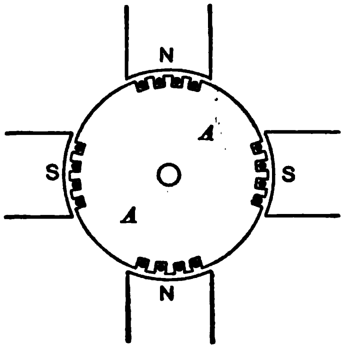

Fig. 15a.

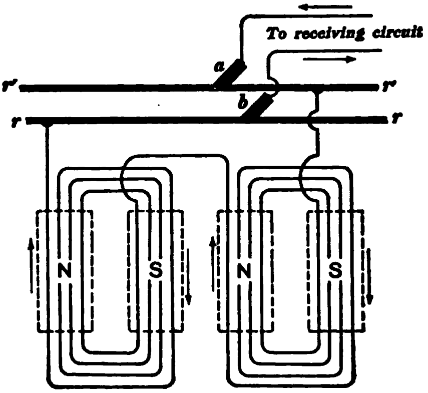

Fig. 15b.

In commercial alternators a large number of wires is used instead of the single wire *W* shown in Fig. 13, and these wires are placed in slots in the periphery of a rotating mass of laminated iron *AA*, Figs. 14 and 15. Thus, Fig. 14 shows four wires placed in four slots and Fig. 15 shows sixteen wires placed in sixteen slots. Figures 14*b* and 15*b* are what are called developed diagrams which show how the wires are connected together and how they are connected to two insulated metal rings *r* and *r'* upon which two brushes *a* and *b* rub and keep the moving wires connected to the outside receiving circuit.

The insulated metal rings *rr'* are mounted at one end of the armature and are called *collector rings*. The laminated structure *AA* with its winding of wire is called the *armature* of the alternator. The field magnet of an alternator is generally an electromagnet which is excited by direct current from a small auxiliary direct-current generator called the *exciter*. The field windings

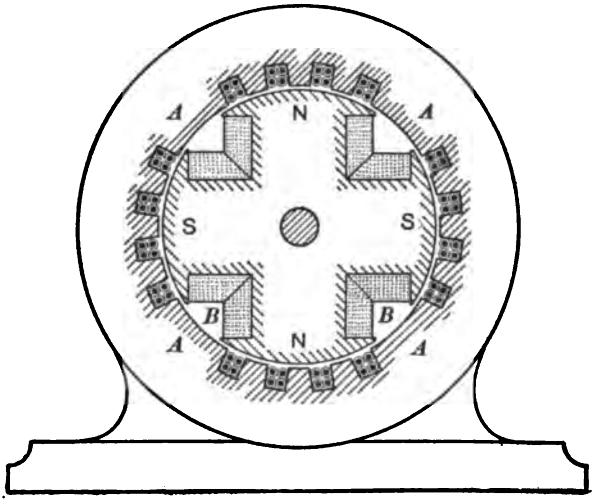

Fig. 16.

BB, Fig. 13, of an alternator are arranged so as to cause alternate poles of the field magnet to be north and south as indicated in the figure.

It is the relative motion only of armature and field magnet which is essential in any dynamo, and alternators are frequently made with a revolving field magnet the outwardly projecting poles of which, *NSNS*, Fig. 16, sweep past the stationary armature wires which are placed in slots around the inner periphery of the ring-shaped structure of laminated iron *AA*. Figure 16 shows sixteen armature slots in each of which four wires are placed.

In the revolving-field type of alternator the direct current for exciting the revolving field magnet is delivered to the revolving structure by brushes which rub on two insulated metal rings, and the terminals of the stationary armature winding are connected directly to the receiving circuit.

The revolving-armature type of alternator is generally used in small electric lighting stations. Figure 17 shows an alternator of this type manufactured by the Fort Wayne Electric Works. The figure shows the small belt-driven exciter, and it shows, although somewhat indistinctly, the rectifying commutator through which a series field winding is connected in the main circuit of the machine as explained in Art. 37.

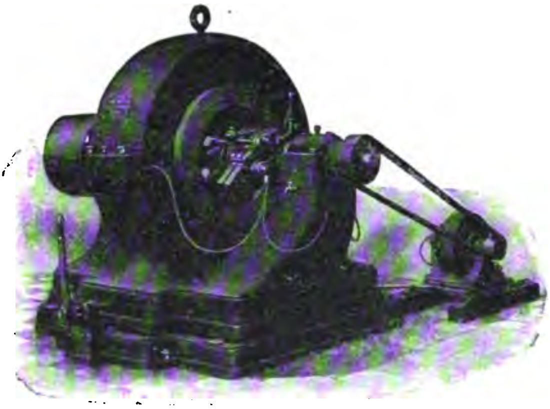

Fig. 17.

Large alternators are usually of the revolving-field type. A small belt-driven machine of this type, built by the Allis-Chalmers Company is shown in Fig. 18. In this figure the projecting poles of the rotating field magnet are clearly visible, and the collector rings which serve to deliver direct current to the field windings are shown near the right-hand pedestal. The figure shows a small direct-current exciter mounted on the end of the main shaft of the alternator.

*The inductor alternator** is an alternator in which a stationary field winding produces magnetic flux which passes through a rotating mass of iron called the inductor, this inductor being so arranged as to carry tufts of lines of force across the windings of a stationary armature structure. The inductor alternator contains no moving wires, field winding and armature winding both being stationary.

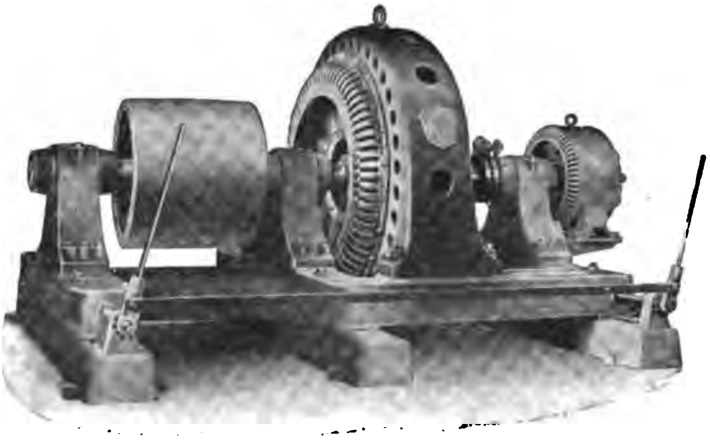

Fig. 18.

Concentrated and distributed windings.—That type of armature winding in which all of the wires on the armature are grouped in p slots, where p is the number of field magnet poles, is called a concentrated armature winding. The slots for concentrated armature windings are usually rather large and each slot contains a large number of wires. Figure 14, however, shows a concentrated armature winding with but one wire per slot.

In the armature windings shown in Figs. 15 and 16, the wires are spread out in p similar *groups of slots*, where p is the number of field magnet poles. This type of armature winding is called a distributed winding.

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*The most familiar example of the inductor alternator is the Stanley-Kelly-Chesney (S.K.C.) alternator of the Stanley Company. This alternator is described on page 7, Franklin and Esty's *Elements of Electrical Engineering*, Vol. II.

In the concentrated armature winding the electromotive force rises suddenly in value as the armature slots pass under the pole pieces, and falls suddenly in value as the armature slots pass out from under the pole pieces; whereas in the distributed armature winding, the electromotive force rises and falls more gradually.

Single-phase and polyphase alternators.—The details shown in Figs. 13, 14, 15, and 16 refer to what is called a single-phase alternator. A polyphase alternator is an alternator upon whose armature two or more entirely distinct windings are placed, each winding being arranged to deliver current to a separate receiving circuit. The polyphase alternator and the polyphase system of distribution are discussed in Chapter X.

Speed and frequency.—The electromotive force of an alternator passes through a set of positive values while a group of armature wires is passing a north pole of the field magnet, and through a set of negative values while the given group of armature wires is passing a south pole of the field magnet. The complete set of values, including positive and negative values, is called a *cycle*, the duration of a cycle is called a *period*, and the number of cycles per second is called the *frequency*. If the field magnet of an alternator has p poles ($p/2$ north poles and $p/2$ south poles), then

$$f = \frac{pn}{2} \tag{26}$$

in which f is the frequency in cycles per second of the electromotive force which is developed by the alternator, and n is the speed of the alternator in revolutions per second. This is evident when we consider that a complete cycle corresponds to the passage of a given group of armature wires across two field poles, a north pole and a south pole, so that there are $p/2$ cycles in one revolution.

When north poles and south poles are similar in shape and size, with similar air gaps, which is usually the case in commercial alternators, the successive sets of positive and negative values of the electromotive force are alike. That is, a cycle is made up of a set of positive values of electromotive force and a similar set of negative values of electromotive force.

Synchronism.—Two alternating electromotive forces or currents are said to be in synchronism when they have the same frequency. Two alternators are said to run in synchronism when their electromotive forces are in synchronism.*

30. The direct-current dynamo.—The following description applies to the direct-current dynamo having an armature of the ring-wound type. An iron ring *ab*, Fig. 19, which is built up of sheet iron stampings, is wound uniformly with insulated wire as indicated in the figure, the ends of the wire being spliced together and soldered so that the winding is endless. This iron ring with its winding of wire is called the *armature*, and it rotates between the poles *N* and *S* of a strong *field magnet* as indicated by the curved arrow. The wires on the outside of the iron ring have electromotive forces induced in them as they move across the pole faces of the field magnet and cut the lines of force. These electromotive forces cannot, however, produce current in the endless wire that is wound on the armature, because exactly equal and opposite electromotive forces are induced on the opposite sides

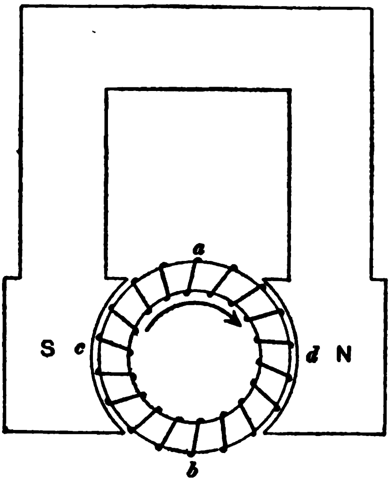

Fig. 19.

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* The term "to synchronize," as used by power-station men, has a double meaning. Thus, to synchronize an alternator preparatory to connecting it in parallel with another alternator, means not only to bring it into synchronism with the other alternator, but also to bring about a certain phase relation between the two. See Chapter XII.

c and *d* of the ring as shown diagrammatically in Fig. 20, in which figure the circle *adbc* represents the endless wire winding on the ring.

A steady, or very nearly steady, current can be taken from the winding on the ring through an outside receiving circuit *l*, Fig. 21, by keeping the terminals of this circuit in metallic contact with the windings on the ring at *a* and *b* as shown in Fig. 21. For this purpose the insulation may be removed from the outer portions of the wire windings on the ring in order that two stationary metal or carbon brushes *SS*, Fig. 21, may make sliding contact with the windings at *a* and *b* as the ring rotates. In practice short lengths of wire are attached to the various turns of wire on the ring and led to insulated copper bars near the axis of rotation as shown in Fig. 22. Sliding contact is then made with these copper bars instead of being made with the turns of wire at *a* and *b* directly. This set of copper bars constitutes what is called the *commutator*.

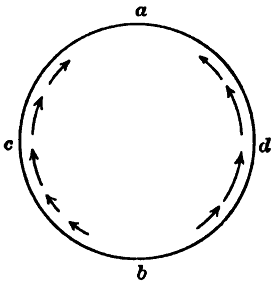

Fig. 20.

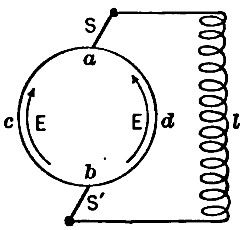

Fig. 21.

From Fig. 21, it is evident that one half of the total current which is delivered to the receiving circuit flows through the windings on one side of the ring, and one half through the windings on the other side of the ring. That is, the windings on the ring present two paths for the flow of current from brush to brush, and the current in each path is one half of the current which is delivered to the receiving circuit. The resistance of the armature winding between brushes is therefore one half of the resistance of one path, since the two paths are similar and in parallel. This statement refers to the two-pole machine.

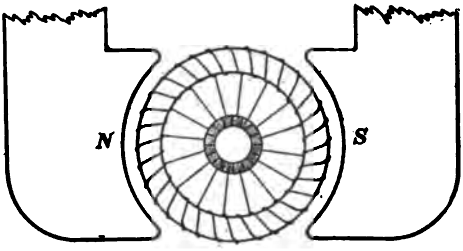

Fig. 22.

The multipolar direct-current dynamo. — Figure 23 shows a ring-wound armature rotating inside of a crown of six inwardly projecting field magnet poles. The electromotive forces which are induced in the windings as they sweep across the pole faces cannot produce current in the endless wire that is wound on the ring, because the electromotive forces induced under the north poles are balanced by the electromotive forces induced under the south poles, as shown diagrammatically in Fig. 24.

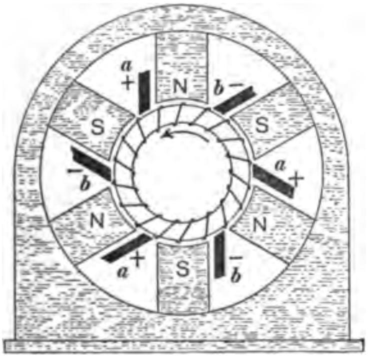

Fig. 23.

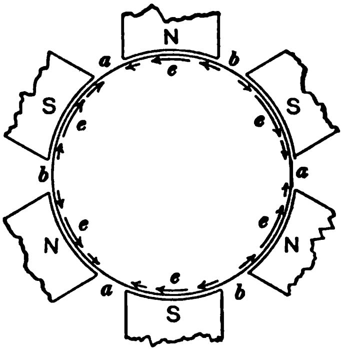

Fig. 24.

In order to utilize the induced electromotive forces *eeeeee*, Fig. 24, for the production of direct current, six brushes *aaa* and *bbb*, Fig. 23, should be used. Three of these brushes maintain contact with the windings at *aaa*, Fig. 24, and through all three of these brushes current flows out of the armature to one terminal of the receiving circuit. The other three brushes *bbb* maintain contact with the windings at *bbb*, Fig. 24; all three of these brushes are connected to the other terminal of the receiving circuit and current flows into the armature through all three. The three brushes *aaa* together constitute the positive terminal of the armature, and the three brushes *bbb* together constitute the negative terminal of the armature.

The armature winding in Fig. 23 presents six paths from the negative to the positive brushes, the current in each path is one sixth of the total current delivered by the machine, and the resistance of the armature winding between brushes is one sixth of the resistance of one path.

*Ring armatures and drum armatures.** — The armature shown in Figs. 19 to 23 is called a ring-wound armature. In this type of armature each wire on the external surface of the armature may *be* a commutator bar or may be connected to a commutator bar, and such an armature is suited to a field magnet having any even number of field poles whatever.

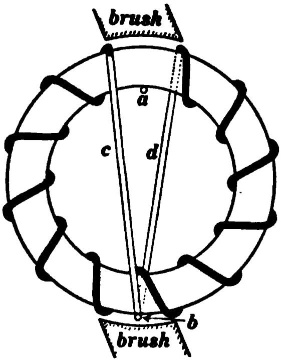

Fig. 25.

The wire on a ring-wound armature passes from one end of the armature to the other on the outside and returns through the interior. In the drum-wound armature, however, the wire passes from one end of the armature to the other on the outside, crosses over at the other end, and returns on the outside. The relationship between the ring-wound armature and the drum-wound armature may be understood with the help of Fig. 25. Each return wire *a* on the interior of the ring-wound armature may be placed at *b* as shown.† In this case it is evident that the conductor *b*, Fig. 25, must not make contact with the lower brush, because if it did, the wires *c* and *d* would be short-circuit connections from brush to brush. Every alternate wire on the outside of a drum-wound armature may *be* a commutator bar or may be connected to a commutator bar. A drum-wound armature which is adaped to a field magnet having a given number of poles is not in general adapted to a field magnet having a different number of poles.

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* A full discussion of armature windings is given in Appendix C of Volume I, Franklin and Esty's *Elements of Electrical Engineering*.

† This statement applies to the two-pole machine.

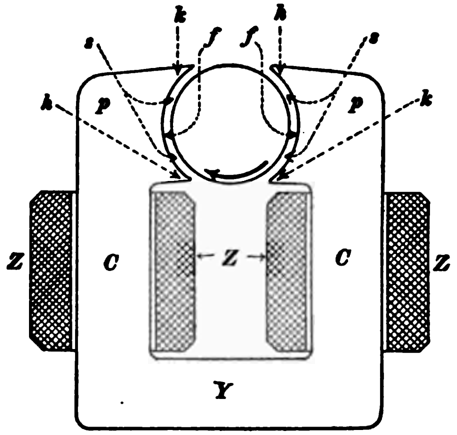

Fig 26.

31. Examples of direct-current dynamos. — Figure 26 shows in outline a typical bi-polar dynamo, and Fig. 27 shows in outline a typical multipolar dynamo. The regions *ss* are called the *air gaps*, *ff* the *pole faces*, *pp* the *pole pieces*, and *hhkk* are called the *pole tips* or *horns*; *hh* are called the *leading* pole tips and *kk* the *trailing* pole tips with reference to the motion of the armature as indicated by the curved arrow. The coils of wire *ZZ* are called the *field coils*, the portions *CC* of the field magnet iron around which the field coils are wound are called the *field magnet cores*, and the portion *Y* of the field magnet iron is called

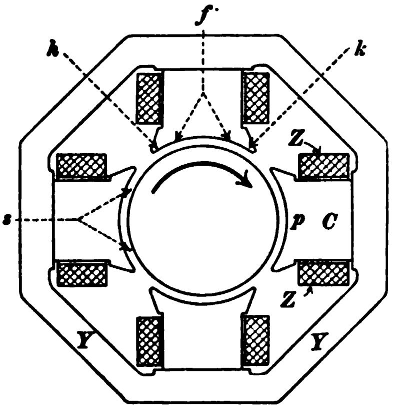

Fig. 27.

the *yoke.* Figure 28 is a general view of an early type* of bipolar direct-current dynamo (motor) manufactured by the Crocker-Wheeler Company. This dynamo has a ring-wound armature, the details of which are shown in Fig. 29. Figure 30 is a general view of a multipolar direct-current dynamo manufactured by the Fort Wayne Electric Works. The field yoke is cast in two parts

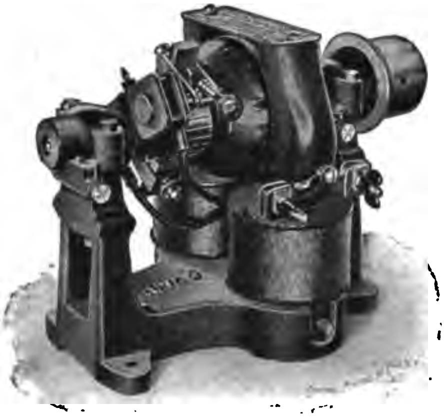

Fig. 28.

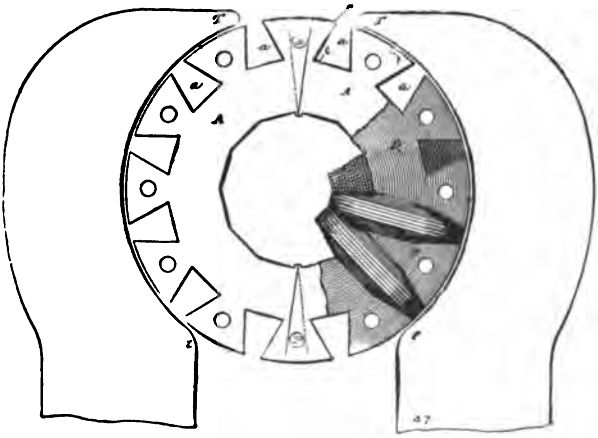

Fig. 29.

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* Motors are now usually enclosed and a general view of such a motor would show nothing but an iron box.

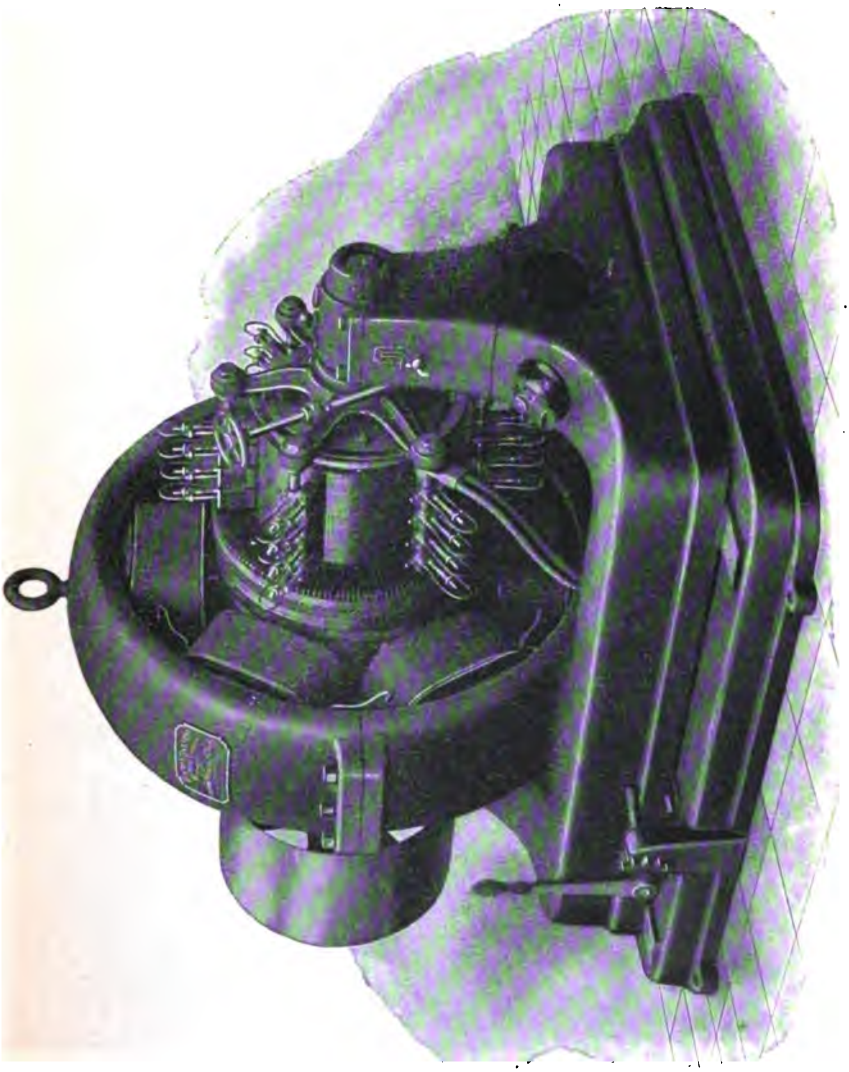

Fig. 30.

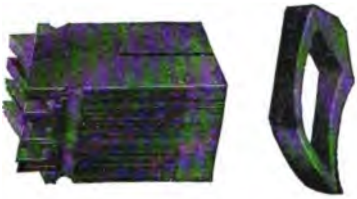

Fig. 32.

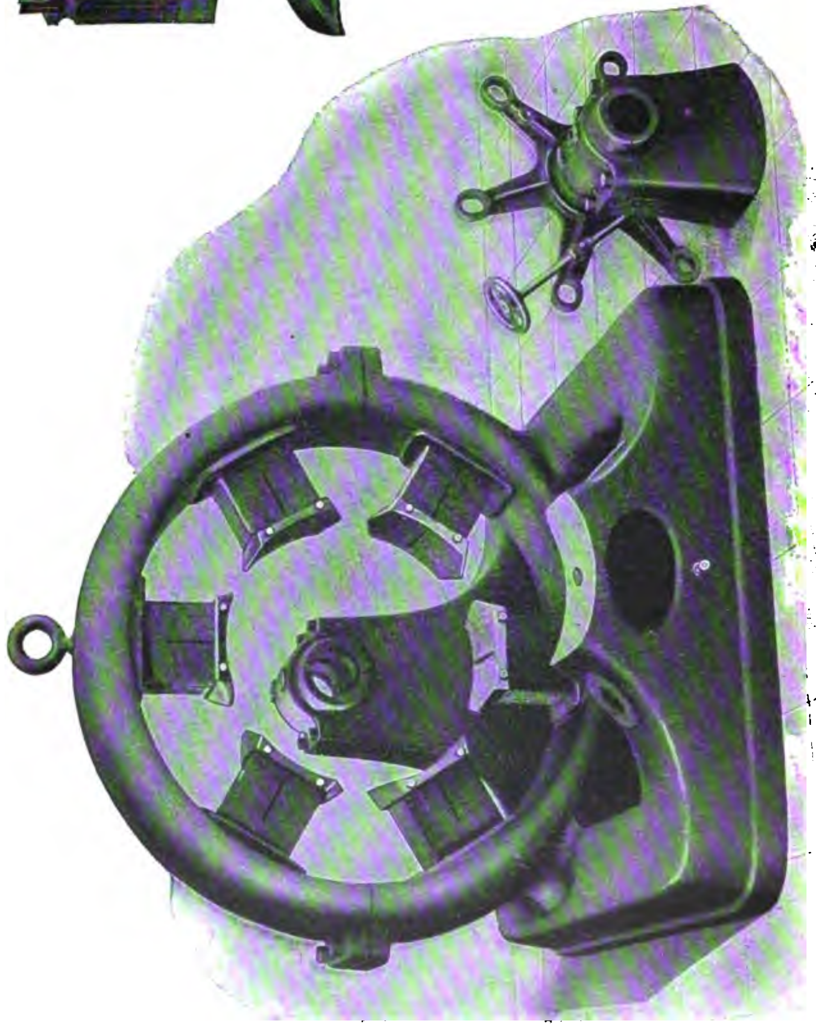

Fig. 31.

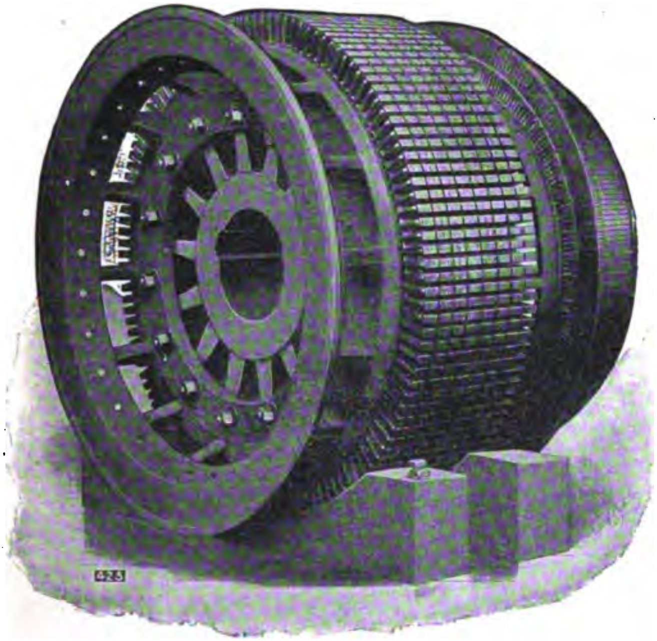

Fig. 33.

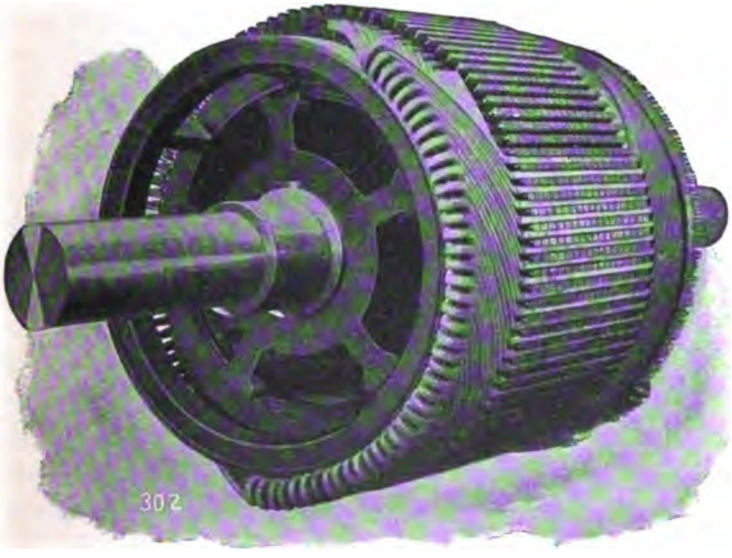

Fig. 34.

which are bolted together so that the upper part of the field may be lifted off when it is desired to remove the armature for repairs. Figure 31 shows the field structure and base of this machine with

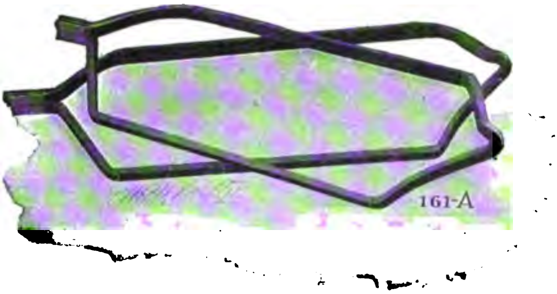

Fig. 35.

the armature and all field windings removed. One of the laminated structures which constitutes a combined field magnet core and pole piece is shown in Fig. 32. These pole pieces are set into the mould and cast-welded in the field magnet yoke. An

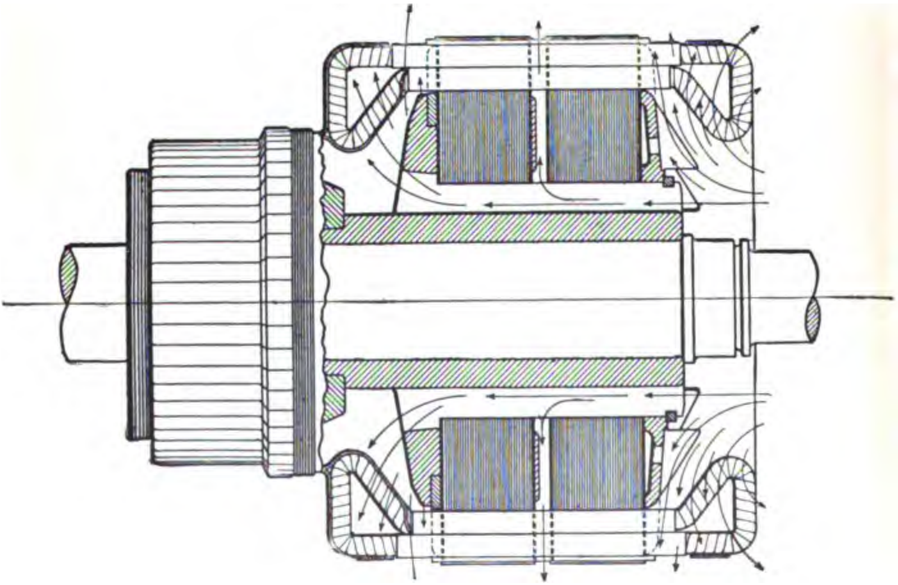

Fig. 36.

enlarged area of pole face is obtained by the use of an open frame of cast iron shown in Fig. 32, which is called the *pole shoe*. This pole shoe is put in place and fixed by bolts after the field coils have been slipped into position on the projecting field cores. The armature body with windings removed is shown in Fig. 33, and the armature with windings partly in place is shown in Fig. 34. Figure 35 shows two of the armature coils. These coils are wound on frames, bound together with tape, and then laid into the armature slots and their terminals soldered to the lugs of the commutator bars *ll*, Fig. 38.

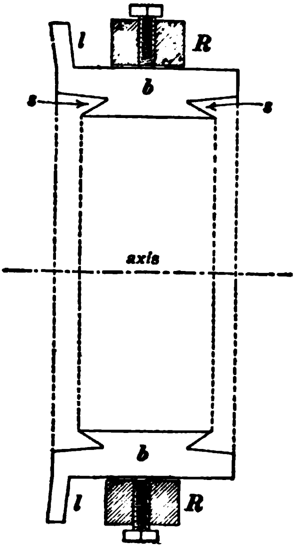

Fig. 37.

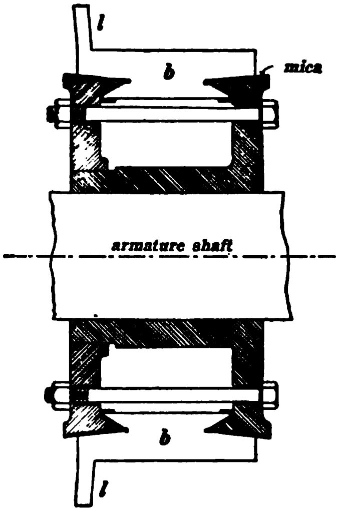

Fig. 38.

The space between the laminated body of the armature and the flange at the near end in Fig. 33, and also the space between the laminated body of the armature and the commutator at the far end in Fig. 33, are for receiving the overlapping ends of the armature coils as shown in Fig. 34. Both of these spaces are left more or less open so that cool air may be drawn in at one end of the rotating armature and driven out through the overlapping ends of the armature windings as shown by the curved arrows in Fig. 36. Furthermore, the laminations of the armature core or body are separated at intervals by corrugated disks, thus forming air ducts from the interior to the exterior of the armature core. Figure 33 shows seven places where the laminations are thus separated. The object of this thorough ventilation of the armature is to avoid the high temperatures which would otherwise be produced by the very considerable heat which is generated in the armature core by eddy currents and magnetic hysteresis, and in the armature windings by the current.

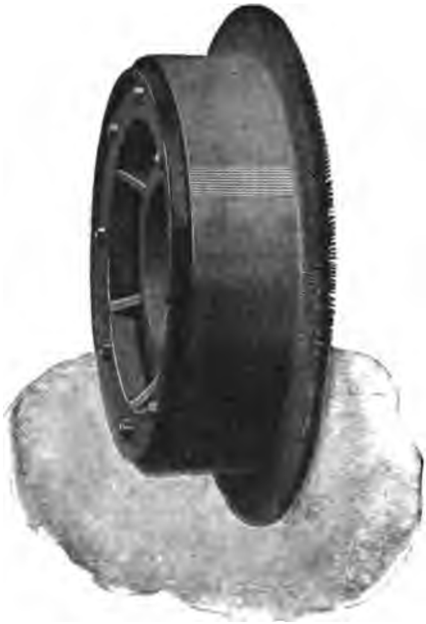

Fig. 39.

The commutator, the function of which is explained in Art. 30, is always made in the form of a rigid self-contained structure. Forged copper bars *bb*, Fig. 37, slightly thinner at one edge than at the other, are built up in the form of a hollow cylinder with accurately gauged mica plates between the bars. This built-up structure is clamped in a massive ring *RR*, placed in a lathe, and the conical seats *ss* are turned out. A steel hub is provided with conical lips which fit into the seats or grooves *ss*, and the ends of the hub are drawn together by bolts as shown in Fig. 38. A thin conical washer of moulded mica separates the steel hub from the commutator bars as shown in Fig. 38. The radial lugs *ll*, one on each commutator bar or segment, serve for the attachment of the wire leads which connect with the armature windings. Figure 39 is a general view of a completed commutator manufactured by the General Electric Company.

The field coils are generally wound on forms and bound solidly together with tape. One pair of field coils for a compound generator is shown in Fig. 40; the lower part of the figure is the fine wire winding (shunt) and the upper part of the figure is the coarse wire winding (series), which, in this case, is made of copper strip bent edgewise.

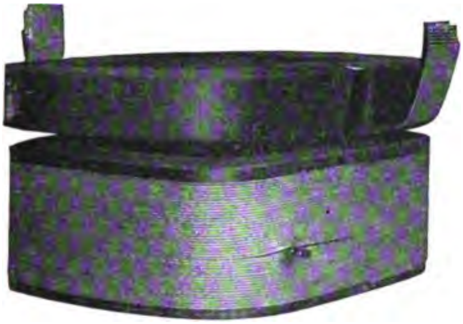

Fig. 40.

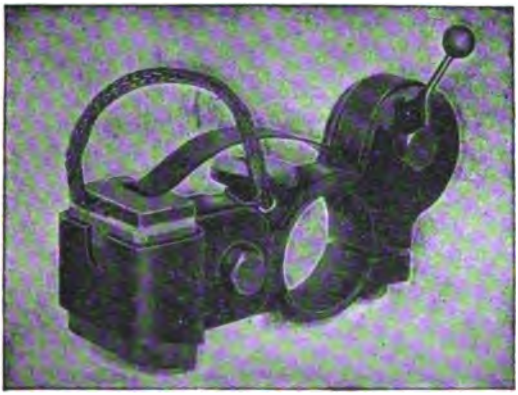

Fig. 41.

Figure 41 shows a brush holder of the General Electric Company. The current is led from the carbon block (the brush) to the metal part of the holder by the flexible cable of fine copper wire which is called a *pig-tail.* This relieves the spring from doing duty as an electrical conductor which would be likely to heat it and cause it to lose its temper. The small lever at the back serves to adjust the pressure with which the spring pushes the carbon block against the commutator.

32. Field excitation. — Some very small dynamos such as the "magnetos" which are used for ringing telephone call-bells, are provided with field magnets of hardened steel which are permanently magnetized. Large dynamos, on the other hand, are always provided with electrically excited field magnets (electromagnets).

Separate excitation. — When the field magnet of a dynamo is excited by current (direct current) from an outside source, we have what is called separate excitation. For example, the field magnets of alternators are nearly always excited by direct current from an auxiliary direct-current generator, and certain types of direct-current generators, such as the low-voltage generators which are used for electroplating, are separately excited.

Self-excitation. — Direct-current generators usually supply their own field current. This method of field excitation, which is called self-excitation, is not suitable to alternators. The field windings of self-excited direct-current generators may be arranged in three ways, giving what are called the shunt generator, the series generator, and the compound generator. The connections of these three types of machine are described in the following articles, and their practical advantages and disadvantages are discussed in Chapters IV and V.

Composite excitation. — When the field magnet of a generator is partly excited by current from an outside source and partly excited by its own current, we have what is called composite excitation. This mode of field excitation is frequently used in moderate sized alternators as explained below.

33. The shunt generator. — The winding on the field magnet of a direct-current generator may consist of many turns of comparatively fine wire having considerable resistance. In this case the terminals of the field winding are connected directly to the brushes of the machine and from two to ten per cent. of the permissible current output of the generator is taken to magnetize or excite the field, the remainder being available for use in the external circuit. In this case the field winding and the outside receiving circuit are in parallel with each other between the brushes, so that the field winding is in the relation of a shunt to the outside receiving circuit. A direct-current generator with its field windings arranged in this way is called a *shunt generator.*

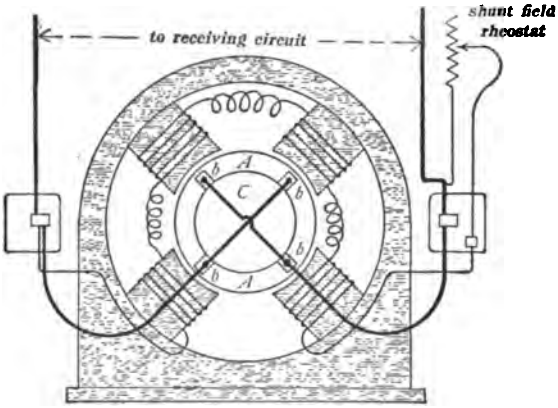

Fig. 42.

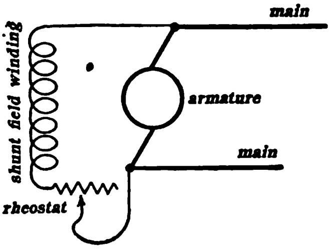

Fig. 43.

When the current output of a shunt generator (driven at constant speed) is increased, the electromotive force of the generator tends to decrease in value and therefore an adjustable resistance is usually connected in the field circuit for controlling the value of the field current and thereby controlling the voltage of the machine. This adjustable resistance is called the *field rheostat.*

Figure 42 shows the arrangement of a shunt generator (four-pole) with its field rheostat; *AA* is the armature, *bbbb* are the brushes, and *C* is the commutator. The brush leads are connected to terminal blocks to which the receiving circuit and the shunt field winding are also connected. Figure 43 is the conventional diagram of connections of the shunt generator.

34. The series generator. — The winding on the field magnet of a direct-current generator may consist of comparatively few turns of heavy wire having a low resistance. In this case the field winding is connected in series with the external receiving circuit, the whole current delivered by the machine flows through the field winding, and from two to ten per cent. of the electromotive force of the machine is used to force the current through the field winding, the remainder being available for forcing cur-

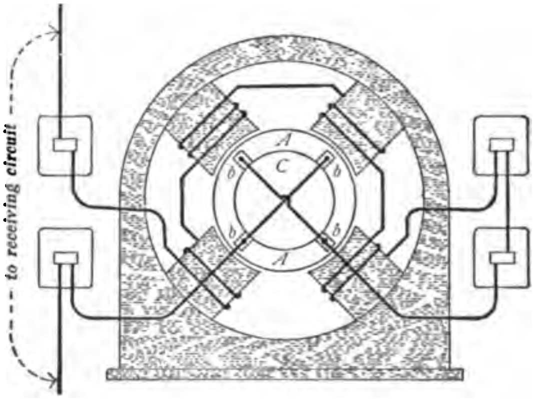

Fig. 44

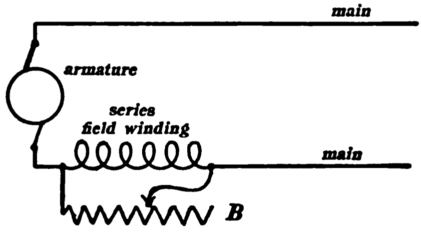

Fig. 45.

rent through the external receiving circuit. A direct-current generator with its field windings arranged in this way is called a *series generator*. Figure 44 shows the arrangement of a series generator (four-pole), and Fig. 45 is the conventional diagram of connections of a series generator. In Fig. 45 an adjustable low resistance *B* is connected between the terminals of the field winding so that the value of the field current may be controlled independently of the total current output of the machine.

35. The compound generator. — The tendency of the electromotive force of a shunt generator (driven at constant speed) to decrease in value with increase of current output may be automatically counteracted by providing the shunt generator with an auxiliary field winding of coarse wire through which the total current output, or a definite fractional part thereof, is allowed to flow. The effect of this auxiliary winding is to increase the field excitation as the current output increases. A direct-current generator arranged in this way is called a *compound generator*. Figure 46 shows the arrangement of a compound generator (four-pole) with a rheostat in circuit with its shunt field winding. The conventional diagram of connections of the "long shunt"

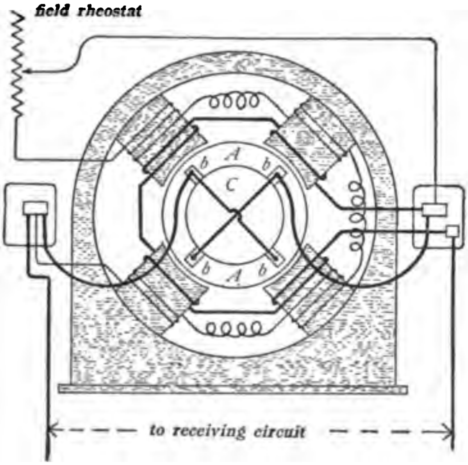

Fig. 46.

compound generator is shown in Fig. 47, and the conventional diagram of connections of the "short shunt" compound generator is shown in Fig. 48. The series field winding of the compound generator usually has a very low resistance, and the difference between the long shunt and short shunt arrangement is of no practical importance, although the calculations of currents, electromotive forces, losses, and efficiencies are slightly different in the two cases.

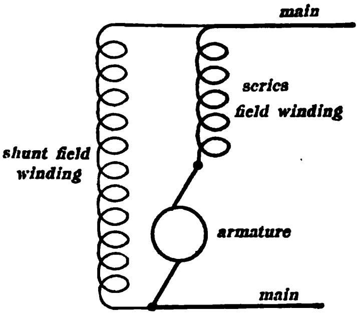

Fig. 47.

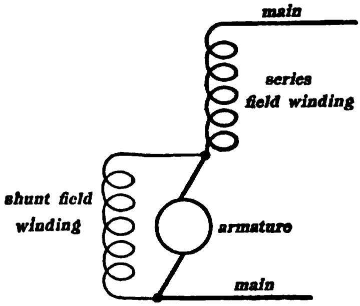

Fig. 48.

36. Field excitation of direct-current motors.—Inasmuch as a motor always receives current from an outside source, there is no such thing as self-excitation of a motor. A *shunt motor* has a fine-wire field winding which is connected to the supply mains in parallel with the armature. A *series motor* has a coarse-wire field winding which is connected in series with the armature to the supply mains. A *compound motor* has a field winding in two parts: one part is of coarse wire and it is connected in series with the armature to the supply mains, and the other part is of fine wire and it is connected to the supply mains in parallel with the armature.

The connections of direct-current motors to the mains are different from the connections of direct-current generators to the mains on account of the *starting resistance* which must be used in connection with the motor as explained in Chapter V.

37. The composite alternator.—Usually the field magnet of an alternator is excited wholly by direct current obtained from an outside source. When the field magnet of an alternator is so excited there is a tendency for the electromotive force between the terminals of the alternator to decrease in value with increase of current output, and it is generally necessary, therefore, to have a switchboard attendant to adjust the exciting current with changes of load on an alternator in order to keep the terminal voltage of the alternator constant.

In the case of a direct-current shunt generator the variations of terminal voltage with change of current output may be counteracted automatically by using an auxiliary (series) field winding through which the current output of the generator flows, thus providing for an increase of field excitation with increase of load. In order to make use of the current output of an alternator for automatically increasing its field excitation with increase of load, it is necessary to introduce a device for reversing the connections of the series field winding at each reversal of the alternating current so that the current may flow always in the same direction through the series field coil and thus have the desired magnetizing action. This reversing device is called a *rectifier*, and an alternator which is provided with a rectifier and a series field winding is called a *composite alternator*.

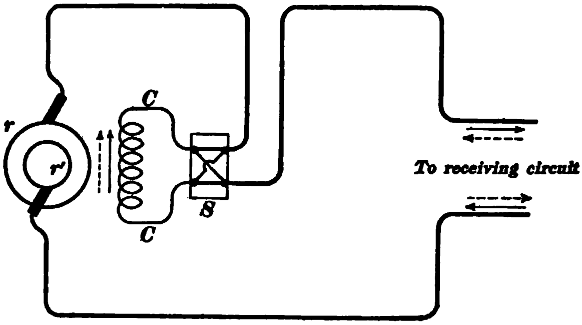

Fig. 49.

The essential features of the connections of a composite alternator are shown in Fig. 49. The series field winding *CC* is connected into the main alternating-current circuit through the rectifier *S* which is essentially a reversing switch, and the current in *CC* flows always in one direction, although the current in the main circuit is an alternating current.

Instead of tapping the coil *CC* directly into the main circuit, it is usually connected to the secondary coil of a small transformer whose primary is in the main circuit, as shown in Fig. 50.

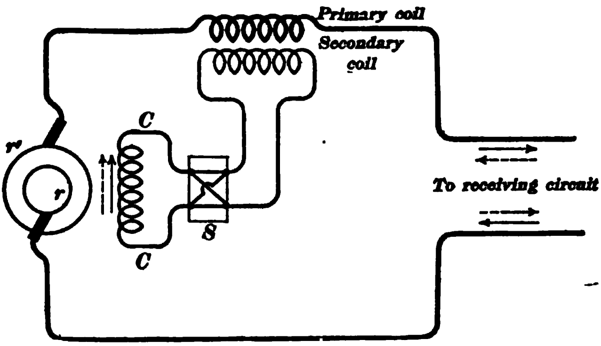

Fig. 50.

This is equivalent in its effect to the insertion of the coil *CC* directly into the main circuit as shown in Fig. 49.

Figure 51 shows the actual arrangement of the composite alternator. The small transformer, the collecting rings *rr'*, and the rectifier *S*. are all mounted together on the armature shaft. The small transformer is in fact usually wound upon a laminated spoke of the armature core. The alternating current flows through the primary coil of the small transformer before it reaches the collecting ring *r*. The rectifier consists of a commutator having *p* insulated segments, where *p* is the number of field magnet poles of the alternator. The odd numbered segments are connected together and to one terminal of the secondary coil of the small transformer, and the even numbered segments are connected together and to the other terminal of the secondary coil of the small transformer. Two brushes at a distance apart equal to the distance * from center to center of adjacent commutator segments, rub on the commutator and deliver rectified current to the series field winding as shown.

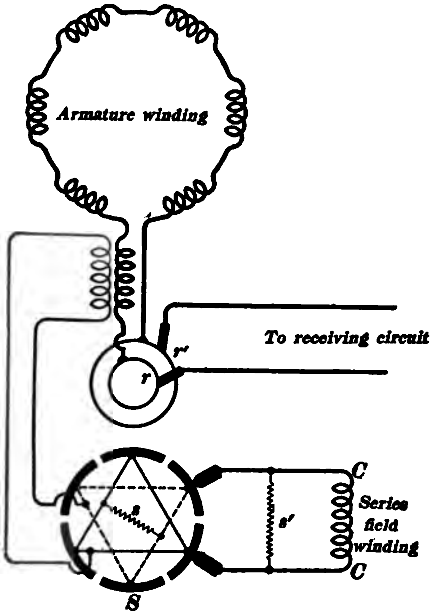

Fig. 51.

It is necessary to shift the rectifier brushes back and forth until the position is found which gives a minimum of sparking. The brushes will then be passing from one segment to the next at the instant when the alternating current is passing through the zero value. The sparking, however, is serious at best and in order to reduce the sparking it is necessary to connect resistances *s* and *s'* as shown. Sometimes only one of these resistances is used. Thus, in the composite alternator which is shown in Fig. 17, the brushes of the rectifier are connected to the series field winding and also to a resistance *s'* which is mounted inside of the bearing pedestal of the machine.

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* Or any odd multiple of this distance.

38. The alternating-current transformer. — In order to be able to appreciate the relative advantages of alternating-current service and direct-current service, the student must have an understanding of the alternating-current transformer. The alternating-current transformer, or simply the transformer, consists of two separate coils of wire wound upon an iron core, which core usually forms a closed magnetic circuit. Alternating current is supplied to one of these coils from an alternator, this alternating current causes rapid reversals of magnetic flux through the iron core, and these reversals of flux induce an alternating electromotive force in the other coil which delivers alternating current to a receiving circuit. The coil to which alternating current is supplied is called the *primary coil* and the coil which delivers alternating current to a receiving circuit is called the *secondary coil.*

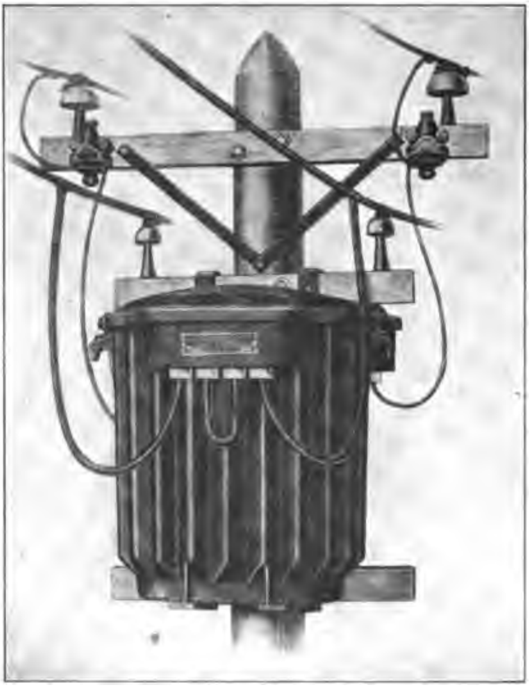

Fig. 52.

Step-up and step-down transformation. — Usually one coil of a transformer has many more turns of wire than the other. When the coil of few turns is the primary coil, the transformer takes *large current at low voltage* and delivers *small current at high voltage.* This is called step-up transformation. When however the coil of many turns is the primary coil, the transformer takes *small current at high voltage* and delivers *large current at low voltage.* This is called step-down transformation. The object of step-up and step-down transformation is explained in Art. 39.

Mounting of transformers. — Transformers are usually inclosed in iron cases, and the case is usually filled with oil. The purpose of the oil is partly to improve the insulation of the coils by impregnating the fibrous insulating material, and partly to facilitate cooling by carrying the heat from the core and coils to the containing case. A special provision must be made for cooling very large transformers. For the cooling of very large transformers, passageways or ducts are left through the core and coils, and oil circulates through these ducts and is itself cooled by a coil of water pipes placed in the top of the containing case. In some instances a blast of air is driven through the transformer by a fan-blower.

Figure 52 shows a small transformer mounted on a pole, taking current from street mains (upper wires in the figure) at 1,100 volts, and delivering current to house mains (lower wires in the figure) at 110 volts. The two small cases attached to the upper cross-arm, and through which the wires leading to the primary coil of the transformer pass, are hollow porcelain insulators with fusible cut-outs mounted inside. The transformer shown in Fig. 52 is enclosed in a cast-iron case which is filled with oil.

The core-type transformer. — A transformer in which the coils of wire surround a more or less elongated core of laminated iron is called a *core-type transformer.* Figure 53 is a sectional view of a core-type transformer. The coarse wire winding S is wound in two parts on the two limbs a and b of the laminated core, and these two parts may be connected in series or parallel with each other.

The fine wire winding P is also wound in two parts (over the coarse wire winding), and the two parts of the fine wire winding may also be connected in series or in parallel with each other.

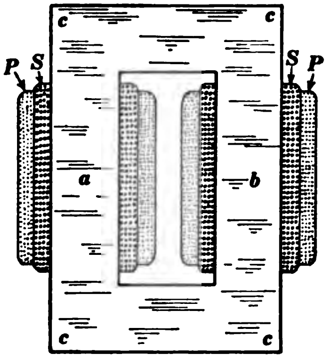

Fig. 53.

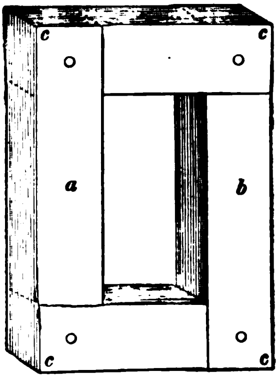

Fig. 54.

The laminated core *ab*, which is shown by itself in Fig. 54, is built up of thin rectangular strips of soft sheet steel which interleave with each other at the corners, thus giving a good magnetic circuit. Figure 55 shows a complete core-type transformer of the General Electric Company without its containing case, and Fig. 56 is a sectional view of the same transformer in its containing

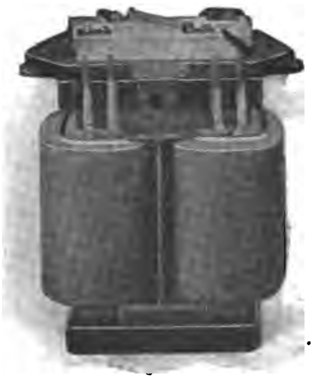

Fig. 55.

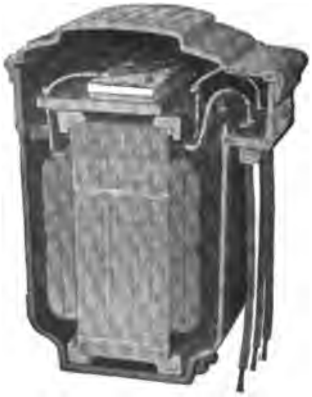

Fig. 56.

case. The four terminals of the coarse wire windings are brought out through the case, and the four terminals of the fine wire windings are led to a connection board on top of the transformer and inside of the case, and two wire leads pass out through the containing case from this connection board.

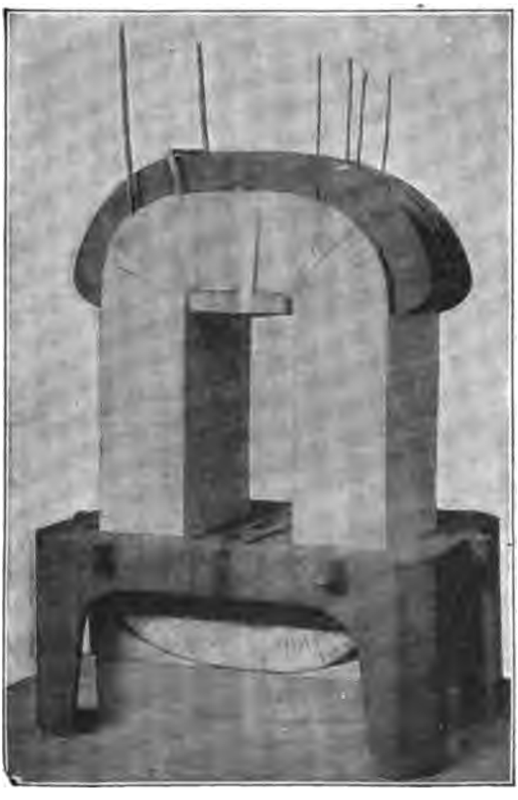

FIG. 57.

The shell-type transformer. — A transformer in which the *coils* form an elongated structure like Fig. 54 and are surrounded by laminated iron, is called a shell-type transformer. Figure 57 shows the assembled windings of a shell-type transformer of the Allis-Chalmers Company, and Fig. 58 shows the coils surrounded by the laminations. Figure 59 shows a large shell-type transformer of the Westinghouse Electric Company, without the containing case, but showing the cover and coils of water-pipe (for cooling) in the assembled position.

39. Direct-current service versus alternating-current service. — The electrical transmission of a given amount of power may be

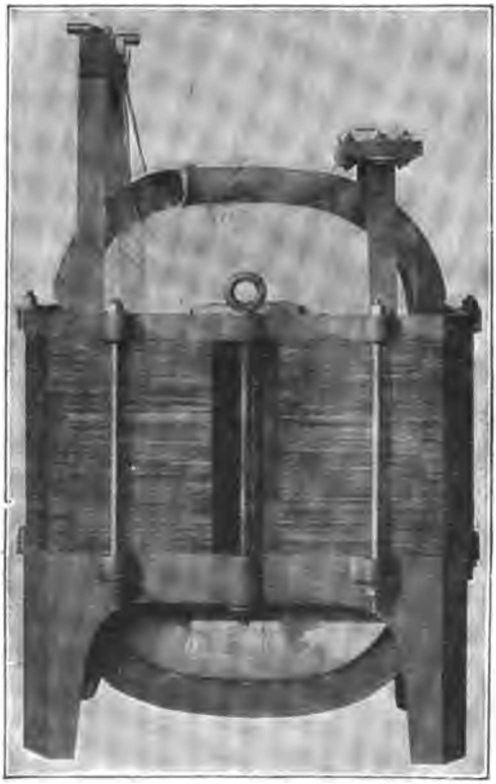

Fig. 58.

accomplished by a large current at low voltage or by a small current at high voltage. In the first case very large and expensive transmission wires must be used or the loss of power in the transmission line will be excessive. In the second case comparatively small and inexpensive transmission wires may be used without excessive loss of power. Therefore it is necessary to use high voltages in the long-distance transmission of power.

High voltages, however, are dangerous under the conditions which ordinarily exist among users of electric power, and many

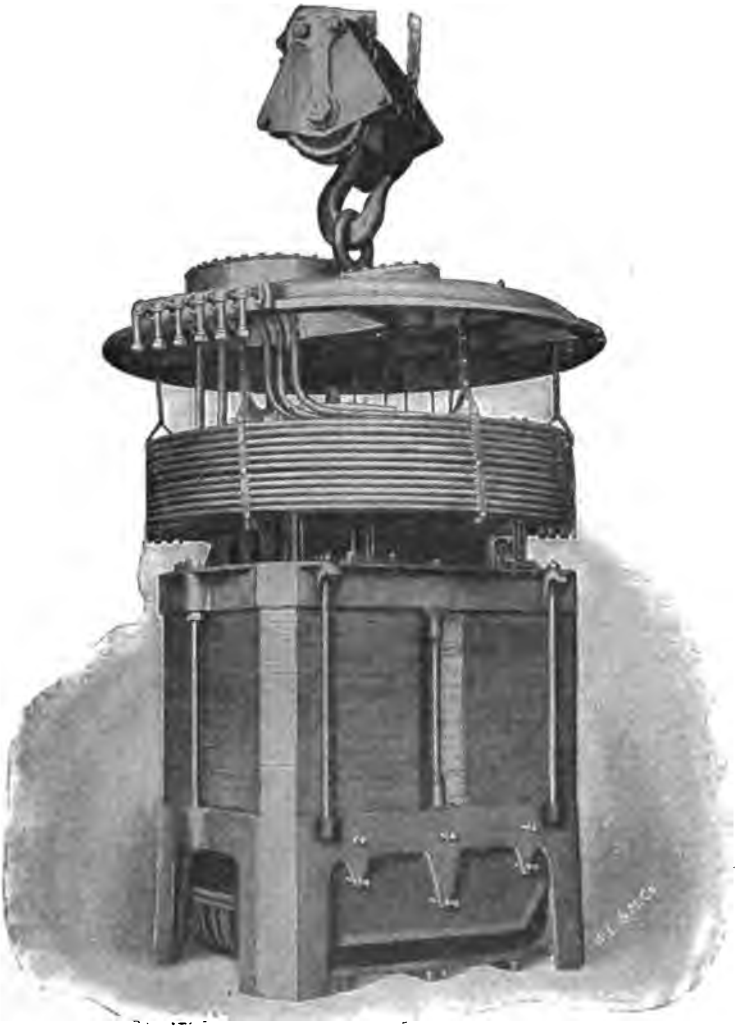

Fig. 59.

kinds of apparatus, such, for example, as incandescent lamps, require low voltages for satisfactory operation. Therefore power which is transmitted at high voltage must be transformed at the receiving station from high-voltage-and-small-current to low-voltage-and-large-current if long-distance transmission is to be successful, and *the advantage of alternating current over direct current lies almost wholly in the cheapness of construction, cheapness of operation, and high efficiency of the alternating-current apparatus that is required for this transformation.*

In the step-down transformation of direct current, a motor would take a small current from the high voltage transmission line and drive a generator which would deliver a large current at low voltage to the service mains. This apparatus would be expensive to construct, it would require constant attention, and its efficiency would never be greater than about 90 per cent.

The step-down transformation of alternating current, however, is accomplished by means of the alternating-current transformer, which is very much cheaper to construct than a motor and a generator of the same capacity, it requires no attention, and its efficiency under full load is usually greater than 97 per cent.

The alternating-current system has a slight advantage over the direct-current system because the alternator, having no commutator, is somewhat simpler than the direct-current generator, and in the alternating-current system one may use the inductor alternator and the induction motor which have no sliding electrical contacts whatever.

Another advantage of the alternating-current system is that the current passes through zero value many times per second so that a circuit breaker operates more effectively on an alternating-current circuit than it does on a direct-current circuit. Indeed, it is claimed that large generating stations, such as those at Niagara Falls, could not be operated at all if it were not for this peculiar effectiveness of the circuit breaker on an alternating-current circuit.

These advantages of the alternating current are to some extent balanced by certain disadvantages which are outlined in the following paragraphs. Where power is to be transmitted over short distances, the one great advantage of the alternating-current system vanishes, and the direct-current system is distinctly the better of the two, especially if a central station is to supply current, not only for lighting, but also for driving motors and for charging storage batteries.

Alternating-current lighting.—The electric lamp depends fundamentally upon the heating effect of the electric current, and in so far as the mere heating effect is concerned the alternating current is as satisfactory as the direct current. There is a tendency, however, for an alternating-current lamp to give an intermittent light because of the cooling of the luminous element of the lamp between the successive pulses of current. The intermittent character of the light is especially marked in alternating-current arc lamps, and glow lamps give a perceptibly intermittent light at low frequencies, especially if the lamp filament is very fine.

The electric arc, too, is peculiarly unstable when it is operated by alternating current, and certain types of arc lamps, some of the luminous-arc lamps for example, cannot possibly be operated by alternating current.

On the other hand the Nernst lamp operates very much better with alternating current than with direct current.

Alternating-current motors. The simple alternating current is very much less satisfactory for motor driving than direct current. The one very great advantage of alternating current over direct current (ease of transformation up or down) has led, however, to the development of several satisfactory types of alternating-current motors, and it justifies in other cases the use of a motor which may not be as easy to start and to operate as the direct-current motor.

The operation of direct-current motors is discussed in Chapters V and VI. The operation of the *synchronous motor* (alternating-current) is described in Chapter XII. The *induction motor* (alternating-current) is described in Chapter XV and the *single-phase alternating-current commutator motor* is described in Chapter XVI.

The induction motor, in the form in which it is most generally used, must be supplied with two or more distinct alternating currents transmitted to the motor over two or more distinct transmission lines. This arrangement is called the *polyphase system* of transmission and it is described in Chapter X.

Conversion of alternating current into direct current. — For many electrochemical processes direct current only can be used. Thus, storage batteries cannot be charged by alternating current. Furthermore, the greater simplicity of the direct-current motor has led to the general use of direct current for operating street railways. When power is transmitted by the alternating-current system and utilized in the form of direct current, the conversion from alternating current to direct current is usually accomplished by means of the *synchronous converter*, or *rotary converter*, as it is often called. The synchronous converter is described in Chapter XIII.

40. The physical basis of the differences between direct-current calculations and alternating-current calculations. — Figure 60 represents a valveless pump P, the piston of which oscillates up and down causing a current of water to surge back and forth through a circuit of pipe, and Fig. 61 represents an alternator A of which the alternating electromotive force causes alternating current to surge back and forth through a circuit of wire.

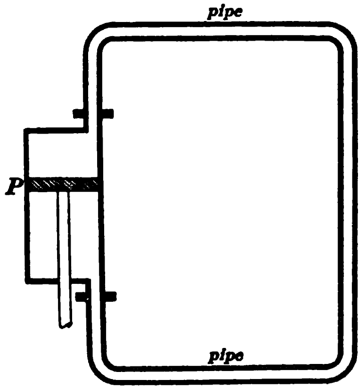

Fig. 60.

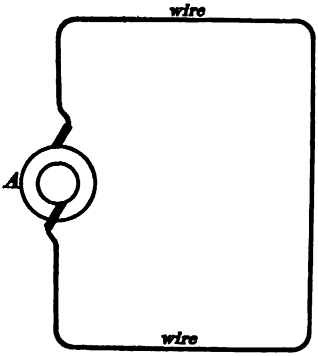

Fig. 61.

The alternating pressure generated by the pump in Fig. 60 must not only overcome the resistance which the current of water encounters in the pipe, but a large part of the pressure developed by the pump must be used to overcome the inertia of the water in the pipe, first in getting the current of water started and then in stopping the current and starting it again in the opposite direction. Similarly, the alternating electromotive force generated by the alternator in Fig. 61 must not only overcome the electrical resistance of the wire, but a large part of the electromotive force developed by the alternator must be used to overcome the electrical inertia or inductance of the circuit of wire, first in starting the electric current and then in stopping it and starting it again in the opposite direction.

The alternating pressure generated by the pump in Fig. 62 must not only overcome the resistance of the pipe and the inertia of the water in the pipe, but a portion of the pressure developed by the pump must be used to distort the elastic diaphragm *DD* which bridges across the chamber *CC*. Similarly, the alternating electromotive force generated by the alternator in Fig. 63 must not only overcome the electrical resistance of the wire and the electrical inertia of the circuit, but a portion of the electromotive force developed by the alternator must be used to produce the electrical stress which is created in the insulating material *DD*

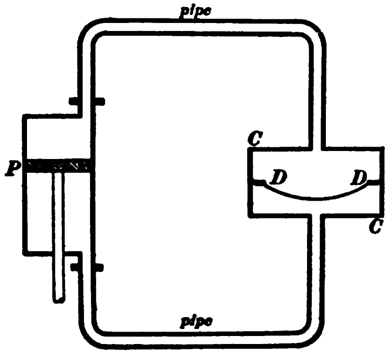

Fig. 62.

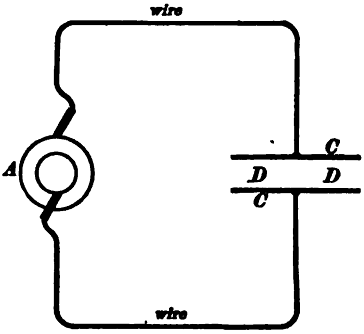

Fig. 63.

between the metal plates *CC* as these plates are electrically charged first in one direction (upper plate positive, lower plate negative) and then in the reverse direction (upper plate negative, lower plate positive).

The metal plates *CC*, Fig. 63, with the insulating material *DD* between them constitute what is called a condenser as explained in Art. 23, and the action of this condenser is exactly analogous to the action of the chamber *CC* in Fig. 62 with its elastic diaphragm *DD*. The elastic diaphragm permits an alternating current of water to surge back and forth through the circuit of pipe, but it does not permit the flow of a steady current of water. Similarly, the condenser *CC*, Fig. 63, permits an alternating electric current to surge back and forth through the circuit of wire, but it does not permit the flow of a steady electric current.*

Direct-current problems and alternating-current problems contrasted.—Direct-current problems are generally the finding of one or two of the quantities, voltage, current, and power when the other, or others, are given and when the circuit conditions are known. Likewise alternating-current problems are generally the finding of one or two of the quantities, voltage, current, and power when the other, or others, are given and when the circuit conditions are known. The difference between direct-current problems and alternating-current problems grows out of the fact that in an alternating-current circuit the relation between voltage, current, and power depends upon the resistance, the inductance, and the capacity in the circuit, whereas in a direct-current circuit, the relation between voltage, current, and power is independent of inductance and capacity. *The relation between voltage, current, and power in an alternating-current circuit can be simply formulated only for the case in which the voltage and current are of the type which is represented by a curve of sines and when the capacity is concentrated.* Therefore, **in most practical alternating-current problems, the voltage and current are assumed to be of this type * and the effects of distributed capacity are ignored.**

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* In Fig. 62 the only appreciably elastic element is the diaphragm *DD*, and in Fig. 63 the only place where an appreciable amount of charge accumulates is on the two metal plates *CC*, the amount of charge that accumulates on the wire being negligible. The metal plates in Fig. 63 constitute what is called a *concentrated capacity* and the elementary theory of alternating currents is limited to the discussion of the effects of concentrated capacity. The hydraulic analogue of distributed capacity is described in Franklin and Esty's *Elements of Electrical Engineering*, Vol. II, pages 21 and 22. Distributed capacity always produces the phenomena of waves.

* This type of alternating electromotive force or current is called harmonic electromotive force or current. See Chapter VIII.

CHAPTER III.

DIRECT-CURRENT AND ALTERNATING-CURRENT MEASUREMENTS.

41. The D'Arsonval-galvanometer-type of direct-current ammeter and voltmeter. — Figures 64 and 65 show the working parts of a direct-current ammeter (or voltmeter). A horse-shoe magnet of steel is provided with soft iron pole pieces *NN* and *SS* between which a soft iron cylinder *C* is rigidly supported by being bolted to the brass strip *A*. In the spaces between the pole pieces and the cylinder *C* move the sides, or limbs, of a small coil of wire which is delicately supported upon a pivot and which carries a pointer which plays over a divided scale. Current is led into this movable coil through the hair spring at one end and through a very flexible conductor at the other end, and the side force which is exerted upon the limbs of the coil by the magnet

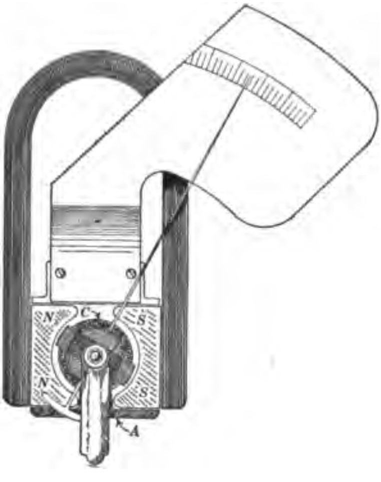

Fig. 64.

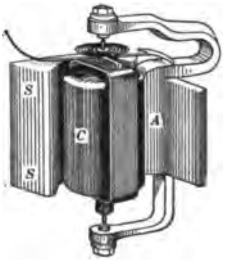

Fig. 65

poles *NN* and *SS* turns the coil until the force is balanced by the reaction of the hair spring.

When this instrument is to be used as an ammeter, a low resistance shunt is connected in parallel with the movable coil so that a large current of any desired value may correspond to the full deflection of the instrument.

When this instrument is to be used as a voltmeter a high resistance is connected in series with the movable coil, and the scale is numbered so as to indicate the value of the voltage between the terminals of the instrument.

42. The difference between direct-current and alternating-current ammeters and voltmeters.* — In direct-current ammeters and voltmeters, the force which acts upon the movable part of the instrument and produces the deflection of the pointer may be any function whatever of the current or electromotive force, provided only that the deflecting force has a perfectly definite value for a given current or electromotive force, thus ensuring that a given current or electromotive force will produce a definite deflection so that the scale may be constructed by marking the positions of the pointer for a series of known values of current or electromotive force.

In alternating-current ammeters and voltmeters, on the other hand, the deflecting force is proportional to the square of the current or electromotive force. That is, as the current or electromotive force increases and decreases in value the deflecting force increases and decreases in value so as to be at each instant proportional to the square of the current or electromotive force at that instant, and the *average force* upon which the *steady deflection* depends is proportional to the *average square* of the current or electromotive force.† Alternating-current ammeters and voltmeters really measure, therefore, the *average value of the square* of an alternating current or electromotive force.

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* All voltmeters, with the exception of the electrostatic voltmeter, are essentially ammeters, that is, the electromotive force to be measured produces a current which actuates the pointer; and the scale, instead of being made to indicate the value of the current, is arranged to indicate the value of the electromotive force between the terminals of the instrument.

† This statement is not applicable to the plunger type of instrument.

When an alternating-current ammeter or voltmeter has been calibrated by direct currents or direct electromotive forces, it indicates the square root of the average value of the square, or so-called *effective value*, of an alternating current or electromotive force.* Consider, for example, an ammeter in which the deflecting force is proportional to the square of the current. The constant deflecting force, due to a direct current C, is equal to kC^2, the instantaneous force due to an alternating current is equal to ki^2, where i is the value of the alternating current at a given instant, and the average force due to the alternating current is equal to $k \times \textit{average } i^2$; but, if the alternating current gives the same deflection as the direct current, the constant force kC^2 must be equal to the average force $k \times \textit{average } i^2$, so that $\textit{average } i^2$ must be equal to C^2, or $\sqrt{\textit{average } i^2}$ must be equal to C. That is to say, although the ammeter really measures the average square of any current flowing through it, the scale numbers (values of C) determined by direct-current calibration, are effective values of alternating current ($\sqrt{\textit{average } i^2} = C$).

43. The electrodynamometer-type of ammeter and voltmeter. — Instruments of this type consist of a fixed coil and a movable coil connected in series. The current which produces the deflection flows through both coils and the fixed coil exerts a force upon the movable coil which deflects it and moves the pointer over a divided scale. In some instruments the force action between the coils is balanced by moving a weight or by twisting a helical spring which is attached to the movable coil. In this case the reading of the instrument is the distance that the weight is moved or the angle through which the spring is twisted.

The electrodynamometer when standardized by direct current indicates effective values of alternating current. — This is evident when we consider that the force action between the coils is proportional to the square of the current, or equal to a constant *k* times the square of the current, and that the constant force action kC^2 due to a direct current *C* must be equal to the average force action $k \times \text{average}\, i^2$ due to an alternating current *i*, if *C* and *i* produce the same deflection.

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* Except in the case of a voltmeter which has an appreciable inductance. This statement is not true of the plunger-type instrument in any case.

The electrodynamometer is the most reliable form of alternating-current ammeter.

When the electrodynamometer is used as a voltmeter, its coils are made of fine wire, an auxiliary non-inductive resistance is connected in series with the coils, and the instrument indicates effective values of alternating electromotive force correctly when it has been calibrated by direct electromotive force *provided the inductance of the instrument is negligibly small.*

44. Hot-wire ammeters and voltmeters. — In instruments of this type the current which produces the deflection flows through a fine stretched wire, and the wire, which is heated by the current, lengthens and actuates the pointer which plays over a divided scale. The hot-wire instrument indicates effective values of alternating current when it has been calibrated by direct current, and it indicates effective values of alternating electromotive force correctly when it has been calibrated by direct electromotive force. The hot-wire instrument has not been considered very satisfactory hitherto on account of the tendency of the hot-wire to take a permanent set, that is, to become permanently elongated.

45. Plunger-type ammeters and voltmeters. — In instruments of this type the current flows through a coil of wire which magnetizes and moves a pivoted or suspended piece of soft iron to which the pointer is attached.

Plunger-type ammeters. — In this type of instrument the force tending to move the pivoted piece of iron is not proportional to the square of the current, and therefore such an instrument does not indicate effective values of alternating current correctly when it has been calibrated with direct current. It is necessary therefore to calibrate a plunger-type ammeter with alternating current which, for the purpose of the calibration, is measured by an electrodynamometer ammeter.

Plunger-type voltmeters. — An instrument which will indicate effective alternating current correctly when calibrated as an ammeter with direct current, will also indicate effective alternating electromotive force correctly when calibrated as a voltmeter with direct electromotive force, provided the inductance of the instrument is negligible. This is exemplified by the use of the electrodynamometer as a voltmeter as explained in Art. 43. The plunger-type instrument, however, does not indicate effective values of alternating current correctly unless specially calibrated as explained above, and, moreover, the inductance of the plunger-type instrument is always fairly large. Therefore, for two reasons, a plunger-type voltmeter should be calibrated with alternating electromotive force which, for the purpose of the calibration, is measured by an electrodynamometer voltmeter of negligible inductance, or by an electrostatic voltmeter.

46. The electrostatic voltmeter. — Two insulated metal plates connected to the terminals of a battery, or to any source of electromotive force, attract each other with a force which is exactly proportional to the square of the electromotive force. This principle is applied in the electrostatic voltmeter which consists essentially of a fixed metal plate and a delicately poised or suspended metal plate which carries a pointer, the two plates being connected to the terminals of the source of the electromotive force to be measured.

Such an instrument, when calibrated by direct electromotive force, indicates effective values of alternating electromotive force. This is evident when we consider that the steady force action $k\mathcal{E}^2$, due to a direct electromotive force $\mathcal{E}$, must be equal to the average force action $k \times$ average e^2, due to the alternating electromotive force e which gives the same deflection as $\mathcal{E}$, that is, $k\mathcal{E}^2 = k \times \text{average } e^2$, or $\mathcal{E} = \sqrt{\text{average } e^2}$.

The great difficulty with the electrostatic voltmeter arises from the excessively weak forces of attraction between two metal plates with electromotive forces less than several hundred volts, so that a low-reading electrostatic voltmeter must be very delicate, and therefore unsuited to the conditions of practical use. For high electromotive forces, however, the electrostatic voltmeter is very satisfactory.

47. The spark gauge. — The high electromotive forces used in "break-down" tests of insulators are usually measured by means of the *spark gauge* which consists of an adjustable spark gap between needle points. This spark gap is varied until the electromotive force to be measured is barely able to strike across it in the form of a spark. The length of the spark gap is then measured and the value of the electromotive force is taken from empirical tables.*

48. The wattmeter. — Power delivered by direct-current mains is usually calculated from ammeter and voltmeter readings, but the power delivered by alternating-current mains is generally not equal to the product of effective voltage times effective current, and therefore the power delivered by alternating-current mains cannot be calculated from ammeter and voltmeter readings alone. The wattmeter is a special form of electrodynamometer and it is suitable for the accurate measurement of power delivered either by direct-current mains or by alternating-current mains. A coil of fine wire A, Fig. 66, is connected to the mains in series with a non-inductive resistance R. This coil of fine wire is delicately suspended inside of a stationary coil of coarse wire B through which flows the current i that is delivered to the lamps or other receiving units L. Let e be the electromotive force between the mains at a given instant and i the current delivered to the receiving circuit at the same instant; then e/R is the current in

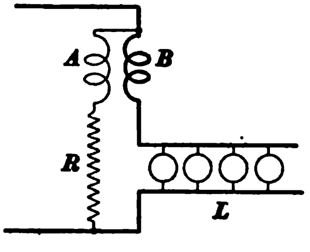

Fig. 66.

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* See page 44, Franklin and Esty's *Elements of Electrical Engineering*, Vol. II.

the fine wire coil A at that instant, and the force action between the two coils A and B at the given instant is proportional to the product of e/R and i, or, since R is constant, it is proportional to the product ei. But ei is the power that is delivered to the receiving circuit at the given instant, so that the force action between the two coils of the wattmeter is proportional at each instant to the power delivered, and therefore the average force action which determines the deflection of the instrument is proportional to the average value of ei or to the average power delivered. The instrument indicates accurately the power delivered by alternating-current mains when it has been calibrated by the use of direct electromotive force and current (inductance of coil A being negligible).

49. Power factor of an alternating-current receiving circuit. — Let E be the effective value of the electromotive force between alternating-current supply mains as measured by a voltmeter, I the effective value of the alternating current delivered to a receiving circuit as measured by an ammeter, and P the power delivered to the receiving circuit as measured by a wattmeter. The product EI is nearly always larger than P, and the ratio $P/(EI)$ is called the *power factor* of the receiving circuit, so that we may write

$$P = EIp \tag{27}$$

in which p is the power factor of the receiving circuit. The power factor of a receiving circuit depends not only upon the nature of the receiving circuit but also upon the frequency and wave shape of the electromotive force of the alternator.

The product EI is sometimes called the *apparent power*, and it is customary to specify the value of EI in *volt-amperes* in order to avoid the suggestion of actual power which would be conveyed by specifying the value of EI in watts.

Examples. — (*a*) An ordinary glow lamp, which is practically non-inductive, takes 0.5 ampere (effective) from 110-volt (effective) alternating-current supply mains, and the power delivered to the lamp is 55 watts. In this case, namely, when the receiving circuit is non-inductive, the actual power P is equal to the apparent power EI and the power factor of the receiving circuit is unity.

(*b*) The electromotive force (effective) between the terminals of an alternator is 1,000 volts, it delivers 100 amperes (effective) of current to operate the lights and motors in a small city, and the actual power as measured by a wattmeter is 89 kilowatts. Therefore the power factor of the entire system is 0.89, inasmuch as the apparent power EI is 100,000 volt-amperes and the actual power is 89,000 watts.

50. The watt-hour meter is an instrument for summing up the total work or energy delivered to a circuit. There are two types of watt-hour meter in general use, namely, the *commutator-motor type*, which is suitable for either direct-current circuits or alternating-current circuits, and the *induction-motor type* which is suitable for alternating-current circuits only. The commutator-motor type of watt-hour meter is exemplified by the Thomson meter and the induction-motor type of watt-hour meter is exemplified by the so-called induction meter of which there are many varieties on the market.*

51. Alternating electromotive force and current curves. The oscillograph. — The successive instantaneous values of the electromotive force of an alternator may be represented by the ordinates of points on a curve, the corresponding abscissas representing elapsed times reckoned from some chosen instant. Such a curve is called the *electromotive-force curve* of the alternator. In a similar manner, the successive instantaneous values of an alternating current may be represented by ordinates and elapsed times by abscissas, giving a *current curve*. The shape of the electromotive-force curve of an alternator depends upon the shape and distance apart of the pole pieces of the field magnet,* and upon the distribution of the armature winding. This is exemplified in some of the following examples, especially in Fig. 71. The shape of an alternating-current curve depends in part upon the shape of the electromotive force curve of the alternator which is delivering the current, and in part upon the character of the receiving circuit. This is exemplified in Figs. 68 and 75.

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* See page 204, Vol. I, Franklin and Esty's *Elements of Electrical Engineering* for a description of the Thomson meter. See page 50, Vol. II, Franklin and Esty's *Elements of Electrical Engineering* for a description of the induction meter.

The oscillograph. — In order that a galvanometer needle may indicate the successive instantaneous values of a rapidly changing current, such as an alternating current, the galvanometer needle must be extremely light and the period of free vibration of the needle must be extremely short. The oscillograph is a galvanometer constructed to meet these requirements. The oscillograph consists of two very fine wires *WW*, Fig. 67, stretched very close together between the poles *NS* of a strong magnet. A very light mirror *M* is attached to the wires, and the current to be measured flows down one wire across the connection *c* and up the other wire, so that one wire is pushed forwards and the other wire is pushed backwards by the magnet, thus deflecting the mirror. A beam of light falls on the mirror and is reflected to a moving photographic plate upon which is left a permanent trace of the movements of the mirror.

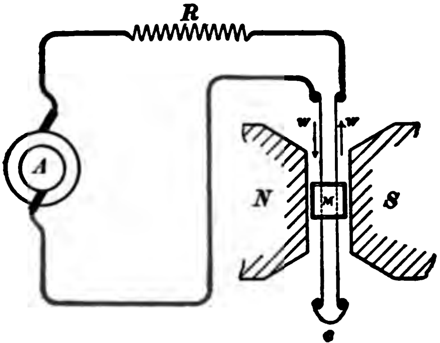

Fig. 67.

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* This is intended to include everything which has to do with the distribution of magnetic flux under the pole pieces of the alternator.

To determine an electromotive-force curve of an alternator. — The oscillograph is connected through a non-inductive resistance R to the terminals of the alternator A, as shown in Fig. 67, and the photographic tracing obtained is the required electromotive-force curve.

To determine an alternating-current curve. — The oscillograph with more or less resistance in series with it is connected to the terminals of a non-inductive resistance through which the given alternating current flows, and the photographic tracing obtained is the required alternating-current curve.

Examples of oscillograph curves. — The curve A, Fig. 68,* is

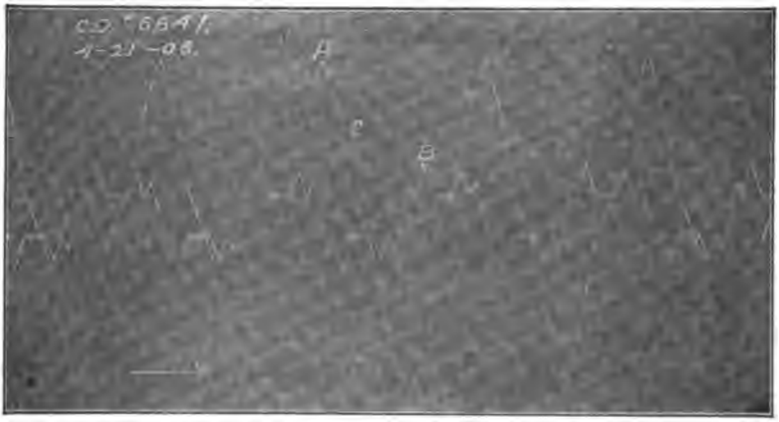

Fig. 68.

the electromotive force curve of an alternator whose armature winding is placed in comparatively few slots, the effect of the slots being to introduce slight fluctuations in the electromotive-force wave as shown by the fine waves in curve A in the figure; the curve B is the curve of current delivered to a condenser; and the curve C is the curve of current delivered to a circuit containing inductance. It is interesting to note that the small fluctuations of electromotive force due to the armature slots are greatly magnified in the current curve B (condenser in circuit) whereas the curve *C* is very smooth (current in inductive circuit).

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* From a photograph furnished by the General Electric Company.

52. Examples of electromotive-force curves illustrating the dependence of form of the curve upon shape and size of pole pieces and distribution of armature winding. — Figure 69*a* represents a single armature wire *a* cutting the uniformly distributed flux under the pole pieces, and the resulting electromotive-force curve is shown in the lower part of the figure. Figure 69*a* represents the magnetic flux as stopping short at the pole tips, and Fig. 69*b*

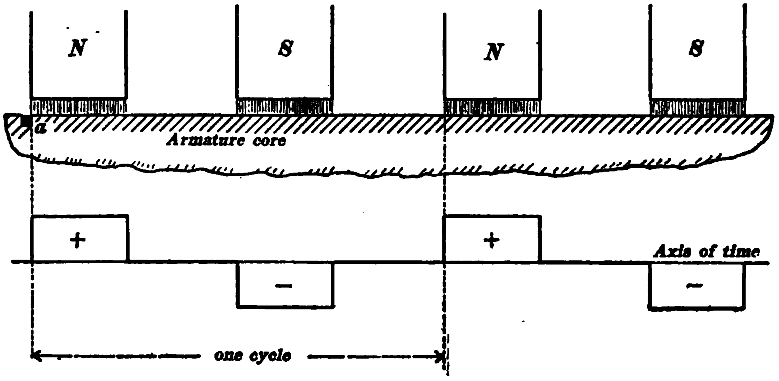

Fig. 69a.

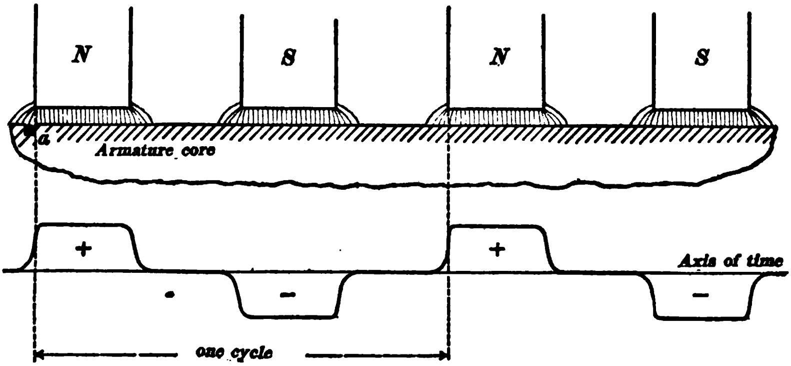

Fig. 69b.

shows the fringe of magnetic flux which always exists beyond the pole tips.

Figure 70*a* represents a band of armature wires *ab* cutting the uniformly distributed flux under the pole pieces, and the resulting electromotive-force curve is shown in the lower part of the figure. Figure 70*a* represents the magnetic flux as stopping short at the pole tips, and Fig. 70*b* shows the fringe of magnetic field which always exists beyond the pole tips. Figure 71 is a more or less fanciful drawing which is intended to show how the positive and negative sets of the electromotive-force values are unlike when the north poles of the magnet are not similar in shape to the south poles.

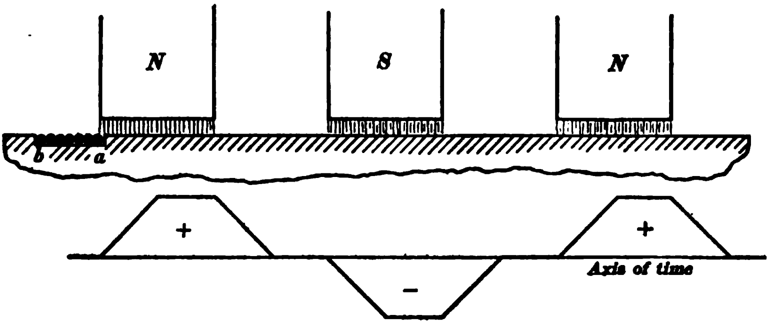

Fig. 70a.

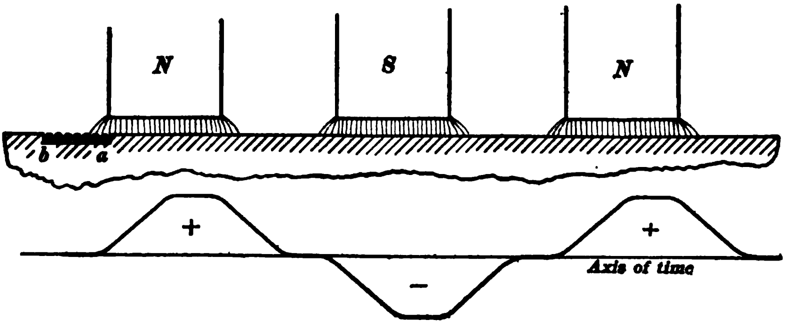

Fig. 70b.

It is important to note in connection with Figs. 69, 70 and 71 that a certain armature wire must be chosen as an index point and that, as the wire moves to the right, the ordinate of the electromotive-force curve which is directly beneath this chosen wire at a given instant represents the value of the electromotive force at that instant. Thus, the wire *a* is chosen as the index point in Fig. 70, and wherever this wire may be at a given instant the ordinate of the electromotive-force curve which is immediately beneath *a* represents the electromotive force at that instant.

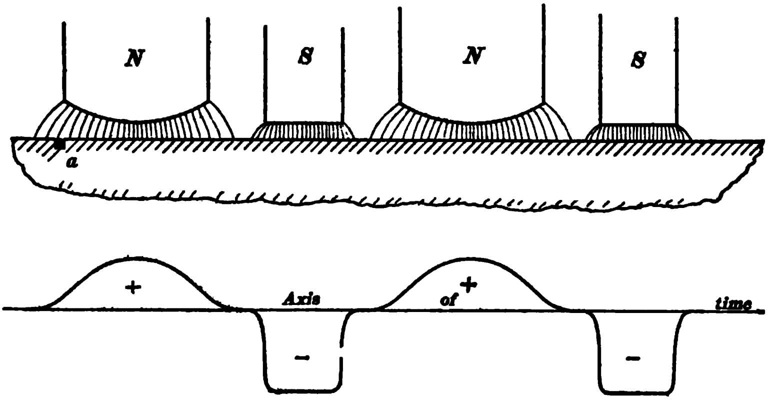

Fig. 71.

Examples of current curves. — When an alternating electromotive force acts upon a circuit which contains resistance only, that is, a circuit which has a negligibly small inductance and which does not include a condenser, the current curve is of exactly the same shape as the electromotive-force curve, inasmuch as the current is in this case equal at each instant to the electromotive force divided by the resistance of the circuit according to Ohm's law.

When an alternating-current circuit has an appreciable inductance (ironless) or when it contains a condenser, *the current curve has the same shape as the electromotive-force curve if the electromotive-force curve is a curve of sines, but not otherwise;* but the current curve is displaced (shifted along the axis of time) with reference to the electromotive-force curve. These voltage-current relations are discussed in detail in Chapter IX.

When an electromotive force or current curve is a curve of sines the electromotive force or current is said to be *harmonic*, otherwise the electromotive force or current is said to be *non-harmonic*.

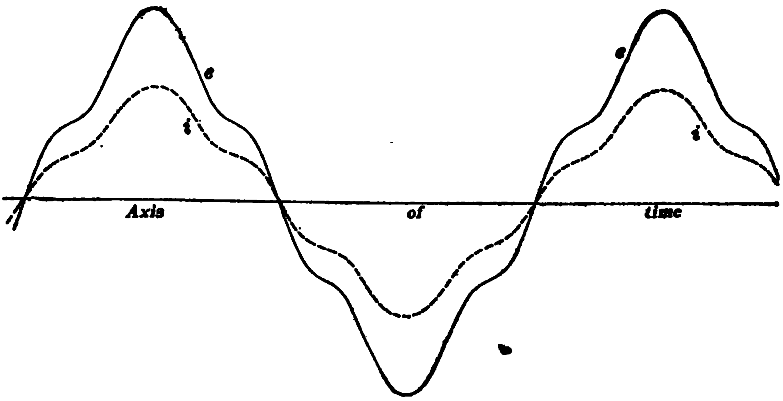

Fig. 72. Circuit containing resistance only.

The full-line curve in Fig. 72 represents a non-harmonic electromotive force, and the similarly-shaped dotted curve represents the current which this electromotive force produces in a circuit which contains resistance only. The full-line curves in Figs. 73 and 74 represent harmonic electromotive forces; the dotted curve

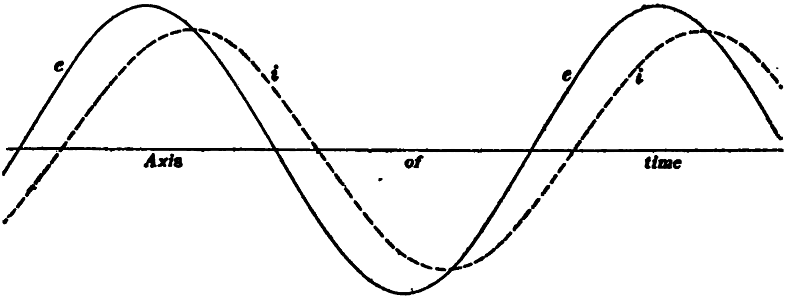

Fig. 73. Circuit having resistance and inductance.

in Fig. 73 represents the harmonic current which the electromotive force produces in a circuit having resistance and inductance; and the dotted curve in Fig. 74 represents the harmonic current which the electromotive force produces in a circuit having resistance and containing a condenser. It is to be noted that the maximum positive value of the current occurs *after* the maximum positive value of the electromotive force in Fig. 73, and *before* the maximum positive value of the electromotive force in Fig. 74. This is usually expressed by saying that the current *lags behind* the electromotive force in a circuit having resistance and inductance, and that the current *leads* or *is ahead of* the electromotive force in a circuit having resistance and containing a condenser.

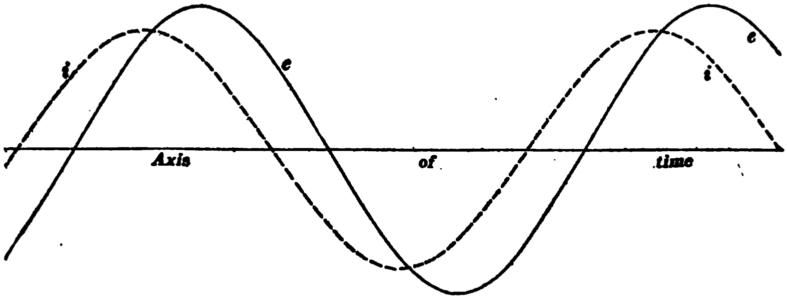

Fig. 74. Circuit having resistance and containing a condenser.

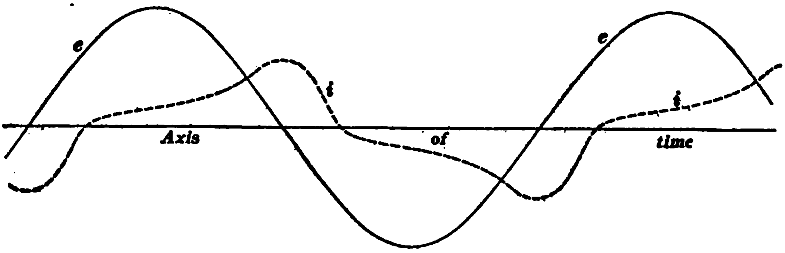

Fig. 75. Coil of wire wound on an iron core.

A harmonic electromotive force does not always produce a harmonic current. Thus, Fig. 75 shows a harmonic electromotive force *e*, and the current *i* which this electromotive force produces in a coil of wire wound on an iron core.

53. Average values and effective values of alternating electromotive force and current. — As pointed out in Art. 42, alternating-current ammeters and voltmeters always indicate effective values; therefore in speaking of an alternating electromotive force as so many volts or of an alternating current as so many amperes, effective values are always meant. Thus, an alternating current of ten amperes is an alternating current of which the average value of i^2 is 100 (amperes)^2. An alternating electromotive force of 1,000 volts is an alternating electromotive force of which the average value of e^2 is 1,000,000 (volts)^2.

The average of the successive values of an alternating electromotive force or current is always zero. This is true even in a case like that shown in Fig. 71, where the successive half-waves are unlike, and the only exception is in the case of the current which is produced by an alternating electromotive force through a circuit which allows current to pass in one direction but not in the other direction. The mercury-arc and the aluminum valve rectifiers are examples of such circuits. The simple average of the positive (or negative) values of an alternating electromotive force or current during half a cycle is of course not zero.

Example. — The successive instantaneous values of an alternating electromotive force at equal intervals during a half-cycle are: 0, 30, 60, 80, 90, 95, 90, 80, 60 and 30 volts. Dividing the sum of these voltages by their number, 10, gives 61.5 volts which is the average value of the alternating electromotive force during the half-cycle. Squaring each of these voltages, adding these squares and dividing by their number gives the average value of their squares, namely, 4,702.5 volts^2, and the square root of this average square is the effective value of the alternating electromotive force, namely, 68.6 volts.

54. Instantaneous and average power delivered by an alternator. — Let e be the value, at a given instant, of the electromotive force of an alternator and i the value at the same instant of the current delivered by the alternator; then ei is the power in watts which is delivered by the alternator at the given instant, and the average value of *ei* is the average power delivered by the alternator.

When the alternator delivers current to a circuit which has resistance only, the average value of *ei* is equal to the product of the effective value of the alternating electromotive force and the effective value of the alternating current; but, in general, the average power delivered by an alternator (average value of *ei*) is less than the product of effective voltage and effective current.

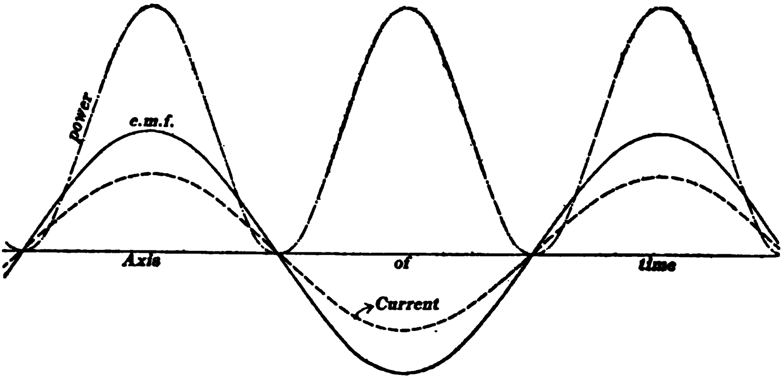

Fig. 76. Current in phase with voltage.

Examples. — The full-line curve in Fig. 76 is the electromotive-force curve of an alternator, the dotted curve is the current curve for the case in which the alternator delivers current to a circuit which has resistance only, and the ordinates of the dot-dash curve represent the successive instantaneous values of the power *ei* delivered by the alternator. In this case the values of *ei* are all positive but they vary from zero to a certain maximum value. That is to say, the alternator delivers power in pulses, one for each half-cycle of electromotive force and current.

The full-line curve in Fig. 77 is the electromotive-force curve of an alternator, the dotted curve is the current curve for the case in which the alternator delivers current to a circuit which has both resistance and inductance, and the ordinates of the dot-dash curve represent the successive instantaneous values of the power *ei* delivered by the alternator. In this case there are two pulses of power delivered by the alternator to the receiving circuit (*ei* positive), and two smaller pulses of power taken back by the alternator from the receiving circuit (*ei* negative) during each complete cycle of electromotive force and current.

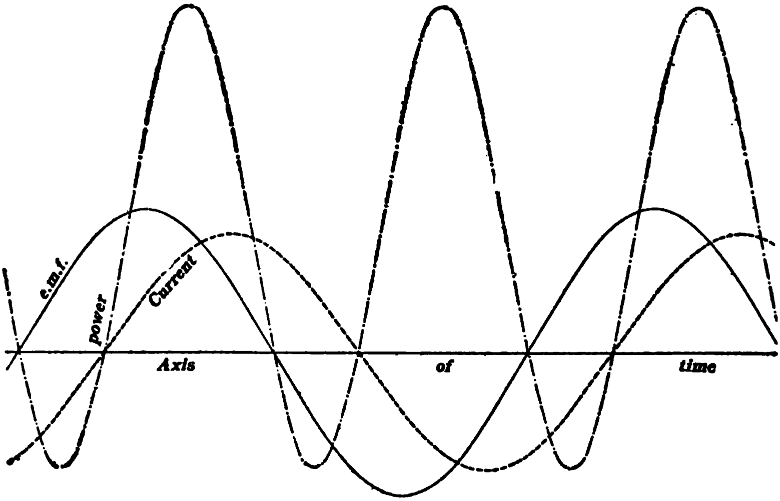

Fig. 77. Current lagging 60° behind voltage.

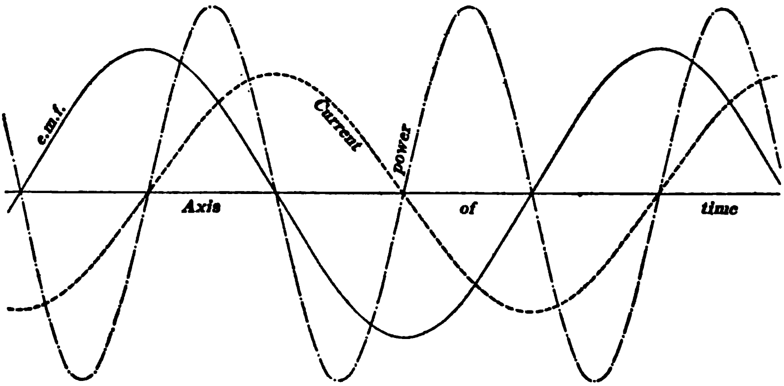

Fig. 78. Current lagging 90° behind voltage.

The full-line curve in Fig. 78 is the electromotive-force curve of an alternator, the dotted curve is the current curve for the case in which the alternator delivers current to a circuit which has inductance but no appreciable resistance, and the ordinates of the dot-dash curve represent the successive instantaneous values of *ei*. In this case there are two pulses of power delivered by the alternator and two similar pulses of power taken back by the alternator from the receiving circuit during each complete cycle. In this case, therefore, the average value of *ei* is zero. A mechanical analogue of this third case is afforded by the movement of the balance wheel of a watch, assuming it to be frictionless. The hair spring does work upon the wheel setting it in motion, and the spring takes all of the energy back again in stopping the wheel. Then work is delivered to the wheel once more in reversing its motion, and this energy is all taken back by the spring as the wheel returns to its starting point, thus completing one cycle of the motion during which two pulses of power have been delivered to the wheel by the hair spring, and two similar pulses of power have been taken back from the wheel.

PART II.

DIRECT-CURRENT GENERATORS AND MOTORS.

CHAPTER IV.

THE OPERATION OF A DIRECT-CURRENT DYNAMO AS A GENERATOR.

55. The fundamental equation of direct-current dynamo. — The equation which expresses the relation between the induced electromotive force in the armature winding of a dynamo, the amount of magnetic flux from each field pole, the number of field poles, the number of conductors on the outside of the armature, the number of current paths in the armature winding between brushes, and the speed of the armature is called the *fundamental equation* of the dynamo on account of its importance in the theory of design and operation of generators and motors.

Let Φ = magnetic flux which enters the armature from each north pole of the field magnet and leaves the armature at each south pole of the field magnet.

p = number of field poles.

Z = number of conductors on the outside of the armature.

p' = number of electrical paths in parallel between the brushes.

E_a = total electromotive force induced in the armature. This is the same as the electromotive force between the brushes, as measured by a voltmeter, when the current in the armature is negligibly small.

n = speed of armature in revolutions per second.

Then

$$E_a = \frac{p\Phi Zn}{p'} \text{ abvolts} \qquad (28a)$$

or

$$E_a = \frac{p\Phi Zn}{p' \times 10^8} \text{ volts} \qquad (28b)$$

Proof. — During $1/pn$-th of a second a given armature conductor sweeps past a field pole, from a to b, Fig. 23 or 24, and cuts Φ lines of force. This conductor therefore cuts lines of force at an average rate which is equal to $\Phi \div 1/pn$, or Φpn lines of force per second, and this is the average electromotive force in the given conductor while it is moving from a to b, Fig. 23 or 24; also this is the *average* electromotive force per conductor in all the conductors between a and b at any *instant*. Now, there are Z/p' conductors * in series in each path between the brushes, and since Φpn is the average electromotive force per conductor, therefore $Z/p' \times \Phi pn$ is the electromotive force between the brushes. That is, $E_a = p\Phi Zn/p'$, in which E_a is, of course, expressed in abvolts.

Equations (28) apply to bipolar or multipolar machines, and to machines having ring or drum armatures with any kind of winding except open-coil † windings, and these equations give the electromotive force induced in the windings between the brushes, provided the brushes are at the neutral points as shown in Fig. 23, and as explained in Art. 68.

56. Constant-voltage and constant-current supply. — Before proceeding to the discussion of the operation of the direct-current dynamo as a generator, it is necessary to consider the two conditions under which a number of distinct receiving units (lamps or motors) may be operated *independently* of each other, so that any lamp or any motor may be put into or taken out of service at will without affecting the other lamps or motors.

(*a*) When a number of lamps, or motors, or both, are connected in parallel with each other across the supply mains, it is necessary to maintain a constant electromotive force between the mains in order that the various lamps and motors may operate independently of each other. This method of supply is called *constant-voltage distribution.* In this system a given lamp or motor is put out of service by disconnecting it from the mains, that is, by simply opening its circuit. When a lamp or motor is put out of service in this way, each remaining lamp or motor takes the same current as before (voltage between mains being constant), and the generator delivers a total current which is less than before by the amount of current which was taken by the disconnected lamp or motor.

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* The conductors which constitute a given path are always distributed evenly over the space ab, Fig. 23, so that the average electromotive force per conductor for all the conductors of a path is the same as the general average for the whole armature. In the simple ring-wound armature $p = p'$, and the conductors which at any instant lie between the points a and b, Fig. 23, constitute one path.

† The essential features of the open-coil armature winding are described on page 40 of Franklin and Esty's *Elements of Electrical Engineering*, Vol. I.

(*b*) When a number of lamps are connected in series, it is necessary to maintain a constant current in the circuit in order that the various lamps may operate *independently* of each other. This method of supply is called *constant-current distribution.* In this system a lamp is put out of service, not by breaking its circuit, but by short-circuiting it, that is, by closing a switch which establishes a by-pass for the current. The closing of this switch generally reduces the resistance of the circuit of lamps as a whole, and therefore the maintenance of a constant current in the circuit requires a decrease of the electromotive force of the generator when lamps are put out of service and an increase of the electromotive force of the generator when additional lamps are put into service.

The constant-voltage system of supply is almost universally employed nowadays both for direct-current distribution and for alternating-current distribution. This system is exemplified by nearly every installation for the long distance transmission of power, by nearly every electric railway system, and by great numbers of plants for supplying current to motors and lamps in our cities.

The constant-current system of supply is advantageous when a fixed number of widely-distributed lamps, arc or incandescent, are to be operated; the lamps are connected in series and supplied with constant current. This system is exemplified by many street-lighting plants in our cities and by the Thury system * of power transmission.

57. Direct-current generators for constant-voltage supply. (*a*) *The shunt generator.*—The voltage of a shunt generator which is driven at constant speed falls off slightly with increasing current output, as shown by the curve in Fig. 79 of which the abscissas represent the current outputs of the generator and the ordinates represent the corresponding values of the voltage of the generator. Such a curve is called the *characteristic curve* of the generator. Shunt generators are very frequently used for constant-voltage supply, and when so used it is necessary for an attendant to repeatedly adjust the field rheostat of the generator so as to keep its voltage constant in spite of variations of current output.

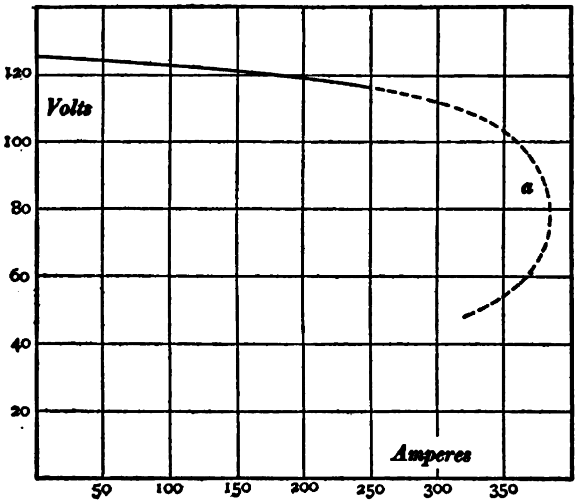

Fig. 79.

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* The Thury system (direct-current) is a constant-current system of distribution. It is exemplified by several large, long-distance, power-transmission plants in Europe. A description of the Thury system is given by Wieshofer in the *Zeitschrift für Electrotechnik* (Vienna), Vol. XVI, pages 5–10, 1898. See also a paper by Cuénod and Thury in the *Bulletin de la Sociétété Internationale des Électriciens*, Vol. XVII, pages 9–93, January, 1900.

(*b*) *The compound generator.*—The tendency of the electromotive force of a shunt generator to fall off with increase of current output may be counteracted by the use of a series field winding as explained in Art. 35. Thus, the ordinates of the curve in Fig. 80 represent the values of the electromotive force of a compound generator and the abscissas represent the values of current output. This curve shows that the compound generator is peculiarly suitable for supplying current at constant voltage.

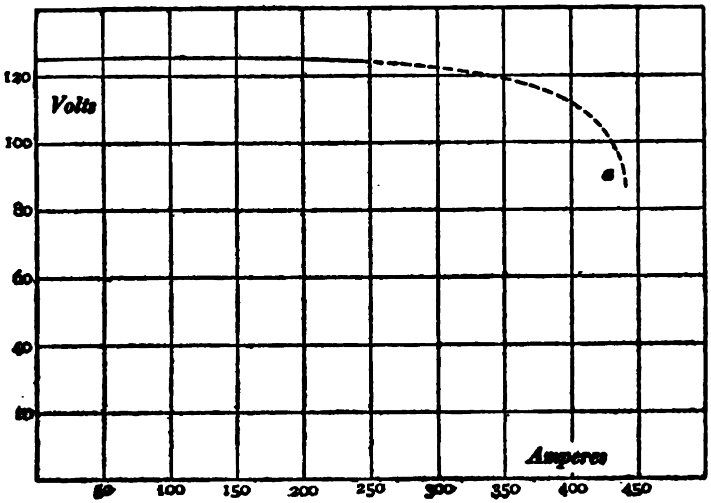

Fig. 80.

Figure 80 is the characteristic curve of what is called a *flat-compounded* generator, that is, a generator in which the zero-load and full-load voltages are equal. The series field winding of a compound generator may be arranged to actually increase the voltage of the generator with increase of current output giving what is called an *over-compounded* generator.

58. Direct-current generators for constant-current supply.—To supply automatically a constant current irrespective of changes of resistance of the receiving circuit, a generator would have to have a characteristic curve like *AB*, Fig. 81. A characteristic curve of a shunt generator is shown in Fig. 79, a characteristic curve of a compound generator is shown in Fig. 80, and a typical characteristic curve of a series generator is shown in Fig. 82. It is evident that neither type of direct-current generator is inherently suited for constant-current supply. As a matter of fact, however, the series generator is always used for constant direct current supply for the following reason :* The voltages used for series, constant-current, arc-lighting circuits are high, ranging from 1,000 to 6,000 volts per lamp circuit, and a high-voltage shunt generator is expensive to construct because of the great quantity of fine and highly in-

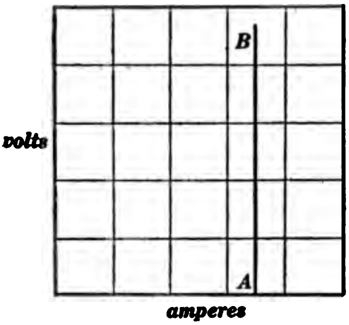

Fig. 81.

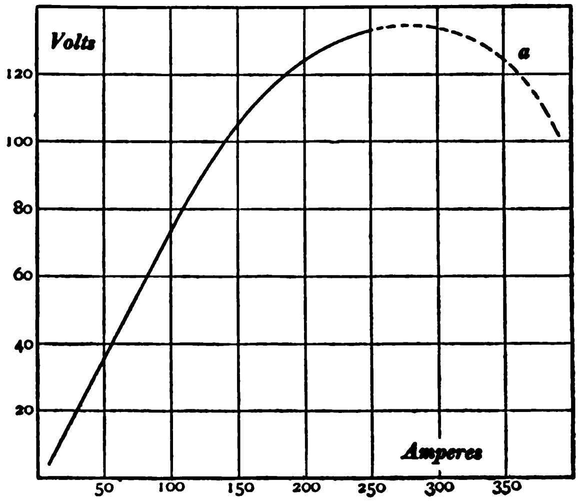

Fig. 82.

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* It is usually stated that the series generator is better suited for constant-current supply than the shunt generator on account of the approximately vertical portion of the series characteristic as shown at *a* in Fig. 82. This same property, however, is exhibited by the shunt generator and by the compound generator, as shown in Figs. 79 and 80.

sulated wire that would be required for its field winding; whereas the high-voltage series generator has comparatively coarse wire in its field winding which need not be so thoroughly insulated as a shunt winding.

A series generator which is used for constant-current distribution is always provided with an automatic regulator for increasing the voltage of the machine very considerably whenever the current output tends to decrease and for decreasing the voltage of the machine very considerably whenever the current output tends to increase.[*]

59. Connections of a shunt or compound generator and its receiving circuit. — Figure 83 is a diagram of connections of a compound generator (short-shunt) with its "field rheostat" *R*, main fuses at *F* and *F*, main switch *M*, ammeter *A*, voltmeter *V*, and circuit-breaker *BB*. These accessory appliances are always mounted in convenient positions on a panel of insulating material, slate or marble, which is called a *switchboard*.[†] The wires *a*, *b* and *c* lead from the generator to the switchboard, and the wires *d* and *e* lead from the switchboard to the street

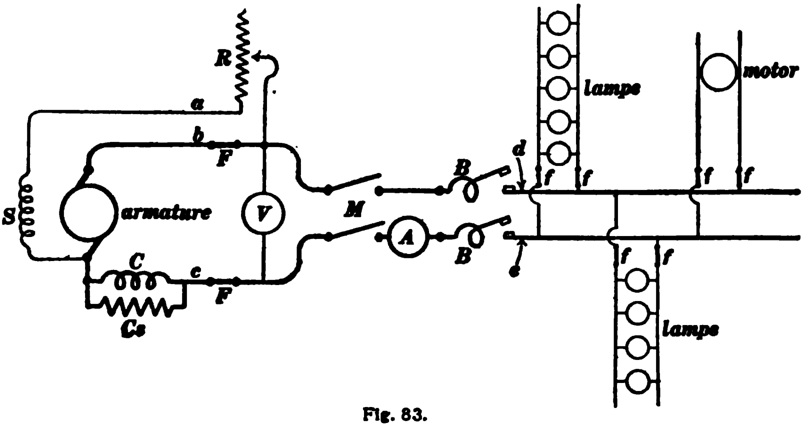

Fig. 83.

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[*] See chapter on Arc Lighting Dynamos in S. P. Thomson's *Dynamo Electric Machinery*.

[†] See Chapter XVII.

mains. The shunt field winding of the generator is shown at *S*, the series field winding at *C*, and *Cs* is an adjustable shunt by means of which a certain fraction only of the current output of the generator may be made to flow through the series field winding.

The wires leading to the individual groups of lamps and to the motor in Fig. 83 are of course smaller than the mains leading out from the switchboard. Fuse links *ff* should be inserted at every point where a smaller wire attaches to a larger one, otherwise if a short-circuit should occur in a small group of lamps, enough current might flow to make the small copper wires red-hot but not enough to open the circuit breaker *BB* or to melt the main fuses *FF*, thus giving rise to a serious fire risk.

The connections of a shunt generator are the same as Fig. 83, except that there is no series field winding.

60. Connections of a series generator and its receiving circuit. — Figure 84 is a diagram of connections of a series generator and a circuit of arc lamps. An ammeter *A* is usually placed in the circuit so that the station attendant can see when the current has the proper value. In Fig. 84 an adjustable resistance *B* is shown connected in parallel with the field winding; by varying this resistance that portion of the fixed current output of the generator which flows through the field winding may be changed at will, and thus the electromotive force of the generator may be controlled so as to cause it to deliver a constant current in spite of variations of the resistance of the receiving circuit. This hand controlling device is not used in commercial arc-lighting plants and it is only mentioned here to give a clear idea of a possible method for controlling a series generator.

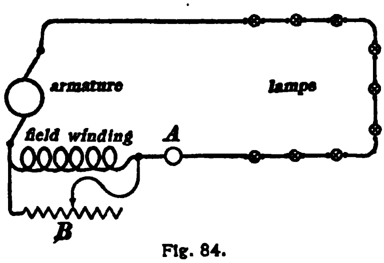

Fig. 84.

61. The building up of a generator at starting.—When a self-excited generator is started, the slight residual magnetism in the field structure induces a small electromotive force in the armature windings. This electromotive force produces a small current in the field winding which *strengthens* the residual magnetism. This strengthened residual magnetism induces an increased electromotive force, this produces an increased current in the field winding which in turn still further increases the magnetism, and so on until the machine is in full action. This process is called *building up;* it usually takes half a minute or more for this building up process to bring a machine up to its full voltage.

If a generator has no residual magnetism it cannot begin the building up process. To get such a machine into operation it is necessary to excite its field magnet temporarily, by direct current from an outside source. This generally leaves enough residual magnetism to start the building up process.

It frequently happens that the current produced in the armature by the residual magnetism flows through the field winding in such a direction as to *weaken* the residual magnetism. In this case the machine cannot build up at all. This failure to build up does not depend upon the direction of the residual magnetism; for if the field current produced by given residual magnetism strengthens (or weakens) the residual magnetism, then if the residual magnetism be reversed, it will produce a reversed current, and this reversed current will strengthen (or weaken) the reversed residual magnetism as before.

Whether the current produced by the residual magnetism flows through the field winding in the direction to strengthen or in the direction to weaken the residual magnetism, depends only upon the direction of driving of the generator, and upon which terminal of the field winding is connected to a given brush. If under given conditions, a generator does build up, it cannot build up if its direction of driving is reversed, or if its field connections are reversed; if, however, its direction of driving and its field connections are both reversed, then it will build up. If, under given conditions, a generator cannot build up, it may build up if its direction of driving is reversed, *or* if its field connections are reversed.

A generator driven in a given direction may build up so that either the one or the other of its brushes is the positive brush, according to the initial direction of the residual magnetism. A generator which is repeatedly started and stopped usually builds up each time in the same direction; but some outside disturbance, such as a momentary current from a lightning discharge, may reverse the residual magnetism of a machine; in this case the machine will build up in a reverse direction the next time it is started.

From the above description of the building up process it may perhaps seem that there is no limit to the field strength and voltage which may be reached by a given self-excited generator driven at a given speed. There is however a definite limit in every case on account of the magnetic saturation of the iron parts of the machine.

62. Output and rating of a generator.—Shunt and compound generators are used for constant-voltage supply as explained in Art. 57. When the voltage of a generator is maintained at a constant value the current delivered by the machine is inversely proportional to the resistance of the receiving circuit. When the resistance of the receiving circuit is large, the current is small, and when the resistance of the receiving circuit is small, the current is large, according to Ohm's law, and the power output is equal to the product of the voltage of the generator and the current delivered by the generator.

Power rating.—From the above discussion it may seem that any desired amount of power, however great, can be delivered by a given generator. In a certain sense this is true,* but when the current output of a generator is increased more and more, the temperature of the machine increases more and more, on account of the heat generated by the current in the windings, and the tendency to spark at the brushes increases. The deterioration of the insulating materials caused by heat sets a more or less definite limit to the permissible rise of temperature of a generator, and the smooth running and durability of the commutator depend upon sparkless commutation. Therefore, there is a practical limit to the output of a given generator beyond which either the heating or the sparking becomes excessive. This practical limit to the output of a generator determines what is called its *rated output.*

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* There is an actual limit to the possible power output of a generator because of the fact that the terminal voltage falls off (even in a series generator) when the current becomes excessive.

63. Conditions which affect the electromotive force of a generator. (*a*) **Primary conditions.** *Speed and field excitation.*—The electromotive force of a given generator depends upon its speed n, and upon the armature flux Φ according to equation (28). The armature flux depends in its turn upon the degree of excitation of the field magnet. Therefore, aside from the comparatively small disturbing action of the current in the armature, the voltage of a generator depends upon its speed and its field excitation.

The dependence of the voltage of a generator upon the value of the armature flux Φ is strikingly shown by the following experiment: Given a generator driven at a constant speed, with a voltmeter connected across its terminals, the receiving circuit being disconnected. When a heavy slab of iron is laid across the pole pieces, the voltage, as indicated by the voltmeter, drops considerably on account of the fact that a portion of the flux from the field magnet poles now passes through the iron slab instead of passing through the armature, and Φ is therefore decreased. The iron slab acts as a magnetic shunt in parallel with the armature core of the generator. When the slab is removed the voltage rises to its original value.

(*a*) *Secondary condition.* — The disturbing action of the current in the armature of a generator is comparatively small in its effect upon the voltage of the generator, and therefore it is permissible to consider the voltage of a generator as primarily dependent upon its speed and its field excitation, and as only secondarily dependent upon the armature current. The effect of armature current upon voltage is twofold. In the first place the current in the armature of a generator, under ordinary operating conditions, opposes the magnetizing action of the field windings, and thus tends to lessen the armature flux Φ, and thereby to lessen the actual electromotive force induced in the armature. In the second place a portion of the electromotive force actually induced in the armature is used to overcome the resistance of the armature (and of the series field winding through which the current flows before it reaches the terminals of the machine). Electromotive force lost in this way is called *internal drop*, and because of this internal drop, the electromotive force between the terminals of the generator is less than the electromotive force actually induced in the armature.

The effect of internal drop upon the terminal voltage of a generator may be easily formulated as follows:

Let I_a be the current flowing through the armature;
I_s the current flowing through the shunt field winding;
I_c the current flowing through the series field winding of a series or compound machine;
I_x the current flowing through the external circuit;
E_a the total electromotive force induced in the armature;
E_x the electromotive force between the terminals of a generator;
R_a the resistance of the armature, including resistance of brushes and of contacts between the brushes and the commutator;
R_s the resistance of the shunt field winding;
R_c the resistance of the series field winding; and
R_x the resistance of the external circuit.

The loss of electromotive force in the armature due to armature resistance is R_aI_a, and the loss of electromotive force in the series field winding on account of its resistance is R_cI_c, so that the total internal drop is $R_aI_a + R_cI_c$ and the relation between E_x and E_a is:

$$E_x = E_a - R_aI_a - R_cI_c \qquad (29)$$

A voltmeter connected to the terminals of a generator indicates E_x. The value of E_a cannot be directly measured by a voltmeter except when I_a and I_c are negligibly small, in which case $E_a = E_x$.

In case of the shunt generator there is no series field winding, so that R_c is zero and the term R_cI_c drops out of equation (29)

In case of the series generator $I_a = I_c = I_x$ since there is but one electric circuit.

In case of the long shunt compound generator

$$I_a = I_c = I_x + I_s$$

as is evident when we consider, referring to Fig. 47, that the whole armature current flows through the series field winding, and then divides between the shunt field winding and the external circuit.

In case of the short-shunt compound generator

$$I_x = I_c = I_a - I_s,$$

as is evident when we consider, referring to Fig. 48, that the armature current divides at the brushes, flowing partly through the shunt field winding, and partly through the series field winding and the external circuit.

64. Comparison of separately excited generator, shunt generator, compound generator and series generator in regard to variations of terminal voltage. *Separately excited generator.*—The field current of a separately excited generator is constant, so that the armature flux Φ is constant except for the comparatively slight decrease due to armature reaction when the machine is delivering current. Therefore:

(*a*) The total induced voltage E_a, of a separately excited generator falls off in proportion to the speed at which the machine is driven, and the terminal voltage E_x, which is always approximately equal to E_a, likewise falls off nearly in proportion to the speed. This is evident from the fundamental equation $E_a = \Phi Z'n$.

(*b*) The external voltage E_x, of a separately excited generator, falls off with increase of current, on account of internal drop R_aI_a, and on account of the lessening of Φ by armature reaction.

Shunt generator. — The field current of a shunt generator falls off when the terminal voltage E_x decreases, inasmuch as the shunt field windings are connected between the terminals of the machine. Therefore:

(*a*) A decrease in speed of a shunt generator causes a much greater decrease in E_a and E_x than in the case of the separately excited generator, because of the decrease of I_s which accompanies the decrease of E_x.

(*b*) An increase in current output of a shunt generator causes a much greater decrease in E_x than in the case of the separately excited generator, because of the decrease of I_s which accompanies the decrease of E_x.

The compound generator. — (*a*) When a generator has a compound field winding, the current output of the machine flows through the series field coil,* and the field excitation of the machine is therefore increased with increased current output. If the series field coil has a sufficient number of turns of wire, this increase of field excitation may not only counterbalance the demagnetizing action of the current in the armature, but it may more than counterbalance this demagnetizing action and cause Φ to actually increase with increase of current output, thus increasing the total induced electromotive force E_a.

1. This increase of E_a can be made sufficient to compensate for the internal drop $R_aI_a + R_cI_c$, thus keeping E_x approximately constant with increase of current output. In this case the machine is said to be *flat-compounded.*

2. This increase of E_a can be made more than sufficient to compensate for the internal drop. In this case the voltage between the terminals of the machine increases with increase of current output, and the machine is said to be *over-compounded.*

(*b*) The dependence of the voltage of a compound generator

*** This statement applies specifically to the short-shunt compound dynamo.**

upon speed is, for a given current output, nearly the same as in the case of the shunt generator.

Series generator. — (*a*) The voltage of a series generator is zero or nearly zero,* when the current output is zero, and it increases rapidly with the current output. This is evident when we consider that the field winding is in series with the external circuit.

(*b*) For a given current output, the total induced voltage E_a of a series generator is proportional to the speed, and the terminal voltage E_x is very nearly proportional to the speed.

Adaptability of the various methods of field excitation to special purposes. — Separate excitation is very often used in dynamo testing. When it is desired to obtain a voltage which fluctuates as little as possible with changes of speed, separate excitation is used in commercial installations.

Generators which are used for charging storage batteries, and generators which are used for electroplating, are liable to have their field magnetism reversed when the machine is stopped without disconnecting it from the external receiving circuit. In such service the reversal of field magnetism is very much less likely to occur when the generator is separately excited. Therefore separately excited generators are sometimes used for charging storage batteries, and usually for electroplating.

Direct-current generators in central stations for lighting or power are nearly always self-excited, inasmuch as modern engines and water wheels give very nearly constant-speed driving, and therefore the use of an auxiliary generator for supplying current for exciting the field magnets of the main generators is not usually warranted by the greater constancy of voltage obtainable thereby. Alternating-current generators are always separately excited, since the alternating current delivered by an alternating-current generator is not suitable for field excitation, unless rectified by a special commutator as in the "composite field" alternator.

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* Residual magnetism produces a small electromotive force even when the series field current is zero.

65. Voltage regulation of generators.—Suppose that a shunt generator (or a compound generator) is used for delivering current for "constant voltage" distribution, and suppose that the station attendant does *not* keep the voltage of the generator constant by manipulating the field rheostat as explained in the next article, then the extent to which the voltage of the generator may depart from the normal voltage becomes an important consideration. The maximum possible change of voltage under these conditions expressed as a percentage of normal full-load voltage is called the percentage regulation of the generator.

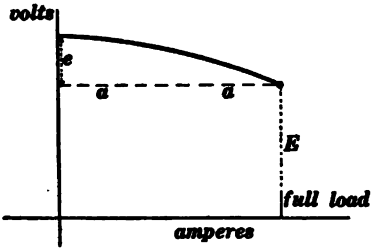

Fig. 85a.

Percentage regulation of a shunt generator.— A shunt generator operating at full load and at normal voltage has its current output reduced to zero, speed of driving and resistance of field circuit being kept constant. Under these conditions the voltage of the machine increases by the amount e as shown in Fig. 85a. The percentage regulation of the machine is found by expressing e as a percentage of the normal full-load voltage E. Thus, if the increase of voltage e is 16.5 volts and the normal full-load voltage 110 volts, then the percentage regulation is 15 per cent.

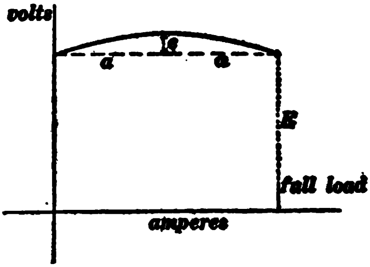

Fig. 85b.

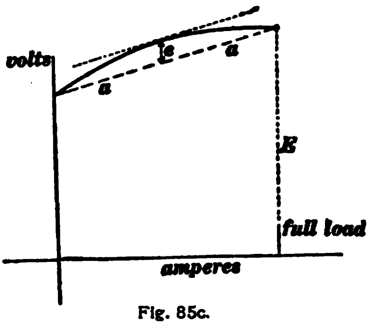

Fig. 85c.

The percentage regulation of the flat-compounded generator is equal to the maximum deviation *e* in Fig. 85*b*, expressed in per cent. of normal full-load voltage. An ideal flat-compounded generator would give a horizontal straight-line characteristic like *aa* in Fig. 85*b*, and *e* is the maximum departure of the actual characteristic from this ideal characteristic.

The percentage regulation of an over-compounded generator is equal to the maximum deviation *e* in Fig. 85*c* expressed in per cent. of the normal full-load voltage. An ideal over-compounded generator would give an inclined straight-line characteristic like *aa* in Fig. 85*c*, and *e* is the maximum departure of the actual characteristic from this ideal characteristic.

Over-compounded generators are used where current is to be delivered to lamps or motors over a long transmission line and where it is desired to maintain automatically an approximately constant voltage *at the lamps or motors* in spite of the increased loss of electromotive force in the line (*line drop*) with increase of current. For example, electric railway generators are usually over-compounded so as to give 500 volts at the generator terminals when the current output is zero, and 550 volts at the generator terminals when the current output is at the normal full-load value. In this case the machine is said to be 10 per cent. over-compounded inasmuch as the full-load voltage is 110 per cent. of the no-load voltage.

66. Control of voltage of shunt and compound generators by manipulation of field rheostat. — The term *voltage regulation* refers to inherent changes of voltage due to variations of current output of a generator. The term *voltage control* refers to the changes of voltage of a generator due to deliberate adjustments by an attendant. The voltage of a shunt or compound generator is always controlled by means of a field rheostat in circuit with the shunt field winding. Thus *R*, Fig. 83, is such a field rheostat. If it is desired to *increase* or *decrease* the voltage of a shunt or compound generator, resistance must be *taken from* or *added to* the shunt field circuit by manipulating the field rheostat, thus *increasing* or *decreasing* the current in the shunt field winding and thereby *increasing* or *decreasing* the field excitation of the machine.

The usual form of field rheostat consists of coils or zigzags of wire or metal ribbon connected in series, and arranged with a number of contact points over which a metal arm may be moved so as to include any desired portion of the wire or ribbon in the circuit. The insulation of the wire or ribbon is generally made of fire-proof material such as mica or asbestos, and special provision is made for cooling either by ventilation or by large expanse of radiating surface. A very compact form of field rheostat is that which is manufactured by the Ward Leonard Electric Company. It consists of zigzags of moderately fine resistance wire imbedded in a thin layer of fused enamel on the back of a cast-iron plate, and this cast-iron plate is deeply ribbed in front to facilitate radiation.

67. Adjustment of a compound generator. — A compound generator always has a low resistance shunt, usually made of German silver ribbon, connected across the terminals of its series field winding. Thus, Fig. 83 shows a compound generator (short-shunt) with a shunt *Cs* connected between the terminals of its series field winding *C*. A compound generator is always provided with a greater number of turns of wire in its series field winding than is strictly necessary, and by changing the resistance of the shunt *Cs* the machine may be adjusted in the factory for flat-compounding or for any desired degree of over-compounding. This adjustment should be made when the machine is at its normal running temperature.

68. Brush lead. — Figure 86 represents a two-pole generator with ring armature. The fine curved lines show the trend of the magnetic flux from the north pole of the field magnet into the armature core, and from the armature core into the south pole of the field magnet.

The turns of wire as they pass the points *a* and *b*, Fig. 86, have zero electromotive force induced in them, whereas the electromotive force induced in an individual turn of wire becomes greater and greater on either side of the points *a* and *b* up to the pole tips, and approximately equal electromotive forces are induced in all the turns which are under the pole faces. Figure 87 is a diagram, similar to Fig. 20, in which the circle represents the endless circuit formed by the windings on the ring armature. The arrows in Fig. 87 show the directions of the electromotive forces induced in the various turns of wire on the armature, and the lengths of these arrows are intended to show roughly the relative values of these induced electromotive forces.

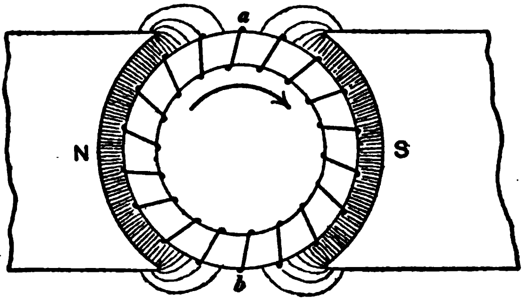

Fig. 86.

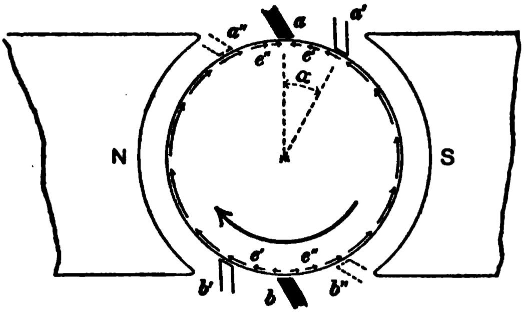

Fig. 87.

Figures 86 and 87 show the state of affairs when the current in the armature windings is small. When the armature current is large, the trend of the magnetic flux is altered from that shown in Fig. 86 to that shown in Fig. 88, and the electromotive forces in the various turns are altered from what is shown in Fig. 87 to what is shown in Fig. 89.

The line *ab*, Figs. 86, 87, 88, and 89, is called the ***neutral axis*** of the armature. The line connecting the brushes is called the *axis of commutation.** When the brushes of a generator are shifted by moving the rocker arm, the electromotive force between the brushes, as indicated by a voltmeter, reaches a maximum value when the axis of commutation coincides with the neutral axis. In fact the whole of the electromotive force E_a, which is induced in each half of the armature, is available between the brushes when the brushes are at the points *a* and *b*, Figs.

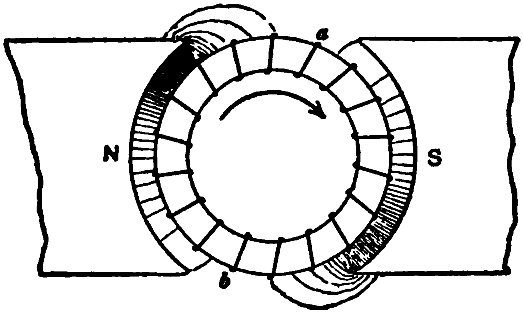

Fig. 88.

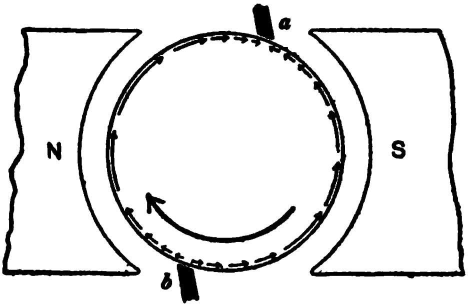

Fig. 89.

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* The leads which connect the armature conductors to the commutator bars are supposed to be radial, for the sake of simplicity of statement. In commercial machines these leads are nearly always curved like segments of a spiral.

87 and 89. When the brushes are shifted from the neutral axis to the points a' and b', Fig. 87, for instance, the available electromotive force between them is reduced in value to $(E_a - 2e')$, where e' is the electromotive force induced in the portion of the armature winding between a and a', or in the portion between b and b'. The fundamental equation of the dynamo, equation (28), was derived on the assumption that the brushes stand at the points a and b, Figs. 86 and 88.

The angle between the neutral axis and the axis of commutation is called the *angle of lead* of the brushes. When the shift of the brushes from the neutral axis is in the direction of rotation of the armature, the brushes are said to have a *forward lead*, and when the shift of the brushes is in the opposite direction, the brushes are said to have a *backward lead.* Thus the angle α, Fig. 87, is the angle of forward lead of the brushes when they are at the points a' and b'.

The sparking at the brushes of a generator is a minimum when the brushes have a slight forward lead, the amount of lead depending upon the current output. The sparking at the brushes of a motor is a minimum when the brushes have a slight backward lead, the amount of lead depending upon the current intake of the motor, that is to say, upon the motor load. A well-designed dynamo, however, will operate satisfactorily from zero load to full load without any change in the position of the brushes.*

69. Directions for starting and stopping a single shunt or compound generator. — The arrangement of a single shunt or compound generator for supplying current to lamps or motors is shown in Fig. 83. In the starting of a single-generator plant like this, the procedure is simple, there being no necessity for the special precautions required in the starting and stopping of one of a set of generators in parallel, as pointed out in a subsequent article. To start the generator shown in Fig. 83 open the main switch, if it is not already open, turn the hand wheel of field rheostat so as to include all of its resistance in the field circuit, start the engine and generator and bring them slowly up to full speed, close the main switch, and then cut out resistance from the field rheostat until the generator builds up to the exact voltage desired. Raising the voltage above the proper steady value is likely to damage the incandescent lamps, and is to be carefully avoided. To stop the generator, turn the hand wheel of the field rheostat so as to include all of the resistance of the rheostat in the field circuit, open the main switch, and stop the engine. In case of an overload, due to short circuit out on the mains or to the connecting of too many lamps or motors, the circuit breaker will open the circuit. Before closing the circuit breaker again, open the main switch, then close the circuit breaker, and finally close the main switch; by proceeding in this way the circuit breaker will operate instantly if the short-circuit on the line still exists.

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* Armature reaction and the phenomena associated with commutation are discussed in Chapter VII.

70. The operation of generators in parallel.—A generator operates at its maximum efficiency at or near full load. Therefore when the total output of a station varies from hour to hour, as it always does in electric-lighting and street-railway service, it is desirable to use several generators which may be put into service one after the other as the station output increases, and disconnected one after the other as the station output decreases; the object being to maintain nearly full load at all times upon the generators which are in service. This is especially desirable when each generator is driven by a separate engine, inasmuch as the engines also may then be operated under the most economical conditions. This use of a number of generating units in a station is also advantageous in that a spare unit (engine and generator) may be installed at a moderate cost to serve in case of a breakdown, whereas a single-unit station would have to be completely duplicated in order to provide for such an emergency.

When a number of shunt or compound generators are installed in a station as above explained, they are always operated in parallel.* Series generators are seldom used in combination, but when they are so used they are always operated in series.

The proper division of the total load among a number of shunt generators connected in parallel depends simply upon the adjustment of their respective shunt-field rheostats. On the other hand, a number of compound generators connected in parallel as shown in Fig. 90 constitute an unstable system. To obviate this instability and ensure the proper distribution of load among a number of compound generators operating in parallel, the so-called *equalizing arrangement of connections* is used. This equalizing arrangement consists simply of the connection of the series field coils of all the generators in parallel with each other, independently of the parallel connections of the generator armatures, as shown in Fig. 91. Two distinct conditions must be satisfied by the arrangement

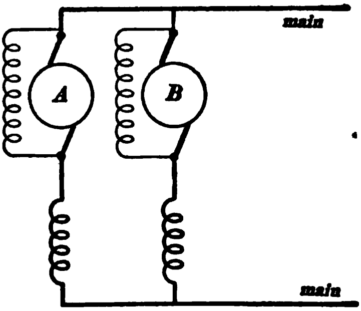

Fig. 90. (Unstable.)

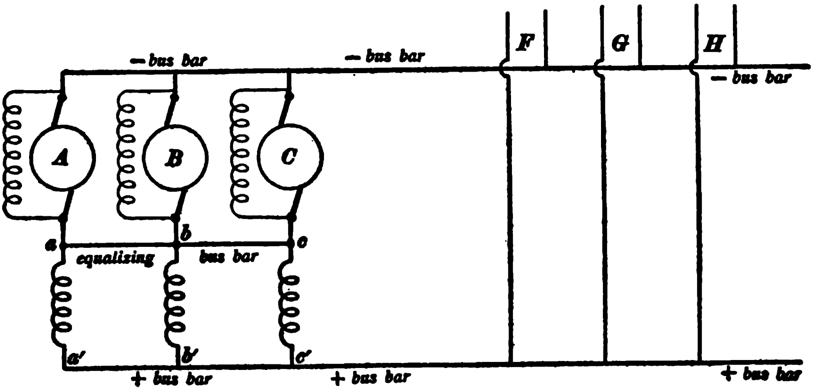

Fig. 91. (Stable.)

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* When the equipment of a power station consists of a number of alternating-current generators, the generators are nearly always operated in parallel. See Chapter XII.

shown in Fig. 91 to ensure the proper division of the load among the several generators, namely:

(*a*) Each compound generator must be separately adjusted to give the same degree of compounding, by means of a German silver shunt *Cs* (not shown in Fig. 91) connected in parallel with its series field coil as explained in Art. 67.

(*b*) The total resistances aa', bb' and cc', Fig. 91, must be inversely proportional to the full-load current outputs of the respective generators. This ensures the proper division of the total current among the several series-field coils.

The use of the equalizing arrangement makes it possible to operate in parallel compound generators which are very different in size and in design.

The instability of two or more compound generators connected in parallel may be shown as follows: Consider two over-compounded generators connected in parallel as shown in Fig. 90. Suppose that the machines A and B are running steadily and that each is giving half of the total current that is being delivered to the mains. If machine A then runs momentarily at a slightly increased speed a momentary increase of its induced electromotive force is produced, which causes an increase in the current which it delivers, and a corresponding decrease in the current delivered by B.* The increase of load current in machine A causes a further increase of the induced voltage of this machine, inasmuch as it is understood to be over-compounded, and this increase of induced electromotive force causes a further increase of load current in A. At the same time the lessened current in B causes a decrease of its induced electromotive force which tends to decrease its current still more, and so on. This unequal distribution of load between A and B when once started goes to an extreme and leads to an excessive load current in machine A (or

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* This is due to the fact that the two machines form together a closed circuit of very low resistance, and a slight difference in the electromotive forces of the two machines causes a large current to flow around this circuit adding to the load current of one machine and subtracting from the load current of the other.

B) and a negative load current in machine *B* (or *A*), that is one machine becomes a motor and takes power from the other.

Never open the shunt field switch of a generator which is operating in parallel with others. To do this makes the generator armature a short-circuit for the other machines and the armature is likely to be burned out.

Directions for starting and stopping shunt or compound generators which operate in parallel are given in Chapter XVII.

CHAPTER V.

OPERATION OF THE DIRECT-CURRENT DYNAMO AS A MOTOR.

71. Conditions which determine the direction of running of a motor. — A self-exciting dynamo, with given connections of field winding to brushes, must be driven in a certain direction in order that it may build up as a generator, as explained in Art. 61. The conditions which determine the direction in which a dynamo must be driven in order that it may build up as a generator also determine the direction in which a machine will run as a motor when it is supplied with current.

The shunt dynamo, with given connections of field to brushes, runs as a motor in the same direction in which it must be driven in order to build up as a generator. This may be made evident as follows: The heavy arrow in Fig. 92 shows the direction of the electromotive force and current in the armature of a shunt generator, and the dotted arrows show the direction of the currents in field and in mains; the heavy arrows in Fig. 93 show the direction of the current delivered to a shunt motor, and the dotted arrows show the direction of the currents in field and armature. Figures * 92 and 93 show at a glance that the current in the armature of a shunt machine is reversed with respect to the

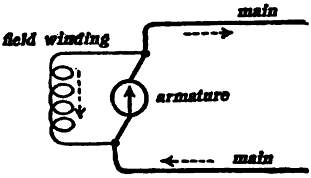

Fig. 92. Shunt Generator.

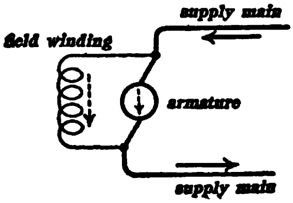

Fig. 93. Shunt Motor.

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[*] There are four possible combinations of arrows in Figs. 92 and 93, and each combination has the property here specified.

field current, when the function of the machine is changed from generator to motor, so that the mechanical force with which the field acts upon the armature is reversed; but the force action opposes the motion of the generator, and helps the motion of the motor. Therefore since the force action is reversed, the motion must be in the same direction, whether the machine operates as generator or as motor.

The series dynamo, with given connections of field to brushes, runs as a motor in a direction opposite to that in which the machine must be driven to enable it to build up as a generator. This is evident from Figs. 94 and 95 which show that, with given connections of field to brushes, the relative direction of current in armature and in field is the same, whether the machine is operating as a generator or as a motor. Therefore the mechanical force with which the field acts upon the armature is unchanged in direction when the function of the machine is changed from generator to motor; but the force action opposes the motion of the generator and helps the motion of the motor. Therefore the motion must be reversed, since the force action is unchanged.

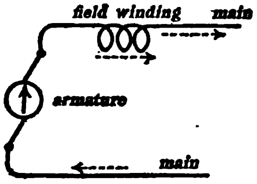

Fig. 94. Series Generator.

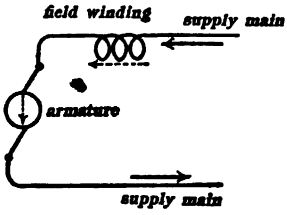

Fig. 95. Series Motor.

Direction of running of a motor. — The direction of running of a motor, with given connections of field winding to brushes, is the same, irrespective of the direction of the current supplied to the machine. This is evident when we consider (*a*) that the reversal of the supply current, which is represented by the heavy arrows in Figs. 93 and 95, reverses the current both in the armature and in the field, and (*b*) that the reversed field of course exerts a force upon the reversed armature currents in the same direction as before, thus leaving the direction of rotation unchanged. To reverse the direction of running of a motor, the connections must be changed so that the current through armature (or field) is reversed while the direction of the current in the field (or armature) remains unchanged. This requires the reversal of the armature connections, or a reversal of the field connections, but not of both.

72. Fundamental equation of the motor. — In the generator the *voltage* is determined by the fundamental equations (28) and (29), inasmuch as all the other quantities in these equations are fixed in value by the conditions under which the generator is used. In the case of the motor, on the other hand, the *speed* is determined by these fundamental equations, inasmuch as all the other quantities in these equations are, as a rule, fixed in value by the conditions under which the motor is used. The equation which expresses the speed of the motor in terms of the other quantities which appear in equations (28) and (29) is called the *fundamental equation of the motor.*

Let E_x be the electromotive force applied at the terminals of a motor. This is called the *impressed electromotive force.* This electromotive force is partly used to overcome the resistance R_a of the armature. The part so used is equal to R_aI_a, where I_a is the current flowing through the armature. The remainder of the impressed electromotive force balances the electromotive force, E_a $(= \Phi Z'n)$ which is induced in the rotating armature by the field flux, and it is numerically equal thereto. Therefore

$$E_x = \Phi Z'n + R_aI_a \tag{30a}$$

in which E_x is the impressed electromotive force acting on a motor, R_a is the armature resistance,* I_a is the current flowing through the armature, Φ is the magnetic flux entering the armature from one field pole, n is the speed of the motor in revolutions per second, and Z' is written for the factor $pZ/(p' \times 10^8)$, in equation (28).

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* In the long-shunt compound motor the combined resistance of armature and series field coil, $R_a + R_c$, is to be used for R_a. It is hardly necessary to formulate the very slightly different behavior of the short-shunt compound motor.

The electromotive force $\Phi Z'n$, which is induced in the rotating armature of a motor, is opposite in direction to the impressed electromotive force, E_x and to the current, I_a. Therefore $\Phi Z'n$ is called the *counter electromotive force* of the motor.

The most convenient form of equation (30*a*) is that which gives speed in terms of the other quantities. Solving equation (30*a*) for speed, we have

$$n = \frac{E_x - R_a I_a}{\Phi Z'} \tag{30b}$$

Another useful form of this equation is that which expresses I_a in terms of the other quantities, namely,

$$I_a = \frac{E_x - \Phi Z'n}{R_a} \tag{30c}$$

From this equation it is evident that the current I_a is not in general equal to E_x divided by R_a, inasmuch as only a small part of E_x is used to overcome resistance. When the motor is not running, that is, when $n = 0$, then I_a is equal to E_x divided by R_a, and the value of I_a is excessively large. To avoid this excessive flow of current when n is small, an auxiliary resistance must be connected in series with the armature as explained in Arts. 78 and 80.

Torque. — Another equation of considerable importance in the discussion of the motor is the expression for torque. The value of the torque with which the field magnet acts upon the armature, may be derived by considering the side push on each armature wire, multiplying each side push by its lever arm, namely the radius of the armature, and adding these results together. This method of derivation, however, repeats many of the essential considerations involved in the derivation of the fundamental equation of the dynamo as given in Art. 55. It is simpler to derive the expression for torque directly from the fundamental equation (30) with the help of Lenz's law as follows:

The rate at which work is done, in watts, in forcing the current I_a through the armature, in opposition to the counter electromotive force $\Phi Z'n$, is equal to the product $I_a \times \Phi Z'n$; and, according to Lenz's law, it is equal to the mechanical power developed by the torque T with which the field magnet acts upon the rotating armature. Now, the mechanical power in watts developed by a torque T (in pound-inches) acting upon a body making n revolutions per second is

$$P = \frac{2\pi \times \overset{\text{ratio}\frac{\text{cm}}{\text{in.}}}{2.54} \times \overset{\text{ratio}\frac{\text{grams}}{\text{pounds}}}{453.6} \times \overset{\text{accel. gravity}}{980}}{10^7} \cdot nT \text{ watts}$$

or

$$P = 0.71nT \text{ watts}$$

Therefore by Lenz's law we have

$$I_a \times \Phi Z'n = 0.71nT$$

whence

$$T = \left\{\begin{array}{l}\text{constant} \times \Phi\, I_{\text{armature}} \\ 1.41\Phi Z' I_a \text{ pound-inches}\end{array}\right. \qquad (31)$$

in which $Z' = pZ/(p' \times 10^8)$, I_a is the armature current in amperes, Φ is the magnetic flux entering the armature from one field magnet pole, p is the number of magnet poles, p' is the number of paths in parallel through the armature between brushes, and Z is the total number of conductors on the outside of the armature.

The net effective torque at the pulley of a motor is less than T in equation (31) by the amount of the torque required to overcome friction and other so-called "stray power" losses.

73. Siemens' law of efficiency. — The dependence of the efficiency of a motor upon the conditions of driving is complicated by the loss of power in field excitation, by friction, and by eddy currents and hysteresis. This matter is fully discussed in Chapter VI. A very simple expression for the efficiency of a motor, due to Siemens, is especially useful for purposes of general discussion. The formula is only approximate, however, inasmuch as it is based on the assumption that the only loss of power in the motor is that which is expended in heating the armature windings by the armature current; that is to say, field loss and friction losses are ignored. The total power delivered to a motor, neglecting field loss, is equal to $E_x I_a$, where E_x is the impressed voltage, and I_a is the current flowing through the armature. Part of this power is consumed in heating the armature windings, and part is used in forcing the current through the armature in opposition to the electromotive force E_a which is induced in the armature windings by the motion of the armature. This latter part is equal to $E_a I_a$ and it is, according to Lenz's law, all converted into the mechanical power which turns the armature.

Therefore, ignoring the power used for field excitation, the intake of electrical power by the motor is $E_x I_a$; and, ignoring the losses of mechanical power in the motor, by bearing friction, air friction, and magnetic friction,* the output of mechanical power is $E_a I_a$, so that the approximate efficiency is $E_a I_a \div E_x I_a$ or E_a / E_x. This expression for efficiency, known as Siemens' law, always gives an overestimate of the efficiency of a motor; and when a motor is running under a small load the ratio E_a / E_x may be many times as great as the actual efficiency.

74. Constant-voltage and constant-current motor driving.—The discussion of the electromagnetic behavior of a generator, that is, the discussion of the variation of its voltage with its current output, is impossible unless the mechanical conditions of driving are specified. In fact, constant-speed driving is nearly always taken as the basis for the discussion of generator characteristics.

Similarly, the discussion of the mechanical behavior of a motor, that is, the discussion of the variation of its speed with its power output or with its torque, is impossible unless the electrical conditions of driving are specified. Usually the discussion of the mechanical characteristics of a motor is based upon the assumption, either of a constant voltage between the motor terminals or of a constant current flow through the motor. In practice motors are always used under conditions which approximate to constant-voltage driving; constant-current driving is never used. Therefore, the following discussion of the behavior of motors is based upon the assumption of constant, or approximately constant, voltage between the supply mains. In this discussion the shunt motor is considered first and in detail; the compound motor is treated as a modification of the shunt motor; and the series motor is considered last.

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* The effect of eddy currents and hysteresis in a dynamo armature is to create a drag which opposes the motion of the armature, very much like ordinary mechanical friction.

When a motor is delivering mechanical power, it is said to be *loaded*, and its output of mechanical power is called its *load*. The rated *full load* of a motor is the load for which the armature and field currents are as large as permissible without excessive rise of temperature in any of the motor parts, or excessive sparking.

75. The shunt motor. — The field winding of a shunt motor consists of many turns of fine wire and it is connected in parallel with the armature to the supply mains. The essential features of the connections of a shunt motor to constant-voltage supply mains are shown in Fig. 96. *It is to be particularly noted that Fig. 96 shows the connections of the motor after it has been started and brought up to full speed. Special connections are made at starting as explained in Art. 78.* The shunt field coils are usually connected directly to the supply mains without including a field rheostat. The field current is therefore constant, and the only variation of the armature flux Φ is the slight variation which is due to the demagnetizing action of the armature current.

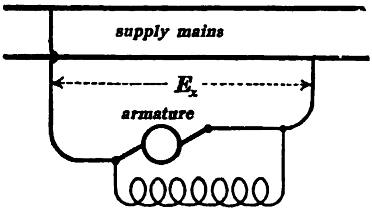

Fig. 96.

Shunt motor unloaded. — When a motor is unloaded the torque necessary to drive it is small, and therefore the armature current is small, according to equation (31). Consequently the part,

R_aI_a, of the impressed voltage which is used to overcome the resistance of the armature, is *negligibly small*, since both I_a and R_a are small. Therefore, neglecting R_aI_a, equation (30*b*) may be written

$$n = \frac{E_x}{\Phi Z'} \text{ at zero load} \tag{32}$$

Inasmuch as $\Phi Z'n$ is the counter-electromotive force of the motor, we see from equation (32) that a shunt motor at zero load runs at a speed such as to make its counter-electromotive force E_a sensibly equal to the impressed electromotive force E_x.

Shunt motor loaded. — When a load is thrown on a shunt motor which has been previously running at zero load, the tendency is to lower the speed slightly and thereby to decrease the counter electromotive force $\Phi Z'n$. As the counter electromotive force decreases, the armature current increases, according to equation (30*c*), and the torque increases according to equation (31), thus enabling the motor to carry its load. The drop in speed of a shunt motor, from zero load to full load, ranges from 2 per cent. of zero-load speed for large-sized motors, to 10 per cent. or more for small motors.

Tendency, in a shunt motor, for the demagnetizing action of the armature current to counteract the tendency of the speed to decrease with increase of load.— When a shunt motor is loaded it is necessary for the counter-electromotive force to decrease sufficiently to permit enough current to flow through the armature to develop the torque required to carry the load. Now, if Φ were invariable, the necessary decrease of counter-electromotive force could be produced only by a drop in speed, but as a matter of fact the flux Φ is decreased slightly by the demagnetizing action of the current in the armature, and consequently the necessary decrease of counter-electromotive force $\Phi Z'n$ is brought about in part by this decrease of Φ, and in part by a decrease in speed, so that the actual decrease in speed is less than it would be if Φ were invariable.

The dependence of the speed of a shunt motor upon the value of the armature flux Φ is strikingly shown by the following experiment: A shunt motor is supplied with current from constant-voltage supply mains and runs at a certain speed, as indicated by a tachometer. A slab of iron is then laid across the pole pieces, a considerable portion of the flux Φ passes through the slab, so that the armature flux is consequently reduced, and the speed of the motor immediately increases as indicated by the tachometer. This rise in speed is due to the fact that the reduction of Φ reduces the counter-electromotive force $\Phi Z'n$, of the motor, which results in an increase of current through the armature, according to equation (30*c*). This increased current produces more torque than is required to carry the load on the motor, so that the motor speeds up until the increase of its counter-electromotive force reduces the current to the value corresponding to the required torque.

An interesting and important fact in connection with a shunt motor is that such a motor runs at a lower speed when first started than after it has been running for some time. This is due to the fact that the rise of temperature of its field coils increases their resistance, decreases the field current, and decreases Φ. This rise of speed often amounts to as much as 5 per cent. in commercial machines.

Dependence of speed upon brush lead. — It was pointed out in Art. 68 that the electromotive force between the brushes of a generator running at a given speed with given field excitation, is reduced in value by shifting the brushes from the neutral axis, and that the electromotive force is a maximum when the brush lead is zero. In the case of a shunt motor supplied from constant-voltage mains, the speed is a minimum when the brushes are in the neutral axis, and the effect of shifting the brushes either forwards or backwards, is to increase the speed. This effect is particularly noticeable at zero load. When the motor is heavily loaded the matter is greatly complicated by the magnetizing or demagnetizing action of the armature current.

The explanation of this variation of speed of a shunt motor with brush lead is as follows: When the brushes are shifted from the neutral axis, the net counter-electromotive force between the brushes is decreased, speed remaining unchanged. As a result, the impressed voltage E_x forces an increased current through the armature, thus producing an increased torque which causes the motor speed to increase until the counter-electromotive force reaches a value sufficiently large to reduce the current to the value required to supply the necessary driving torque.

76. Speed regulation of the shunt motor. — A motor which is designed for supplying mechanical power at constant speed is called a "constant speed" motor. Thus a shunt motor is called a "constant speed" motor, although the maintenance of a strictly constant speed by such a motor can be accomplished only by the manipulation of a rheostat. The range of variation of speed of a "constant speed" motor with change of load, when the rheostat is left wholly unchanged, is an important practical matter. *The change of speed from full load to no load expressed as a percentage of full load speed, is called the "speed regulation" of the motor,* voltage of supply, and resistances of armature circuit and of field circuit being kept constant. Thus, if the speed of a motor rises from 1,000 revolutions per minute at full load to 1,050 revolutions per minute at no load, its speed regulation is 5 per cent.

It is to be noted that the word *regulation* is applied to changes of speed inherent in the machine itself, while the word *control* is applied to changes of speed due to deliberate adjustments by an attendant.

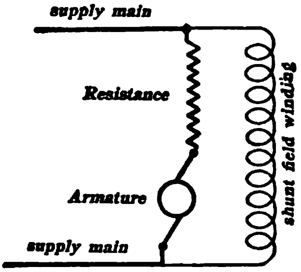

Fig. 97.

Influence of armature resistance upon the speed regulation of a shunt motor. — When the resistance of the armature of a shunt motor is large, or when a considerable resistance is connected in series with the armature as shown in Fig. 97, then the speed of the motor falls off greatly with increase of load. The cause of this great drop in speed under load may be made evident from equations (30*c*) and (31) as follows: The very considerable current which is necessary to develop torque sufficient to enable the motor to carry a load, demands a large value in the numerator of equation (30*c*) when R_a is large, and this large value of $E_x - \Phi Z'n$ necessitates a great drop in speed in order that $\Phi Z'n$ may be considerably less than E_x. It is to be especially noted, however, that the zero-load speed of a shunt motor is not perceptibly affected by a moderately large value of R_a, inasmuch as I_a is very small at zero load. It is for this reason that the speed of a shunt motor (with rapidly varying load) cannot be satisfactorily controlled by inserting resistance in series with the armature. When the speed is cut down in this way while the motor is loaded, the speed increases greatly when more or less of the load is thrown off.

77. Speed control of the shunt motor. — The peculiar and valuable properties of the shunt (and compound) motor are: (*a*) that it runs at a definite speed at zero load, that is to say, it has no tendency to race when the load is thrown off; and (*b*) that its speed drops but little below zero-load speed even when it is fully loaded. For many purposes, however, for example, in the driving of lathes by motors, it is desirable to run the motor fast or slow at will. This speed control is accomplished in the case of the shunt motor by five different methods, as follows:

(1) *Armature-rheostat method.* — When a load on a motor is constant, its speed may be controlled by inserting resistance in series with its armature, as explained in Art. 76.

(2) *Field-rheostat method.* — From equation (32) it is evident that the zero-load speed (and consequently the full-load speed to nearly the same extent) may be raised by decreasing Φ, or lowered by increasing Φ. These changes of Φ may be brought about by manipulating a rheostat in circuit with the field winding of the motor, increasing or decreasing the resistance of the shunt field circuit and thus decreasing or increasing the field current, and thereby decreasing or increasing Φ.

This method of speed control is limited as follows: (*a*) It is not feasible to increase Φ much beyond a certain value on account of what is called the ***magnetic saturation*** of the iron through which this flux Φ passes. To increase Φ to this saturation value requires an excessive value of field current and may involve excessive heating of the field magnet coils. (*b*) When Φ is decreased more and more, the operation of the motor becomes unsatisfactory; the torque that can be developed by the machine [see equation (31)] becomes very small when Φ is small, and the magnetizing action of the current in the armature begins to preponderate over the weakened magnetizing action of the field windings, and serious sparking at the brushes results. Experience shows that the maximum practicable range of speed control of the shunt motor by means of the field rheostat is about as follows: A shunt motor which runs at speed n when its field magnet is near saturation (largest feasible value of Φ) will operate more or less satisfactorily down to a value of Φ equal to about half the saturation value, and the corresponding speed is of course $2n$.

The operation of a shunt motor at low degrees of field excitation (small values of Φ) is greatly improved by the use of what are called *inter-poles*, and the so-called inter-pole motor is generally used where it is desired to obtain the widest possible range of speed control of a shunt motor by the field rheostat method. The inter-pole motor is described in Art. 97.

(3) *Method by altering the reluctance of the magnetic circuit.* — If the reluctance of the magnetic circuit of a shunt motor is increased, the flux Φ is decreased, the field excitation being constant. The speed of the motor will, therefore, be increased as explained above. If the increase of reluctance is produced by widening the gap space, then the sparking difficulties which usually accompany a decrease of Φ are largely obviated by the increased width of the gap space (see chapter VII).

(4) *Multivoltage speed control.* — From equation (32) it is evident that the zero-load speed (and consequently the full-load

speed to nearly the same extent) may be made high or low by supplying current to the armature at high or low voltage E_x, the field flux Φ being kept constant. The range of speed control of a shunt motor by this method is limited only by the speed the motor can stand mechanically, and by the safe limit of impressed voltage E_x (excessive voltage may break through the insulation of the motor windings or cause an arc to form from brush to brush). In carrying out this method of speed control, current is supplied over a number of mains, for example, *A*, *B*, *C* and *D*, Fig. 98, between which constant electromotive forces of any chosen values are maintained by a set of generators. The field winding of the shunt motor is permanently connected to one pair of these supply mains, and arrangements are made, by means of a suitable "controller," for easily and quickly connecting the motor armature to any pair of mains at will.

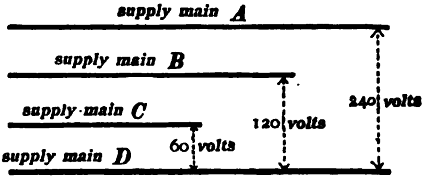

FIG. 98.

This multivoltage method of speed control gives a series of distinct and widely separate speeds. For example, with the voltages indicated in Fig. 98 the available speeds would be n, $2n$ and $4n$, where n is the speed corresponding to $E_x = 60$ volts. The multivoltage method is usually combined with the field rheostat method, so that by the latter method the intermediate speeds may be obtained.

(5) *The Ward Leonard method of speed control.*—A wide range of speed control of a shunt motor by means of a field rheostat cannot be realized, inasmuch as very low speeds cannot be reached because of the saturation of the iron parts of the motor as explained above, and inasmuch as very high speeds are not practicable because of the trouble from sparking when the field excitation of the motor is very small; the impressed voltage E_x being constant in value. Now, on the other hand, a generator does

not tend to spark badly when run at low field excitation, because when the field excitation is low, the voltage between commutator segments is low also. Therefore a given generator may be satisfactorily used to give a range of values of electromotive force from zero, when its field excitation is zero, to a maximum value, when its field magnets are saturated. The Ward Leonard method of speed control makes use of this property of a generator as follows: An auxiliary constant-speed motor takes current from the supply mains and drives a generator G at constant speed. The field winding of this generator G is provided with a field rheostat which has a sufficient range of resistance to change the field excitation of the generator from its full value to nearly zero. The motor which is to be driven at variable speed has its field connected to the supply mains so that its field excitation is maintained at a constant value, while its armature is supplied with current from the generator G. With this arrangement it is practicable to supply the variable speed motor with current at any desired voltage from zero up to the greatest voltage obtainable from the generator G.

Comparison of methods of speed control. — The armature-rheostat method of speed control is fairly satisfactory when the motor load does not vary rapidly. The disadvantage of this method, aside from the great fluctuation of speed with load, is that the use of a rheostat in series with the armature involves a very considerable waste of power due to the I^2R loss in the rheostat. The advantages of this method are: that it is simple and cheap, that it can be arranged to give fine gradations of speed control by having many resistance steps in the rheostat, and that it permits of a speed control ranging from full speed to zero speed, provided the motor load is fairly constant.

The control of speed by a field rheostat does not involve much waste of power, inasmuch as the field current is very small in comparison with the armature current of a shunt motor. Additional advantages of this method are: simplicity and cheapness, the absence of great fluctuations of speed with load, and the possibility of fine gradations of speed control when a finely subdivided rheostat is used. The disadvantage of this method is that it has a limited range, as already explained.

The control of speed by the change-of-reluctance method possesses all of the advantages of the field rheostat method using a very finely subdivided rheostat, and it saves the cost of such a rheostat and gives a wider range of speed control because of the partial elimination of sparking as before explained. On the other hand the change-of-reluctance method requires a motor of a special and rather expensive design.

The multivoltage method of speed control has the great disadvantage that it is expensive, requiring as it does, several generators,* an elaborate system of wiring, and a complex controller for each motor. The disadvantage that it does not provide for a finely graded speed control is entirely overcome by combining with it the field-rheostat method for giving intermediate speeds as already explained. The combined multivoltage and field-rheostat system of speed control is extensively used in machine shops for driving machine tools where it is desired to make arrangements quickly to drive the tool at that speed which will accomplish a particular kind of work in the shortest time.

The advantage of the multivoltage method over the armature rheostat method of speed control in securing a greater constancy of speed under variations of load, is well illustrated by the experimental data in the following table. These observations were made upon a motor-driven lathe turning a piece of steel $3\frac{3}{4}$ inches in diameter. The cutting speed, which has been reduced to a low value by an armature rheostat, rises from 60.8 to 210 feet per minute when the depth of cut is reduced from $\frac{1}{8}$ inch to zero; whereas, when the speed has been reduced to the desired low value by using $E_x = 60$ volts instead of $E_x = 236$ volts, the speed rises only from 60 to 71 feet per minute when the depth of cut is reduced to zero.

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* In practice a single main generator is used, and this main generator operates one or more motor-generator sets, technically called balancers.

Depth of Cut. (Feed 21 to 1.)	ARMATURE RHEOSTAT CONTROL. (Rheostat Adjusted to Give a Desired Speed with the Full Depth of Cut.				MULTIVOLTAGE CONTROL. (Speed Reduced to Desired Value with the Full Cut by Using a 60-Volt Supply Instead of a 236-Volt Supply.)			
	Amperes. Armature.	Volts.		Cutting Speed in Feet per Minute.	Amperes. Armature.	Volts.		Cutting Speed in Feet per Minute.
		Armature.	Line.			Armature.	Line.	
$\frac{1}{8}$ inch.	53.5	53	236	60.8	48	52	60	60
$\frac{1}{16}$ "	38.5	105	236	121.5	34	54	60	66.2
$\frac{1}{32}$ "	27.3	143.2	236	165.5	28	55	60	66.5
$\frac{1}{64}$ "	25.7	146.6	236	172.5	25	56	60	66.8
Zero.	16.2	179	236	210	17	57	60	71
	Field current 0.82 amp.				Field current 0.82 amp.			

The Ward Leonard system of speed control is perhaps more expensive than the multivoltage system, but it is the most complete method of all, giving as it does a finely graded control of speed from full speed down to zero speed. This system is used to a considerable extent for operating guns and turrets on warships where completeness and fineness of control is of prime importance, and where the question of cost is secondary.

A disadvantage which is common to all methods of speed control of an electric motor, where the motor is to supply approximately the same amount of power at all speeds, is that the motor must be very large to be able to deliver the requisite amount of power when it runs at the lowest speed, and that the capacity of the motor is not fully utilized when it runs at higher speeds. A much smaller and cheaper motor could take the place of the large variable speed motor, if the small motor were driven constantly at full speed, and if the variations of speed of the driven machine were accomplished by mechanical means; and the smaller motor would run at a higher efficiency. The use of variable speed motors is justified, however, in many cases because of the great ease and quickness with which speed changes can be accomplished.

78. The shunt motor starting rheostat. — From equation (30*c*) it is evident that an excessively large current $I_a [= (E_r - \Phi Z'n)/R_a]$ will flow through the motor armature at starting if the armature terminals are connected directly to the supply mains, inasmuch as $\Phi Z'n$ is zero at starting, since n is then zero, and the resistance R_a of the armature is small. It is necessary to prevent this excessive flow of current at starting by connecting an auxiliary resistance in series with the motor armature (*not in series with the field*). While the motor is speeding up, the value of $\Phi Z'n$ increases, the current tends to decrease, and it is permissible to gradually cut out the auxiliary resistance. When the motor has reached nearly full speed, the auxiliary resistance is wholly cut out. This auxiliary resistance, when conveniently arranged for being put in series with the armature and slowly cut out as the motor speeds up, is called a *starting rheostat.*

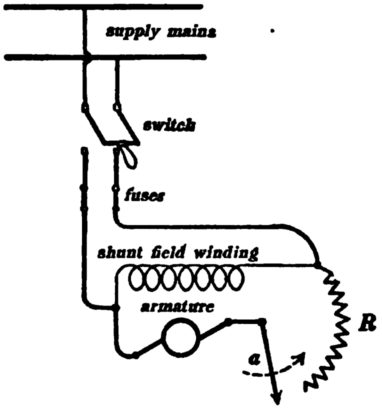

Fig. 99.

Figure 99 shows the simplest arrangement of a shunt motor with a starting rheostat, the motor being connected to the supply mains through a double-pole single-throw switch. The closing of this switch connects the field circuit of the motor to the mains, the first movement of the rheostat arm a connects the armature to the supply mains through the whole of the starting resistance R, and the continued movement of the rheostat arm slowly cuts out the starting resistance.

Starting rheostat with automatic overload release and dead-line release.* — The most frequent causes of trouble with a shunt motor are as follows: (*a*) The motor may be running normally when, for some reason or other, the supply of current fails, the line wires becoming "dead." When this happens the motor of course stops, but the connections to the supply mains remain intact, with all of the resistance cut out of the starting rheostat. When the supply of current is again established under these conditions, an excessive amount of current flows through the motor armature and it is likely to be burned out. In service, a motor should be protected from damage in this way by a dead-line, or low-voltage, release which automatically throws the arm of the starting rheostat back to the starting position when the supply of current fails.

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* The name applied to this device by the manufacturers is the "starting rheostat with overload and no-load release." This name is, however, misleading, inasmuch as what is called the "no-load release" operates only when the line becomes dead, but does not operate when the load on the motor is reduced to zero.

(*b*) The motor may be greatly overloaded. When this happens the motor takes excessive current from the supply mains, and the motor may be overheated or even burned out. In service, a motor should be protected from damage in this way by an overload release which automatically throws the arm of the starting rheostat to the starting position when an excessive current flows through the motor armature.

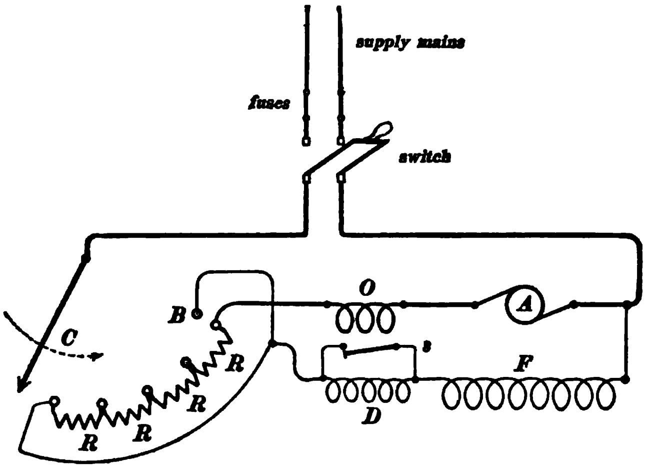

Fig. 100.

The electrical connections of a shunt motor starting-box with automatic dead-line release and overload release, are shown in Fig. 100. The motor armature is represented by *A*, and the shunt field winding by *F*. To start the motor, the switch is closed, and the rheostat arm *C* is slowly moved in the direction of the dotted arrow until it reaches *B*. The first move of the arm *C* connects the shunt field winding directly to the mains, and at the same time it connects the armature to the mains through the resistances *RR*. The continued movement of the arm cuts the resistances out of the armature circuit. In the particular arrangement shown in Fig. 100 the resistances *RR* are left in series with the field winding as they are cut out of the armature circuit, but this has but little effect upon the field excitation, inasmuch as the resistances *RR* are very small in comparison with the resistance of the shunt field winding, but the resistances *RR* are finally cut out of the field circuit when the arm *C* touches the contact point *B*.

When the rheostat arm *C* is pushed to the right in starting the motor, it stretches a spring, and the rheostat arm is held in the running position by an electromagnet, of which the winding *D* is in series with the field winding *F* of the motor. When the lines become dead, current ceases to flow through *D* and *F*, the electromagnet releases the rheostat arm, and the stretched spring pulls the arm back to the starting position.

The overload release is actuated by an electromagnet of which the winding *O* is in series with the motor armature. The armature * of this electromagnet *O* is held back by a spring, the tension of which is so adjusted that the armature may be moved by a prescribed value of current through *O*. When this value of current is reached, the movement of the armature of *O* actuates a small switch *s* which short-circuits the winding *D*. This causes the electromagnet *D* to lose its magnetism, and release the arm *C* as before.

Figure 101*a* is a general view of one of the General Electric Company's shunt motor starting rheostats with an automatic dead-line release, and Fig. 101*b* is a complete diagram of the connections of this starter to a motor, and, through a double-pole switch and two fuses, to the supply mains. Figure 101*a* shows the rheostat arm in the starting position, and Fig. 101*b* shows the arm in the running position. To start the motor, close the switch, and move the rheostat arm *slowly* to the right until the block of iron on the rheostat arm comes against the retaining electromagnet, of which the winding (*D*, Fig. 100) is in series with the field winding of the motor, as shown in Fig. 101*b*.

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* The armature of an electromagnet is the strip of iron which is attracted by the magnet.

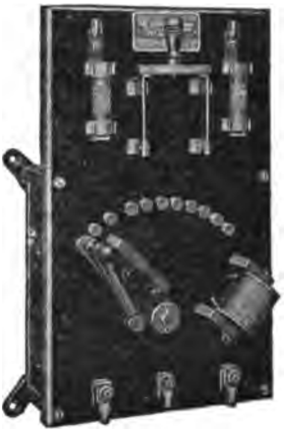

Fig. 101a.

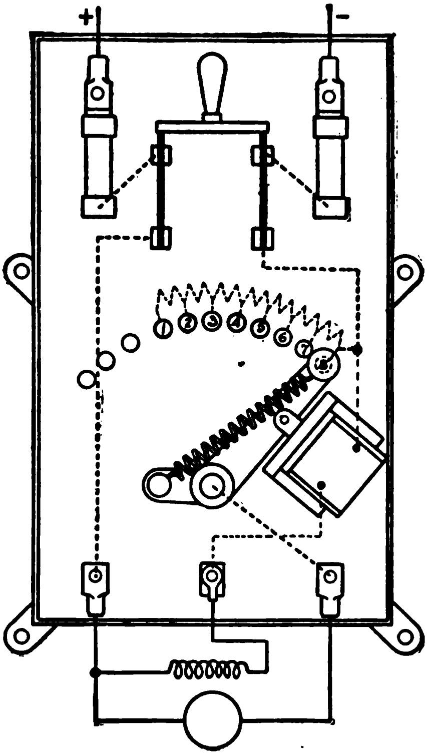

Fig. 101b.

Figure 102 is a view of one of the General Electric Company's automatic double-release (dead-line and over-load) starting rheostats for a shunt motor. The figure shows the rheostat arm in the starting position. This starter is usually connected to the supply mains through a double-pole switch and two fuses. To start the motor the switch is closed, and the rheostat arm is moved *slowly* to the running position, where it is held by the retaining electromagnet. The connections of this starting-box are shown in Fig. 100, and discussion of that figure gives details of its action.

Precautions to be observed in the operation of a shunt motor. — Always start cautiously. If the motor does not start promptly with the first move of the arm, it is likely that the field circuit is broken, or that the load on the motor is excessive. Do not allow the motor to run long with the rheostat arm *not* in the running position, as the resistance in a starting rheostat overheats if it is left permanently in circuit with the armature. To stop the motor, open the switch and leave the starter arm to set itself automatically at the starting position.

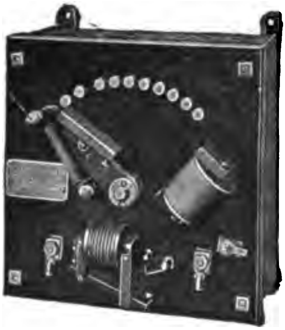
Fig. 102.

79. The compound motor.—The shunt motor is generally used for constant-speed driving, as explained in the foregoing articles. The compound motor is nearly always made so that the greater part of its field excitation is due to the shunt field winding. Such a motor departs slightly from the shunt motor in its speed characteristics, and in the following discussion the compound motor is considered as a slight variation of the shunt motor. There are two arrangements of the compound motor, according to whether the current in the series field winding *opposes* the magnetizing action of the shunt field winding, or *helps* it. The first arrangement is called the *differential compound* winding, and the second arrangement is called the *cumulative compound* winding. Figure 103 shows typical speed-torque characteristics * of a shunt motor, of a cumulative compound motor, and of a differential compound motor. The full-line portions of the curves correspond to the region between zero load and full load.

The differential compound motor. — When a differential compound motor is loaded, the current through the armature and the series field winding increases,† and, by opposing the magnetizing action of the current in the shunt field winding, reduces the armature flux Φ, and thus decreases the counter-electromotive force $\Phi Z'n$, if the speed n remains constant. *If this decrease of* Φ *is just sufficient to give the decrease of counter-electromotive force which is necessary to permit enough current to flow through the armature to enable the motor to carry its load*, then the motor speed will not fall off with increase of load, at least not until the load becomes excessive. The object of the differential compounding of a shunt motor is to prevent a decrease of speed with increase of load, or even to cause an actual increase of speed with increase of load up to full load. The shunt motor, however, runs at a speed which is nearly enough constant for most purposes, and therefore the differential compound motor is seldom used.

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* See Art. 81.

† Constant voltage supply is understood.

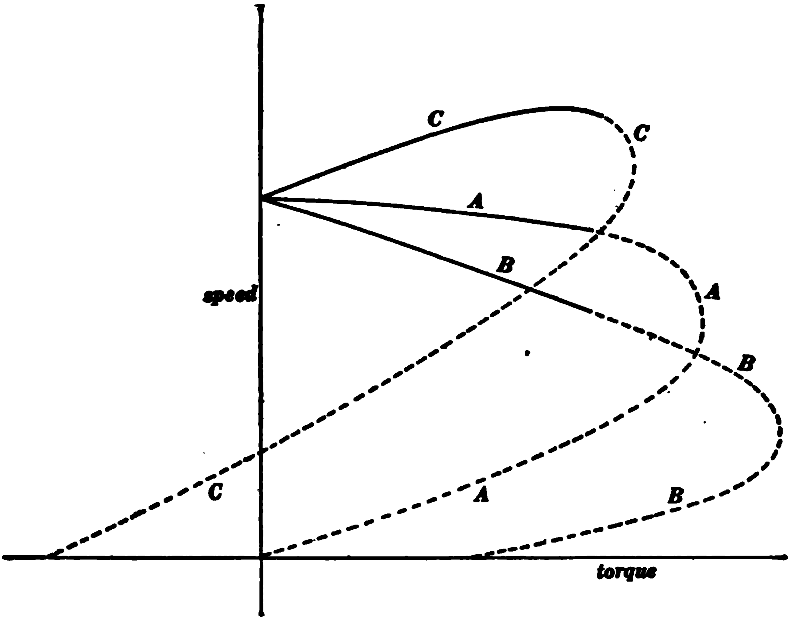

Fig. 103. A = shunt-motor; B = cumulative-compound motor; C = differential-compound motor.

The cumulative compound motor. — The speed of this motor decreases very considerably with increase of load.* This is evident when we consider that an increase of load involves an increase of current in armature and series field winding, and therefore an increase of armature flux Φ (due to increase of field excitation), so that *n* must be considerably reduced in value to enable the impressed voltage E_x to force current through the motor, according to equation (30*c*). The cumulative compound motor is especially suitable for driving a machine, like a punch, in which energy has to be stored in a heavy fly-wheel and used during the very short time that the machine is in operation; the drop in speed of the motor permits a corresponding decrease of speed of the fly-wheel and enables the fly-wheel to give off a portion of its kinetic energy. If a shunt motor were used in this case the great reduction of speed would cause it to take an excessive amount of current which would be likely to damage it.

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* Constant voltage supply is understood.

80. The series motor. Figure 104*a* shows the relation between speed and torque of a shunt motor with constant impressed voltage between its terminals. At zero torque (zero load) the shunt motor runs at definite speed, and with increasing torque (increasing load) the speed falls off slightly as explained in Arts. 75 and 76. The dotted portion of the curve in Fig. 104*a* corresponds to motor loads in excess of what the motor will develop without overheating.

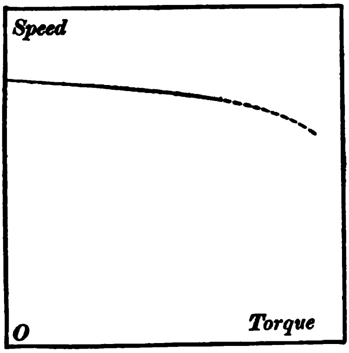

Fig. 104a.

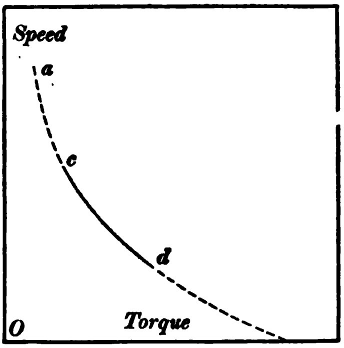

Fig. 104b.

The speed-torque characteristic of a series motor with constant voltage impressed between its terminals is shown in Fig. 104*b*.

The dotted portion of the curve below *d* corresponds to current values so great as to produce excessive heating of the motor, and the dotted portion of the curve above *c* corresponds to excessive speeds. When a series motor (with constant voltage impressed across its terminals) is unloaded, its speed may rise to a value sufficiently great to destroy the armature. Therefore, series motors are used with constant voltage supply only where the motor load is always and inseparably connected to the motor, and where constancy of speed with variations of load is not desired. Thus, series motors are much used for driving fans, where the fan is inseparably attached to the motor, and for driving electric automobiles and street cars. Figure 105 shows the connections of a series fan-motor with its starting rheostat. At starting considerable resistance is connected in series with the motor, and as the motor speeds up this resistance is slowly cut out. When a street car is operated by a single series motor, the connections of the motor to the trolley wire and to the rail are essentially the same as the connections shown in Fig. 105, except that provision is made for quickly reversing the armature connections (or the field connections) in order to reverse the direction of running of the motor.

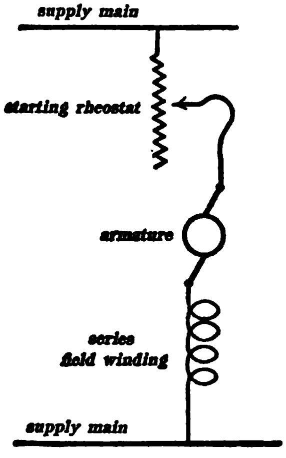

Fig. 105.

In street railway service, where stopping and starting is frequent, the motor runs for a great portion of the time at low speed with a considerable resistance in series with it. Under these conditions most of the energy supplied to the motor circuit is lost as heat in the rheostat, and the efficiency of the motor is very low. Some idea of the efficiency may be obtained with the help of Siemens's law. (See Art. 73.) The approximate efficiency E_a/E_x is small at starting, inasmuch as the counter-electromotive force

$E_a(=\Phi Z'n)$ is small when the speed is small. By using two motors on a car, and arranging to have them connected in series at starting and at low speeds, the counter-electromotive force is doubled, and the efficiency is nearly doubled.

The two motors in series are of course started with resistance in circuit as shown in Fig. 106, and this resistance is slowly cut out in steps as the speed increases. When the speed has reached, say, 10 miles per hour and the resistance is all cut out, the connections are quickly changed * to those shown in Fig. 107. That is, the motors are quickly connected in parallel, and starting

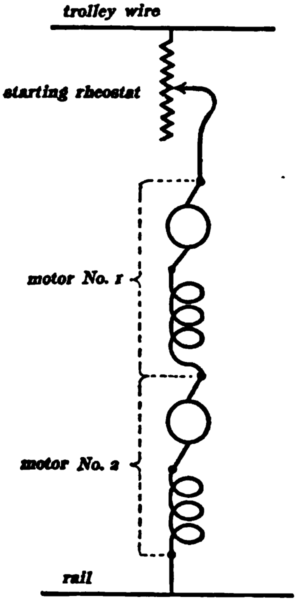

Fig. 106.

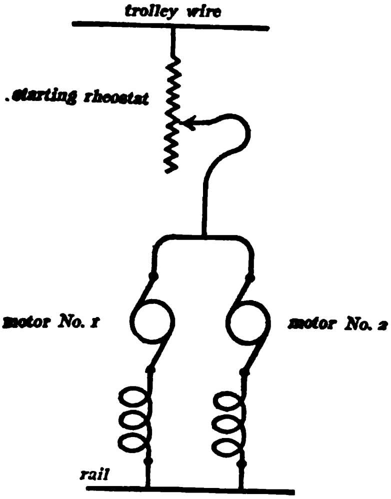

Fig. 107.

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* This change of connections is actually carried out in four distinct steps made in rapid succession as follows: (*a*) Some resistance is reconnected in series with the two motors. (*b*) One motor, No. 2, Fig. 106, is then short-circuited, and this short-circuit constitutes a connection from motor No. 1 to the rail. (*c*) The rail terminal of motor No. 2 is then disconnected. (*d*) This terminal of motor No. 2 is then connected to the rheostat terminal of motor No. 1, giving the connection shown in Fig. 107. During operation (*b*) motor No. 2 cannot build up as a generator, inasmuch as it is running in the wrong direction. (See Art. 71.)

resistance is connected in series with them. The speed then continues to rise, and as it rises the starting resistance is again slowly cut out.

The use of two series * motors in this way is called the "series-parallel system of control." All of the successive connections

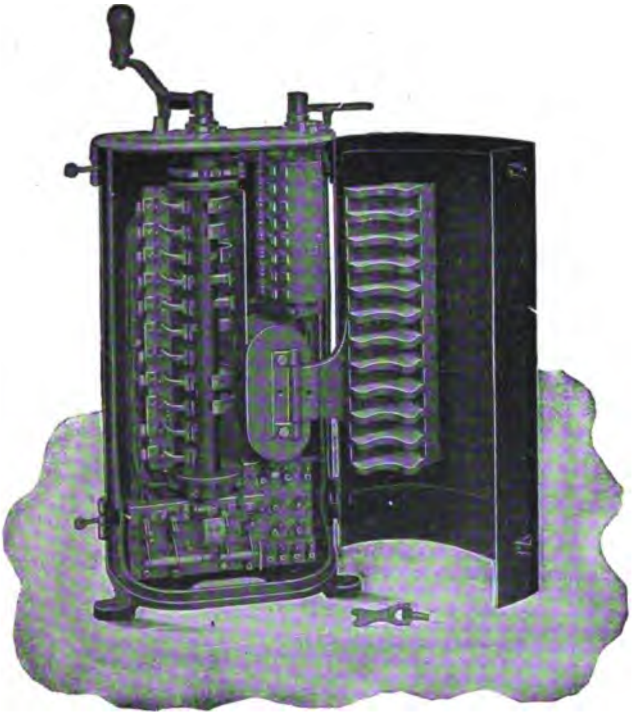

Fig. 108.

described above are made in the proper order by turning a crank which actuates a spindle upon which metal sectors are mounted as shown in Fig. 108. This figure shows a controller spindle

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* Sometimes four series motors are used on one car. With such a four-motor equipment it is customary to connect the motors as two units, each unit consisting of two motors permanently connected in parallel. These two units are then treated as two single motors, and operated by the series-parallel controller exactly as explained above; that is, at starting the two units are in series, and at full speed the two units are in parallel.

with eleven sectors (some of which are hidden behind the spindle), and eleven contact-fingers which touch the sectors when the spindle is turned. The small cylinder at the upper right hand in the figure is arranged to make the necessary changes of connections of the armature terminals for reversing the direction of driving of the motors, by turning the small lever at the top. The large oval structure is an electromagnet, one pole piece of which is spread out in the region behind the controller spindle, and the other pole piece is hinged to the core of the electromagnet, so that it may be swung back, as shown in the figure, to give access to the controller spindle and contact fingers. This swinging pole piece carries twelve plates of a compressed asbestos insulating compound which serve as fire-proof partitions between the various pairs of sectors and fingers. The object of the electromagnet is to "blow out" the arcs which form between the sectors and fingers when connections are broken by turning the spindle, and the asbestos plates serve to prevent the arcs from flashing from sector to sector or from finger to finger. The apparatus as a whole is called a *series parallel controller*. Figure 108 shows a series parallel controller of the General Electric Company.

Figure 109 shows a typical street railway motor. The motor, commutator and all, is enclosed in a steel case to protect it from dust and moisture. The field poles, usually four in number, project inwardly from this steel case which serves as a field magnet yoke. The figure shows the lower half of the motor case dropped, thus exposing the armature and commutator to view. The motor case is supported at one side by the car axle, the two axle bearings being shown in Fig. 109, and on the opposite side the case is supported by a spring which rests on the truck frame. This gives the heavy motor structure that freedom of motion which is necessary when the car is running over a rough track at high speed, but at the same time the car axle and the armature shaft are at a fixed distance apart, so as to accommodate the spur gears which transmit power from the armature to the axle. The spur gear shown in Fig. 109 is attached to the projecting end of the armature shaft. The wire-leads from the brushes and from the series field windings are brought out of the steel case through rubber bushings, and connect through flexible insulated cables to the rheostats and controlling devices. The brushes are not supported in a rocker arm but in rigid holders attached to the inside of the case, and when the brushes are badly worn and need to be replaced, they may be reached through a hand hole which is covered by a metal lid one end of which shows in Fig. 109.

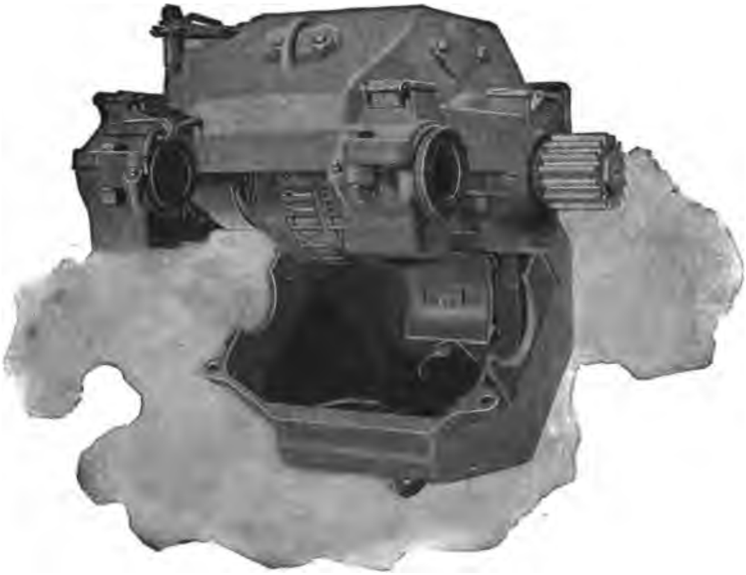

Fig. 109.

81. Characteristic curves of motors. — The behavior of a motor under given electrical conditions of driving is most clearly represented by a curve showing the relationship between any two of the following variable quantities : speed, torque, and mechanical power output. When the current is supplied from constant-voltage mains, it is often convenient to show by a curve the relationship between the *current* and any one of the three mechanical factors :

speed, torque, and power output. Such a curve is called, in general, a *characteristic curve* of the motor. In speaking of a characteristic curve it is customary to designate it in terms of the quantities which are represented by its coördinates, always naming its ordinates first and its abscissas second. For example, a *speed-torque characteristic* would be a curve of which the ordinates represent speeds and the abscissas represent torques.

Figure 103 shows typical speed-torque characteristics of a shunt motor (curve *A*), of a cumulative-compound motor (curve *B*), and of a differential-compound motor (curve *C*). The full-line portions of these curves correspond to normal conditions of running, and the trend of the full-line portions of these curves is fully explained in Arts. 76 and 79.

Figure 104*b* is a typical speed-torque characteristic of a series motor. The general trend of the speed-torque characteristic of the series motor when supplied from constant voltage mains may be explained as follows:

When the motor speed is small, the counter-electromotive force developed in the armature is small, so that a large current flows through the series field winding and through the armature. Therefore the field excitation is large, the flux Φ is large, and the torque is large according to equation (31). As the speed increases, the torque falls off on account of decrease of current due to increase of counter-electromotive force, and on account of the decrease of Φ due to the decrease of current.

The characteristic curves which are most frequently used in connection with street railway motors are: (*a*) the speed-current curve, in which the ordinates represent the speed of the car in miles per hour (instead of the speed of the motor armature in revolutions per minute); and (*b*) the tractive-effort-current curve, in which the ordinates represent the net tractive effort in pounds (instead of the torque developed by the motor armature). Such characteristic curves involve the gear ratio of the motor and the diameter of the driving wheels on the car.

Figure 110 shows the speed-current characteristic of a single

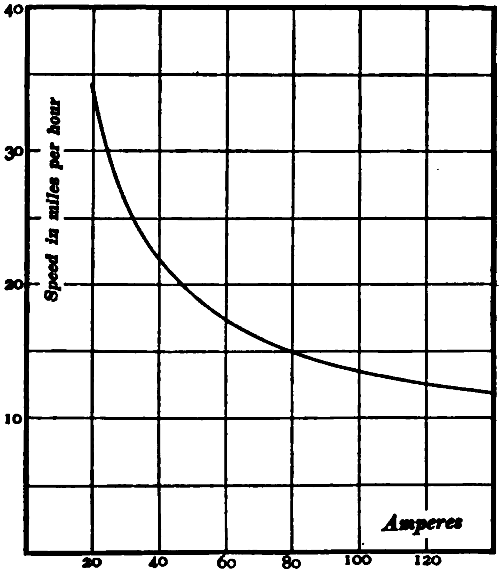

Fig. 110.

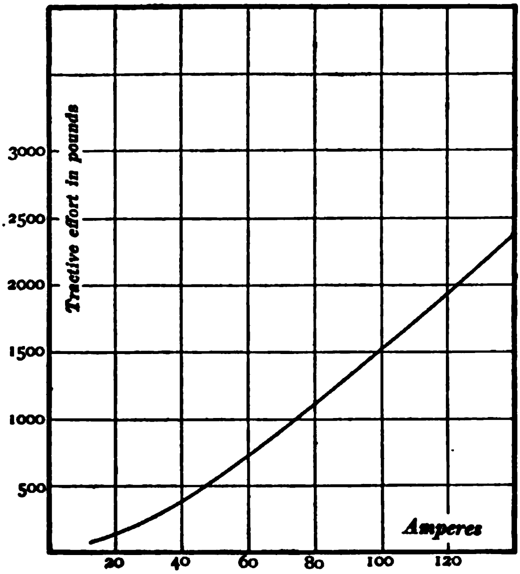

Fig. 111.

Westinghouse railway motor (No. 68), voltage between trolley wire and rail being 500 volts, gear ratio 14:68, diameter of wheels 33 inches, all of the resistance being cut out of the rheostat. This motor is rated to take continuously 33 amperes at 400 volts without overheating. Figure 111 shows the tractive-effort-current curve of the same motor under the same conditions.

The speed-current and tractive-effort-current characteristics of series motors are of great importance in electric railway engineering, since by their means the details of performance and power consumption of an electric car or train with a given equipment of motors may be predetermined; or conversely, with given train schedule and given road-bed a proper selection of motors for the given service conditions may be made. The following example illustrates the application of series motor characteristics to the calculation of the amount of power delivered to the car wheels by a given motor equipment, and the efficiency obtained under specified conditions.

Example. — A twenty-ton car is equipped with two similar series motors of which the characteristic curves, with 500 volt supply (no starting resistance in circuit), are shown in Figs. 110 and 111. With the motors connected in parallel and all starting resistance cut out, it is required to find the full-speed (zero acceleration) at which the car will climb a 5.95 per cent. grade. It is also required to find the current taken by each motor, the total power delivered to both motors, the net power represented by the total tractive effort at the given speed, and the efficiency of the car equipment under the specified conditions.

Solution. — A tractive effort of 2,380 pounds would be required to draw a frictionless 20-ton car up a 5.95 per cent. grade, and, allowing 6 pounds of tractive effort per ton of car to overcome friction, we have a total tractive effort of 2,500 pounds, or 1,250 pounds per motor. Therefore, from Fig. 111, we find that a current of 85 amperes is required for each motor, or a total current of 170 amperes; and the speed corresponding to 85 amperes is found from Fig. 110 to be 14.4 miles per hour. Therefore, the power delivered to the car is 170 amperes × 500 volts = 85 kilowatts, and the power which is represented by the total tractive effort of 2,500 pounds at a speed of 14.4 miles per hour is

$$\frac{2{,}500 \times 14.4 \times 5{,}280}{3{,}600} \times \frac{746}{550} \times \frac{1}{1{,}000} = 71.6 \text{ kilowatts}$$

whence the efficiency is

$$71.6 \text{ kilowatts} \div 85 \text{ kilowatts} = 0.842$$

Note. — In the solution of a problem like the above the frictional resistance (which is taken arbitrarily to be six pounds per ton) must be estimated by means of an empirical formula based on experimentally determined tractive efforts on level tracks at various speeds. See *Electric Railways*, by Ashe and Keiley, pages 23–25, D. Van Nostrand & Co., 1905.

CHAPTER VI.

POWER LOSSES AND EFFICIENCIES.*

82. Motor and generator losses. — The losses of power in a generator or in a motor are: the field loss F, the armature loss A, and the stray power loss S. The field loss is due to the heat generated in the field windings by the field current, that is, no power is required to excite the field magnet except the power used to overcome the resistance of the field winding. The armature loss, as here defined, is due to the heat generated by the armature current in the brushes, in the brush contacts, and in the armature windings. Field and armature losses can be accurately calculated as explained below. The stray power loss includes eddy current and hysteresis losses, chiefly in the armature core, and losses due to friction in the bearings and at the brushes, and to air friction. Stray power loss cannot be accurately calculated from any simple data. This loss is usually determined by experimental tests.

(*a*) *Field loss.* — A certain amount of power is consumed in maintaining the field current of a dynamo. All this power goes to heat the field coils in accordance with Joule's law, that is, no power would be consumed in field excitation if it were possible to make the field windings of a material having zero resistance. The field loss in watts may be calculated by means of the following equations:

(1) *For a shunt dynamo.*

$$F = R_s I_s^2 \tag{33a}$$

or

$$F = E_x I_s \tag{33b}$$

or

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* This chapter is arranged to apply specifically to direct-current dynamos and motors. The ideas throughout are applicable with but little modification to alternating-current machines.

$$F = \frac{E_x^2}{R_s} \qquad (33c)$$

in which R_s is the resistance of the shunt winding including the field rheostat, I_s is the current in the shunt winding, and E_x is the electromotive force between the terminals of the machine. The resistance of the shunt winding increases considerably after the machine is started, on account of rise of temperature. In calculating field loss, the resistance of the winding at the steady running temperature of the machine should be used; this is frequently called the *hot resistance*, for brevity.

The field loss of a shunt dynamo (generator or motor) is approximately constant. Thus the field winding of a shunt motor is usually connected directly to the constant voltage mains, and after the steady running temperature has been reached, the field loss is strictly constant, whatever the load on the motor may be. On the other hand, the terminal voltage of a shunt generator varies more or less with its current output, and, inasmuch as the field winding is connected across the terminals of the machine, the field current, and therefore the field loss also, vary slightly. When the terminal voltage of a shunt generator is kept constant with increase of current output, by the manipulation of a field rheostat, the field current and also the field loss change considerably.

(2) *For a compound dynamo*

$$F = R_s I_s^2 + R_c I_c^2 \qquad (34)$$

in which R_s and I_s have the same significance as in equation (33*a*), R_c is the resistance of the series field winding, and I_c is the current in the series field winding.

In applying equation (34) to "long shunt" or "short-shunt" compound dynamos, careful attention must be given to the diagram of connections to insure that the values of I_s and I_c are correctly derived from such data as may be given.

(3) *For a series dynamo*, equation (34) may be used by dropping the term $R_s I_s^2$. The field loss of a series dynamo (generator or motor) varies greatly with the load.

(*b*) *Armature loss.* — The power losses in the armature of a dynamo include the losses in the armature core due to eddy currents and hysteresis. These core losses are, however, usually included under the term "stray power" loss. By armature loss is here meant simply the power which is consumed in the armature windings in accordance with Joule's law, together with the power lost at the brushes on account of contact resistance. The armature loss in watts may be calculated from the equation

$$A = R_a I_a^2 \qquad (35)$$

in which R_a is the resistance of the armature from positive to negative brushes; R_a includes the resistance (hot) of the armature windings, the contact resistances of the brushes, and the resistances of the brushes themselves; and I_a is the total current leaving the armature at the positive brushes, or entering the armature at the negative brushes.

Armature loss varies greatly with the current output of a generator, and with the amount of load on a motor.

(*c*) *Stray power loss.* — This term includes all losses which cannot be satisfactorily calculated from simple data.* These losses are —

(1) Eddy current and hysteresis losses, chiefly in the armature core, due to reversals of magnetization as the armature rotates.

(2) Friction losses in the bearings and at the brushes, and

(3) Air friction loss, or windage, as it is called, due to the fan-like action of the rotating armature.

These losses cannot be separately measured with accuracy. The stray power loss, which is the sum of these losses can, however, be quite accurately determined for a given machine by an experimental test.

When a dynamo runs at constant speed with constant field excitation, the stray power loss is approximately constant, irrespective of the current output of the machine if it is acting as a generator, and irrespective of the load on the machine if it is acting as a motor. Therefore the stray power loss of a shunt generator driven at constant speed or of a shunt motor supplied from constant voltage mains, is approximately constant irrespective of the load on the machine. The stray power loss of a series generator or motor varies greatly with its load.

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* That is, all losses except RI^2 losses.

It is often desired to find the stray power S'', in a dynamo (generator or motor), when run at such speed and field excitation as to give an armature voltage E_a'', having given the experimentally determined value S' of the stray power loss of the dynamo when run at such speed and field excitation as to give an armature voltage E_a'. Now the various component parts of the stray power loss are very different functions of field excitation and speed, and the dependence of S upon field excitation and speed cannot be accurately represented by any simple formula. For small variations of field excitation and speed, however, it is sufficiently exact to assume that the stray power loss is proportional to the electromotive force induced in the armature, from which we obtain the equation

$$S'' = S'\frac{E_a''}{E_a'} \tag{36}$$

83. Experimental determination of stray power loss. — When a dynamo is driven as a motor at zero load and at a prescribed speed, the power P delivered to the armature of the machine exceeds the stray power loss by the amount $R_a I_a^2$, where R_a is the resistance of the armature from brush to brush and I_a is the measured current delivered to the armature. This simple relation makes it possible to determine the stray power loss of a dynamo.

Example. — A given dynamo, rated as a 100-volt, 50-ampere generator, when driven at a speed of 1,200 revolutions per minute, is tested for stray power loss as follows: The field is separately excited by being connected to 110-volt mains in series with an adjustable rheostat. The armature terminals are also connected to the 110-volt mains through an ammeter and an adjustable rheostat, and the machine starts, running as a motor. A voltmeter is connected to the armature terminals, and the two rheostats are adjusted until the machine runs at 1,200 revolutions per minute, and shows an electromotive force of 100 volts between the brushes. The ammeter then shows that 2.4 amperes are flowing through the armature. The resistance of the armature (at running temperature) is found by measurement to be 0.25 ohm.

From these data the power P delivered to the armature is found to be $P = 2.4$ amp. × 100 volts = 240 watts, and the armature loss corresponding to the current of 2.4 amperes is $(2.4 \text{ amp.})^2 \times 0.25$ ohm = 1.44 watts, so that the stray power loss under the condition of the test is 240 watts − 1.44 watts, or 238.56 watts. A slight correction to this result may be applied to find the value of S, corresponding to a speed of 1,200 revolutions per minute and E_a equal to 100 volts, as follows: Under the conditions of the test, 100 volts $= E_x = E_a + R_a I_a$, where $R_a = 0.25$ ohm and $I_a = 2.4$ amperes, so that E_a of the test is equal to 99.4 volts. Therefore, according to equation (36), the value of S, corresponding to $E_a = 100$ volts, is approximately

$$238.56 \text{ watts} \times \frac{100}{99.4} = 239.8 \text{ watts}$$

If it were desired to find from the above test the approximate value of S for a speed of 1,100 revolutions per minute and a field excitation which would give with this speed $E_a = 105$ volts, then equation (36) would give

$$S = 238.56 \text{ watts} \times \frac{105}{99.4} = 251.8 \text{ watts}$$

84. Efficiency of a generator. — The efficiency of a generator is defined as the ratio: output of power divided by intake of power. This ratio is sometimes called the *true* efficiency or the *commercial* efficiency to avoid confusing it with the two *partial* efficiencies defined later.

Calculation of true efficiency. —When the stray power loss of a dynamo has been determined for a given speed and a given voltage, it is easy to calculate the true efficiency of the dynamo for a specified output of power as a generator at the given speed and voltage, the resistances of the various windings of the dynamo being known. This calculation is based upon the equation

$$\text{Efficiency} = \frac{\text{power output}}{\text{power output} + \text{losses}}$$

It is much more instructive to discuss this matter of efficiency calculation by means of several typical numerical examples, than by deriving a general formula.

Example 1. The shunt generator. — Given a shunt generator which delivers 50 amperes of current at 110 volts between its terminals. It is required to calculate the efficiency of the generator under these conditions, having given the following data:

R_s = 44 ohms (hot) including the portion of the field rheostat which must be in circuit to bring the voltage between the terminals of the machine to the specified value.

R_a = 0.14 ohm (hot).

Stray power loss at given speed and voltage = 700 watts.

Solution:

(*a*) Power output $= 110$ volts $\times$ 50 amperes $= 5{,}500$ watts

(*b*) Field loss $= R_s \times \left(\frac{E_x}{R_s}\right)^2 = 275$ watts

(*c*) Armature loss $= R_a\left(50 + \frac{E_x}{R_s}\right)^2 = 386$ watts

(*d*) Efficiency $= \dfrac{5{,}500}{5{,}500 + 275 + 386 + 700} = 0.801$

*Example 2. The compound-generator (long-shunt).** — Given a long-shunt compound generator which delivers 50 amperes of current at 110 volts between its terminals. It is required to calculate the efficiency of the generator under these conditions having given the following data:

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* The calculations for the short-shunt compound generator are nearly the same as for the long-shunt machine. The student should be able to carry out either calculation, knowing the other.

$$R_s = 55 \text{ ohms (hot)}$$

$$R_c = 0.02 \text{ ohm (hot)}$$

$$R_a = 0.14 \text{ ohm (hot)}$$

Stray power loss at given speed and voltage = 700 watts.

Solution:

(*a*) Power output = 110 volts × 50 amperes = 5,500 watts

(*b*) Shunt field loss $= R_s \times \left(\frac{E_x}{R_s}\right)^2 = 220$ watts

(*c*) Series field loss $= R_c(50 + I_s)^2 = 54.1$ watts

(*d*) Armature loss $= R_a(50 + I_s)^2 = 379$ watts

(*e*) Efficiency $= \dfrac{5{,}500}{5{,}500 + 220 + 54.1 + 379 + 700} = 0.803$

Example 3. The series generator. — Given a series generator which delivers 50 amperes of current at 110 volts between its terminals. It is required to calculate its efficiency under these conditions, having given the following data:

$$R_c = 0.12 \text{ ohm (hot)}$$

$$R_a = 0.15 \text{ ohm (hot)}$$

Stray power loss at given speed and voltage = 700 watts.

Solution:

(*a*) Power output = 50 amperes × 110 volts = 5,500 watts

(*b*) Series field loss $= R_c I_c^2 = 300$ watts

(*c*) Armature loss $= R_a I_a^2 = 375$ watts

(*d*) Efficiency $= \dfrac{5{,}500}{5{,}500 + 300 + 375 + 700} = 0.800$

85. Efficiency of conversion and electrical efficiency of a generator. — The total mechanical power expended in driving the armature of a generator is used: (*a*) to supply the stray power loss S, that is to overcome the opposition to the rotation of the armature due to friction, windage, eddy currents, and hysteresis; and (*b*) to overcome the opposition to the rotation of the armature due to the action of the field magnet on the armature conductors on account of the current flowing in them. The portion (*b*) of the total power is, according to Lenz's law, equal to the total electrical power E_aI_a, developed in the armature. The ratio of the total electrical power E_aI_a developed in the armature, to the total mechanical power supplied to the armature, is called the *efficiency of conversion* of the generator. From the above discussion the total mechanical power delivered to the armature is evidently $E_aI_a + S$, so that

$$\text{Efficiency of conversion} = \frac{E_aI_a}{E_aI_a + S} \tag{37}$$

A portion, A, of the total electrical power developed in the armature of a generator is used to overcome armature resistance, a portion F is expended in field excitation, and the remainder $E_aI_a - A - F$ is delivered as useful power to the external receiving circuit. The ratio of the useful electrical power delivered by a generator, to the total electrical power developed in the armature, is called the *electrical efficiency** of the generator. That is,

$$\text{Electrical efficiency} = \frac{E_aI_a - A - F}{E_aI_a} = \frac{E_xI_x}{E_aI_a} \tag{38}$$

The values of A and F may be calculated with the help of equations (33), (34) and (35).

The product of the two partial efficiencies is equal to the true or commercial efficiency. That is, the efficiency of conversion is the fraction of the total mechanical power which is converted into electrical power, and the electrical efficiency is the fractional part of the latter which is delivered to the receiving circuit.

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* Sometimes called the economic coefficient of a generator.

86. Variation of the efficiency of shunt and compound generators with power output. — The shunt field loss and the stray power loss of a shunt or compound generator driven at constant speed are approximately constant. On the other hand, the armature loss (plus the series field loss) is nearly proportional to the square of the current output. The efficiency of the generator is therefore very small when the power output is small, inasmuch as the constant losses, namely, shunt field loss and stray power loss, are then large as compared with the power output. The efficiency of the generator increases with increase of output, passes through a maximum value for a certain output, and then with further increase of output, the efficiency decreases because of the rapid increase of armature loss.

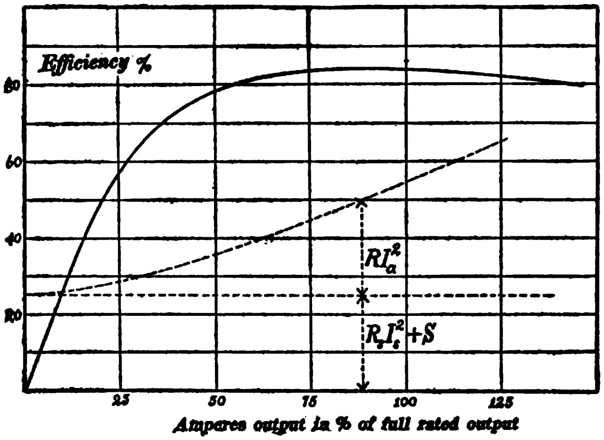

Fig. 112. Efficiency and loss curves of a shunt generator.

The efficiencies of a shunt or compound generator may be calculated for a series of assigned values of power output by the method explained in Art. 84, and a curve may then be plotted showing power outputs as abscissas, and efficiencies as ordinates. Such a curve is called the *efficiency* curve of the generator. Efficiency curves are frequently plotted showing efficiencies as ordinates, and current outputs in amperes as abscissas.

The full-line curve in Fig. 112 is a typical efficiency curve of a shunt generator. The efficiency curve of a compound generator is very similar in form to the efficiency curve of the shunt generator. A generator is usually designed to give its maximum efficiency at its rated full-load output. The ordinate of the horizontal dotted curve represents the approximately constant loss: stray power loss plus $R_sI_s^2$ loss in the shunt field winding. The ordinates of the curved dotted line, measured from the horizontal dotted line, represent the $R_aI_a^2$ loss in the armature.

The maximum efficiency of a shunt or compound generator occurs for that particular current output for which the sum of the variable losses (series field loss and armature loss) is equal to the sum of the constant losses (shunt field loss and stray power loss).

Proof. — Let L be the sum of the constant losses, and let E_x be the terminal voltage of the machine. For the sake of simplicity let us assume $I_a = I_c = I_x$ and let us represent this common current by I. This assumption is approximatety true, inasmuch as the shunt field current is small as compared with the current output, except when the current output is very small. Then we have

Power output $= E_xI$.

Armature loss + series field loss $= (R_a + R_c)\ I^2 = RI^2$, where R is written for $R_a + R_c$.

Therefore,

$$\text{Efficiency} = \frac{E_xI}{E_xI + RI^2 + L}$$

Differentiating this expression with respect to I, assuming L to be constant, and placing the differential coefficient equal to zero, we find

$$L = RI^2$$

which shows that the efficiency is a maximum when the sum of armature and series field losses is equal to the sum of shunt field and stray power losses.

The efficiency of a series generator is also zero at zero current output, it increases with increase of current output, reaches a maximum, and falls off with further increase of current output.

The efficiency of a generator depends upon its size. A generator of one or two kilowatts rated capacity would have an efficiency of about 65 per cent. at best, while very large generators have efficiencies as high as 95 per cent., or even higher.

87. Efficiency of a motor. — The efficiency of a motor is defined as the ratio: output of power divided by intake of power. This ratio is sometimes called the *true* efficiency or the *commercial* efficiency to avoid confusing it with the two *partial* efficiencies defined later.

Calculation of true efficiency. — When the stray power loss of a dynamo has been determined for a given speed and voltage, it is easy to calculate the efficiency of the dynamo when operating as a motor at the given speed and voltage, and supplied with a specified intake * of electrical power; the resistances of the various windings of the dynamo being known.

This calculation is based upon the equation

$$\text{Efficiency} = \frac{\text{power intake} - \text{losses}}{\text{power intake}}$$

It is more instructive to discuss this matter of efficiency calculation by means of several typical examples than by deriving a general formula.

Example 1. Shunt motor. — A given shunt motor is supplied with 50 amperes of current from 110-volt mains, and it is required to calculate the efficiency of the motor, having given the following data:

$$R_s = 44 \text{ ohms (hot)}$$

$$R_a = 0.14 \text{ ohm (hot)}$$

Stray power loss = 700 watts at the given voltage and speed.

Solution:

(*a*) Power intake = 110 volts × 50 amp. = 5,500 watts

(*b*) Shunt field loss $= R_s\left(\frac{E_x}{R_s}\right)^2 = 275$ watts

(*c*) Armature loss $= R_a\left(50 - \frac{E_x}{R_s}\right)^2 = 316$ watts

(*d*) Efficiency $= \frac{5{,}500 - 275 - 316 - 700}{5{,}500} = 0.765$

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* The calculation of the efficiency of a motor for a specified output of power is more complicated than for a specified intake of power or current.

Example 2. Compound motor (long-shunt).[*] — A given compound motor is supplied with 50 amperes of current from 110-volt mains, and it is required to calculate the efficiency of the motor, having given the following data:

$$R_s = 55 \text{ ohms (hot)}$$
$$R_c = 0.078 \text{ ohm (hot)}$$
$$R_a = 0.09 \text{ ohm (hot)}$$

Stray power loss = 700 watts at given voltage and speed.

Solution:

(*a*) Power intake = 110 volts × 50 amperes = 5,500 watts

(*b*) Shunt field loss $= R_s\left(\frac{E_x}{R_s}\right)^2 = 220.0$ watts

(*c*) Series field loss $= R_c\left(50 - \frac{E_x}{R_s}\right)^2 = 179.7$ watts

(*d*) Armature loss $= R_a\left(50 - \frac{E_x}{R_s}\right)^2 = 207.4$ watts

(*e*) Efficiency $= \frac{5{,}506 - 220 - 179.7 - 207.4 - 700}{5{,}500} = 0.762$

Example 3. Series motor. — A given series motor takes 50 amperes of current with an electromotive force of 110 volts between its terminals. Under these conditions the motor has a definite speed and a definite stray power loss. It is required to calculate the efficiency of the motor under the given conditions, having given the following data:

$$R_c = 0.12 \text{ ohm (hot)}$$
$$R_a = 0.15 \text{ ohm (hot)}$$

Stray power loss = 700 watts at the given speed and voltage.

Solution:

(*a*) Power intake = 110 volts × 50 amp. = 5,500 watts

(*b*) Series field loss $= R_c \times (50)^2 = 300$ watts

(*c*) Armature loss $= R_a \times (50)^2 = 375$ watts

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[*] It is not necessary to outline the calculations for the short-shunt machine.

(*d*) $\text{Efficiency} = \dfrac{5{,}500 - 300 - 375 - 700}{5{,}500} = 0.75$

It is important to remember that the stray power loss of a series motor varies greatly with the load on the motor, or in other words, with the intake of current by the motor, and that the stray power loss varies also with the speed.

88. Efficiency of conversion and mechanical efficiency of a motor. — The total electrical power $E_x I_x$ delivered to a motor is used: (*a*) to supply the power lost in heating the field windings, and to supply the armature loss $R_a I_a^2$; and (*b*) to force the current through the armature in opposition to the induced electromotive force E_a. The part (*b*) is equal to $E_a I_a$ and, according to Lenz's law, it is all converted into actual mechanical power (that which supplies the stray power loss and the useful power delivered at the pulley). The ratio of the mechanical power $E_a I_a$ developed in the armature, to the total electrical power $E_x I_x$ delivered to the motor, is called the *efficiency of conversion* of the motor. That is,

$$\text{Efficiency of conversion} = \frac{E_a I_a}{E_x I_x} \qquad (39)$$

A portion, S, of the total mechanical power developed in the motor armature is used to overcome bearing friction, air friction, and magnetic friction, and the remainder $E_a I_a - S$ is delivered at the motor pulley as useful power. The ratio of the useful power delivered by a motor, to the total mechanical power developed in the motor armature, is called the *mechanical efficiency* of the motor. That is,

$$\text{Mechanical efficiency} = \frac{E_a I_a - S}{E_a I_a} \qquad (40)$$

The actual or commercial efficiency of a motor is equal to the product of its efficiency of conversion and its mechanical efficiency, inasmuch as the efficiency of conversion is the fractional part of the delivered electrical power which is converted into mechanical power, and the mechanical efficiency is the fractional part of this total mechanical power which is available at the motor pulley.

89. Variation of the efficiency of shunt and compound motors with load. — The shunt field loss and the stray power loss of a shunt or compound motor supplied with current from constant voltage mains, are nearly constant. On the other hand, the armature loss (plus the series field loss) is nearly proportional to the square of the current intake of the motor. The efficiency of the motor is therefore very small when the load is small, inasmuch as the constant losses, namely, field loss and stray power loss, are then a large part of the motor intake. The efficiency of the motor increases with increase of load, passes through a maximum value for a certain definite load, and then, with further increase of load, the efficiency decreases because of the rapid increase of armature loss.

The efficiencies of a shunt motor or a compound motor may be calculated for a series of assigned values of current or power intake, and from these data a curve may be plotted showing the relation between power output and efficiency. Figure 113 shows a typical efficiency curve of a shunt motor. The efficiency curve of a compound motor is very similar to the efficiency curve of a shunt motor shown in Fig. 113.

The maximum efficiency of a shunt motor (in which the stray power loss is approximately constant) occurs for that particular load for which the armature loss $R_a I_a^2$ is equal to the constant loss, namely, stray power loss plus field loss. The proof of this proposition is very similar to the proof of the corresponding proposition concerning the efficiency of a shunt generator (see Art. 86).

The efficiency of a motor depends upon its size. Thus a very small motor of $\frac{1}{4}$-horse power may have an efficiency under the best conditions of about 60 per cent., while a 20- or 30-horse power motor may have an efficiency of 90 per cent. or more.

90. The use of the separately excited motor as a dynamometer. — One of the most convenient methods for determining the amount of mechanical power required to drive a machine of any kind, is to drive the given machine by a separately excited motor, measure the electrical input of power to the armature of the motor, and subtract from this observed input the stray power loss and the armature loss in the motor, thus finding the net mechanical power delivered by the motor to the given machine.

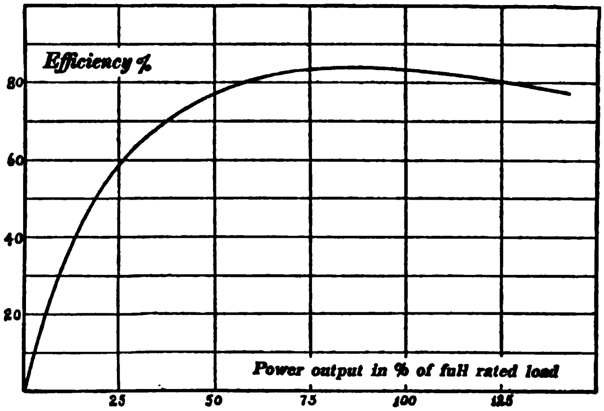

Fig. 113. Efficiency curve of a shunt motor.

Example.—The power required to drive a certain ventilating fan is to be determined. With this end in view, the fan is belted to a shunt motor. The field winding of the motor is connected to the 110-volt supply mains. The armature of the motor is also connected to the supply mains with an ammeter and a rheostat in series with it. The rheostat is adjusted until the fan is driven at the desired speed when the armature current is observed to be 16.4 amperes, and the electromotive force between the brushes, as measured by a voltmeter, is observed to be 92.5 volts.

The belt is then thrown off, a much larger resistance is placed in series with the armature, and this resistance is adjusted until the motor speed is the same as before. The ammeter reading is then observed to be 0.85 ampere, and the electromotive force between the brushes is observed to be 91.6 volts. The resistance (hot) of the armature, as determined by separate measurements, is found to be 0.4 ohm.

Solution:

(*a*) Total power delivered to motor armature to drive the fan = 92.5 volts × 16.4 amp. = 1,517 watts.

(*b*) Stray power loss = 91.6 volts × 0.85 amp. − 0.4 ohm × $(0.85 \text{ amp.})^2$ = 77.6 watts.

(*c*) Armature loss when motor is driving the fan = 0.4 ohm × $(16.4 \text{ amp.})^2$ = 107.6 watts.

(*d*) Mechanical power delivered to the fan = 1,517 watts − 77.6 watts − 107.6 watts = 1,332 watts.

CHAPTER VII.

RATINGS AND GUARANTEES (DIRECT-CURRENT DYNAMOS).

91. Dynamo speeds. — The capacity (power output) of a given sized dynamo, whether it is to be used as a generator or as a motor, depends upon the speed at which the machine is to be run. Roughly speaking the capacity of a dynamo is proportional to its speed. Thus, a given sized dynamo might be rated as a 10-horse power motor or a 15-horse power motor according to whether it is to be run at a speed of 1,000 revolutions per minute or at a speed of 1,500 revolutions per minute, and another dynamo might be rated as a 50-kilowatt generator or as a 75-kilowatt generator according to whether it is to be driven at a speed of 500 revolutions per minute or at a speed of 750 revolutions per minute.

It is of prime importance in making a fair comparison of prices, from different manufacturers, on generators or motors of a given rated output, to consider the speeds at which the machines are to be driven. One manufacturer may very properly charge, for a large, slow-speed machine, a higher price than that charged by another manufacturer for a small, high-speed machine of the same rated output.

The speed at which a dynamo may be driven depends partly upon mechanical conditions, such as completeness of balance of the rotating part, strength of the rotating part to withstand the stresses due to rotation, and satisfactory running of the bearings and brushes; and if a dynamo is to be directly connected to an engine as a generator, or to a printing press or other machine as a motor, the speed at which the dynamo must run is fixed by the speed of the machine to which it is to be connected.

The speed at which a dynamo may be driven depends also to some extent upon electromagnetic conditions as follows:

A shunt dynamo has an inherently proper speed.* Thus a shunt generator should be driven at that speed which will develop sufficient electromotive force in the armature to produce enough current through the shunt field winding to magnetize the field magnet nearly to saturation. If a shunt generator is driven at a speed much lower than this, its field excitation will be low, and its maximum permissible output of power will be considerably less than it would be if its field winding were redesigned to give full field excitation at this reduced speed.

A shunt motor should have its field winding connected to supply mains giving the proper electromotive force to fully excite its field magnet, and under these conditions the armature when connected to the same supply mains, will run at a certain definite speed.

A series dynamo does not have an inherently proper speed. Thus, a series generator may be driven at any desired speed, and, if the receiving circuit has a suitable resistance, the full rated current will be delivered by the machine, its field will be fully excited, and the electromagnetic action of the machine will be at its best. Driven at high speed a series generator will develop a high electromotive force, and driven at low speed the machine will develop a low electromotive force, when the current is at its full-load value.

A series motor does not have an inherently proper speed. Such a motor may be run at full current intake (and consequently at full field excitation) at any speed, although, of course, a high electromotive force will be needed to drive the machine at high speed with full current intake.

Correlation of speed and voltage ratings of shunt dynamos. — From what has been said above, it is evident that a given dynamo, even a given shunt dynamo, may be run more or less satisfactorily at a speed very considerably above or below the rated speed of the machine, but the use in commercial work of certain widely separated standard voltages of supply, 110 volts, 220 volts, and 500 volts, leads to very definite speed ratings of generators and motors. Thus a motor which has a field winding designed to give full field excitation from 220-volt mains, and an armature that is well enough insulated to withstand a 220-volt supply, and strong enough mechanically to run at the speed given by a 220-volt supply, will run at a definite speed, say 1,200 revolutions per minute, under these conditions. Such a motor would be rated as a 220-volt motor at a speed of 1,200 revolutions per minute, and although it would hardly be permissible to run this motor from 110- or 500-volt mains, still the motor would perhaps run satisfactorily from, say, 250-volt mains at a speed, say, of 1,350 revolutions per minute. A shunt dynamo has a *fairly* definite, inherently proper speed, which fact, taken in conjunction with the widely separate standard voltages used in practice, limits a shunt machine (generator or motor) to a certain *very* definite speed.

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* The inherently proper speed of an alternator is the speed which corresponds to the standard frequency for which the machine is designed (see Art. 29).

92. Limitation of output of a dynamo as a generator or as a motor.* — The output of a motor or generator is limited in practice by three distinct things: † (*a*) By overheating and consequent destruction of the insulating materials used in the construction of the machine; (*b*) by excessive sparking at the brushes and the consequent rapid wear, unsatisfactory running, and excessive heating of the commutator; and (*c*) by excessive drop in voltage of a generator or excessive drop in speed of a motor, at excessive loads. That is to say, a dynamo is rated for that load for which it neither heats nor sparks excessively, and for which neither its voltage, if it is to be used as a "constant voltage" generator, nor its speed, if it is to be used as a "constant speed" motor, falls off more than a certain small amount. Heating rather than sparking is the thing that usually limits the output of a machine, especially in the case of completely enclosed machines; while closeness of regulation (voltage regulation in case of a generator and speed regulation in case of a motor, see Arts. 65 and 76) sets a limit to the permissible rating of a machine only in rare special cases where very close regulation is demanded.

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* A formula for estimating the power rating of a direct-current dynamo from its dimensions and speed is given on page 148, Vol. I, Franklin and Esty's *Elements of Electrical Engineering*.

† The output of an alternator is limited in practice by overheating. In special cases the output of an alternator may be limited by drop in voltage at excessive loads. An alternator, having no commutator, is not subject to sparking.

93. Heating.—The ultimate steady running temperature of any part of a dynamo is the temperature at which the part gives off heat (by radiation, convection and conduction) to its surroundings as fast as heat is generated in the part by friction, by eddy currents and hysteresis, or by the heating effect of the electric current. This ultimate steady running temperature is not reached in less time than from six to eighteen hours of steady running under full load, according to the size of the machine. Small machines require a short time, and large machines a long time, to reach this ultimate temperature.

On account of the time required for a dynamo to reach its final steady running temperature, it is evident that a machine may be run for a short time under excessive load without causing undue rise of temperature.

The rated capacity of a machine which is intended for continuous service is always based upon ultimate steady running temperature, and operating engineers usually exact from the manufacturers a definite short-period over-load guarantee, for example, a 50-per cent. over-load for one hour, or a 25-per cent. over-load for three hours, without exceeding a prescribed rise of temperature.

The rated capacity of a machine which is intended for a specific kind of intermittent service may properly be based upon a short-period run. Thus street car motors are usually rated on the basis of a one-hour run, and a street railway motor rated at 25 horse power would overheat if it were operated for much more than one hour at full load.

It is of prime importance in making a fair comparison of prices from different manufacturers on generators or motors of a given rating, to consider this time element, which should always be specified by the manufacturer. One manufacturer may very properly charge for a large machine which will give 100 kilowatts output steadily (or say 125 kilowatts for three hours) a higher price than that charged by another manufacturer for a generator which he rates at 100 kilowatts, if the second manufacturer bases his rating on a three-hour run at full load.*

94. Sparking. — One effect of the sparking of a dynamo is to roughen the commutator and cause the brushes to chatter and to wear the commutator rapidly; another effect is to cause excessive heating of the commutator and brushes. It is not possible to specify precisely the degree of sparking that is permissible, nor is there any satisfactory quantitative sparking test to which a finished machine may be subjected. It is for this reason that ratings and guarantees are based specifically upon rise of temperature. It may be stated in a general way that a dynamo should not show any visible sparking when its output is changed from zero load to full load, or from full load to zero load, the brushes being left in a fixed position.

A dynamo will bear a maximum output rating, with a minimum cost of materials, when it reaches its heating and sparking limits simultaneously.

The cause of sparking, and the means adopted by the designer to obviate sparking are discussed in the following articles on armature reaction and commutation.

95. Armature reaction. — The magnetizing action of the current in a dynamo armature is called *armature reaction.* Armature reaction has a two-fold effect. (*a*) It causes the magnetic flux which enters the armature from the field magnet poles to be crowded toward the trailing pole tips of a generator, or toward the leading pole tips of a motor, as shown in Figs. 114, 115 and 116. (*b*) It opposes (generally) the passage of the flux through the armature and thus reduces the amount of flux.

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* Standard specifications concerning allowable heating of electric machinery are given in the Report of the Committee on Standardization. See *Transactions American Institute of Electrical Engineers* for 1907. See footnote on page 190.

The part (*a*) of armature reaction is called *cross-magnetizing action*, and the part (*b*) is called *demagnetizing action*. The cross-magnetizing action has an important influence upon the sparking of a dynamo, and therefore it has to do more or less with the rating of a given machine. The demagnetizing action, inasmuch as it reduces the flux through the armature, affects the voltage regulation of a generator and the speed regulation of a motor.

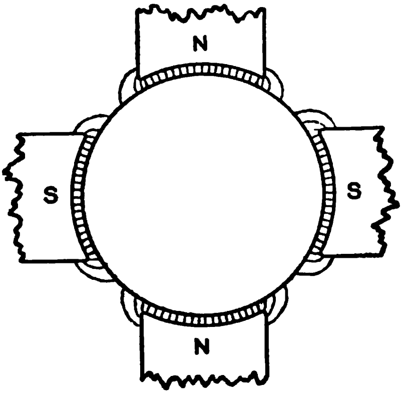

Fig. 114. Undistorted field of a dynamo when armature current is small.

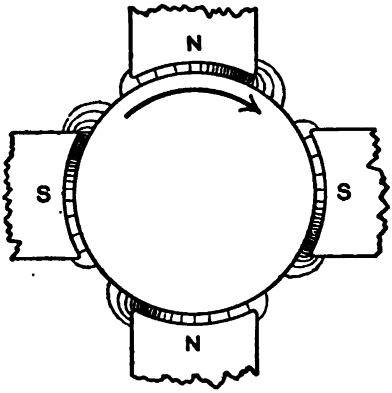

Fig. 115. Distorted field of a generator when its armature current is large.

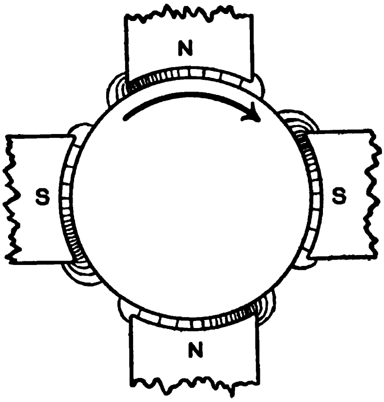

Fig. 116. Same as Fig. 115, but showing motor distortion for same direction of running.

Demagnetizing action and cross-magnetizing action. — The nature of armature reaction and its resolution into the two parts, demagnetizing action and cross-magnetizing action, is most clearly brought out by considering a bipolar dynamo as shown in Fig. 117. The brushes are shown at an angular distance α

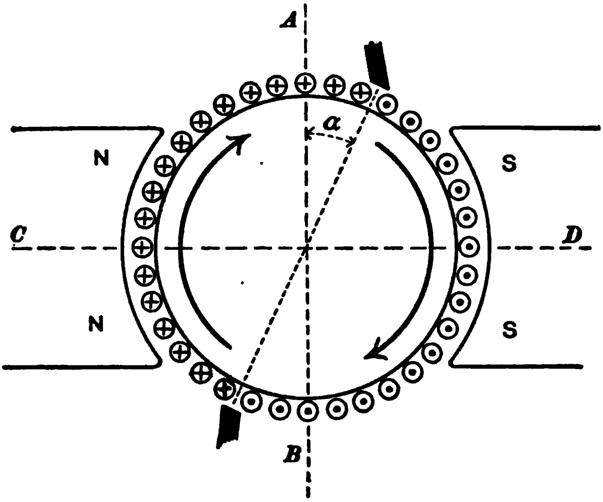

Fig. 117.

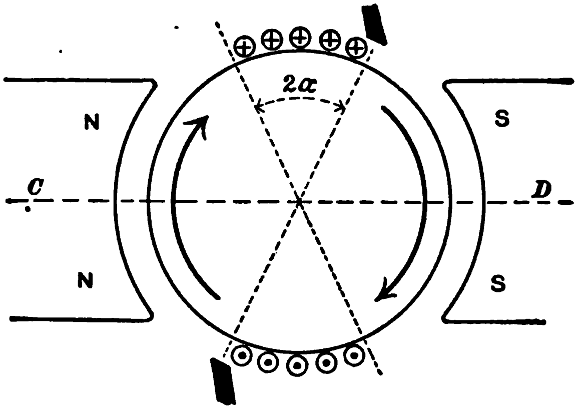

Fig. 118.

ahead of the line AB, which is at right angles to the axis of the field. All of the conductors marked with a dot are supposed to carry outward-flowing current (towards the reader), and all conductors marked with a cross are supposed to carry inward-flowing current (away from the reader).

Figure 118 is like Fig. 117, but it shows only those armature conductors which lie within the angular distance ± *a* from the line *AB*. Each of the conductors in the top group in Fig. 118 may be thought of as paired with a conductor in the bottom group, and the pair may be looked upon as a complete turn of wire, or in other words, all the conductors shown in Fig. 118 may be looked upon as a magnetizing coil surrounding the line *CD* as an axis. The magnetic action of this coil is the demagnetizing action of the armature.

Figure 119 is like Fig. 118 except that it shows only those armature conductors which lie at a greater angular distance than ± *a* from the line *AB*. Each of the conductors on the right in Fig. 119 may be thought of as paired with a conductor on the left, and the pair may be looked upon as a complete turn of wire, or in other words, all the conductors shown in Fig. 119 may be looked upon as a magnetizing coil surrounding the line *AB* (Fig. 117) as an axis. The magnetizing action of this coil is the cross-magnetizing action of the armature.

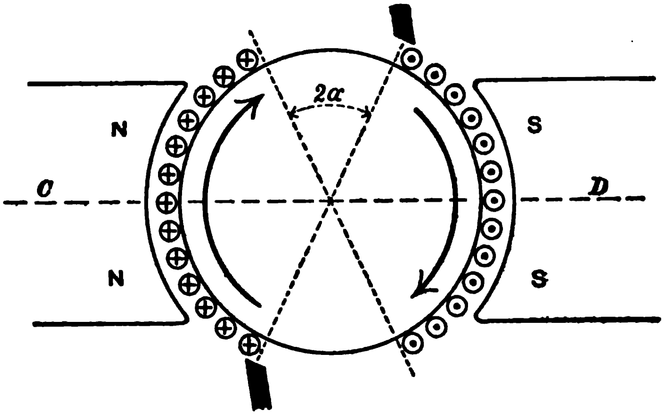

Fig. 119.

The state of affairs shown in Figs. 117, 118 and 119 exists in a generator, as may be seen by applying the rule which correlates direction of a current in a wire, direction of field in which the wire is placed, and direction of side-push on the wire. Furthermore, the demagnetizing action of the conductors shown in Fig. 118 is positive, that is, it is a genuine demagnetizing action which opposes the passage of the flux Φ through the armature, as may be seen by applying the rule which correlates direction of current in a coil and direction of magnetizing action of the coil. To summarize, we have in Fig. 117 a generator of which the brushes have a forward lead from the line *AB* and the armature has a demagnetizing action. In a generator with brushes having a backward lead from the line *AB*, the armature has a magnetizing action which helps the passage of the flux Φ through the armature. In a motor, on the other hand, forward lead of brushes gives magnetizing action, and backward lead of brushes gives demagnetizing action of the armature.

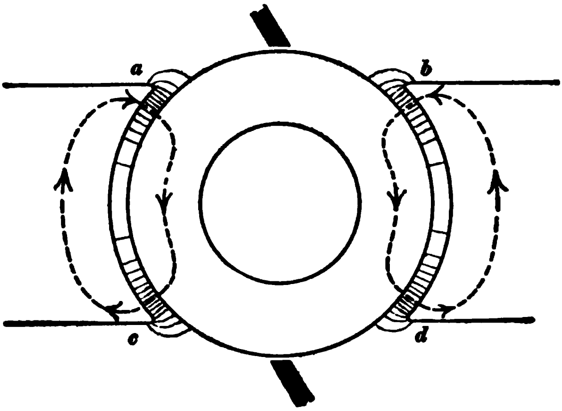

Fig. 120.

The effect of the cross-magnetizing action in the armature of a dynamo may be made more easily intelligible with the help of Figs. 120, 121 and 122. Figure 120 shows the magnetic flux through armature and field poles due to the magnetizing action of the armature alone (field excitation zero), Fig. 121 shows the magnetic flux due to the action of the field coils alone (armature current zero), and Fig. 122 shows the composite effect due to field coils and armature acting together. At the pole tips *a* and *d* the flux is in the same direction, in Figs. 120 and 121, and therefore the resultant flux density under these tips is great, as shown in Fig. 122. At the pole tips *b* and *c* the flux is in opposite directions in Figs. 120 and 121, and therefore the resultant flux density under these tips is small, as shown in Fig. 122.

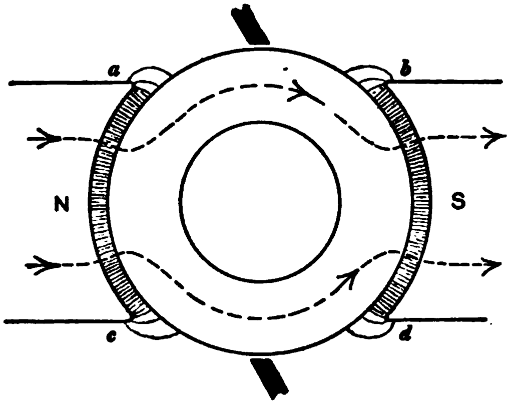

Fig. 121.

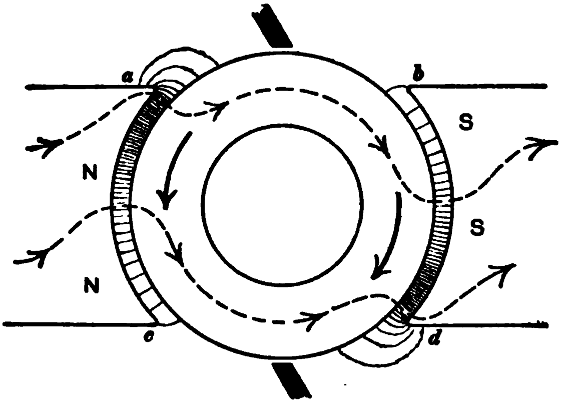

Fig. 122.

Figure 123 shows a multipolar machine with the brushes at an angle α ahead of the diagonal lines *AB*.* The conductors at a greater angular distance than $\pm\,\alpha$ from the line *AB* balance each other's magnetizing action on the magnetic circuit which is shown by the heavy dotted line Φ. That is to say, to each conductor in the group *g* there is a corresponding symmetrically located conductor in the group *h* with an oppositely flowing current in it. On the other hand, the magnetizing action of the conductors which lie within the angular distance $\pm\,\alpha$ from the line *AB* is unbalanced, and this unbalanced magnetizing action is the demagnetizing action of the armature on the magnetic circuit represented by the dotted line Φ.

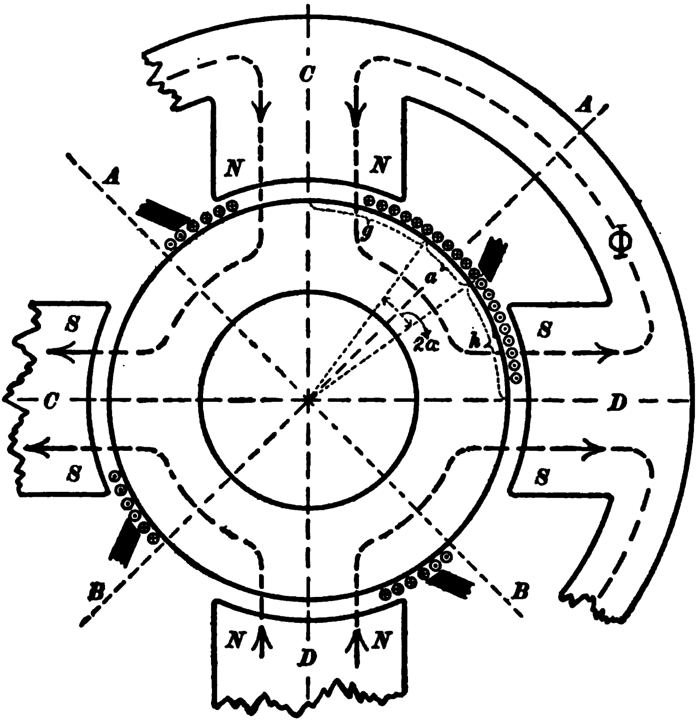

Fig. 123.

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* Which bisect the angles between the lines *CD* which are drawn from the center of the armature to the middle points of the pole pieces.

The value of this demagnetizing action, expressed in ampere-turns, is found by multiplying the number of conductors * within the space 2α by the current flowing in each. That is,

$$D = \frac{a}{180} \cdot \frac{ZI_a}{p'} \tag{41}$$

in which D is the demagnetizing ampere-turns (per magnetic circuit) of a dynamo armature, a is the angle of forward lead of the brushes, Z is the total number of armature conductors, I_a is the total armature current, and p' is the number of paths into which the current I_a divides in flowing through the armature. Equation (41) is true whatever the number of field magnet poles may be.

The cross-magnetizing action of the armature is represented in the above discussion as being due to all the armature conductors which lie at a greater distance than $\pm a$ from the lines AB. It is evident, however, that the conductors which are beyond the pole tips cannot contribute to the magnetic flux shown under the pole tips in Fig. 120. In fact the cross-magnetizing action of the armature depends only upon the number of armature conductors which lie under a pole face as shown in Fig. 124.

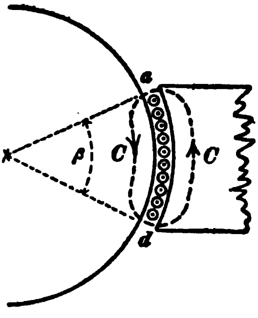

Fig. 124.

The intensity $\mathcal{H}'$, of the magnetic field under the pole tips in Fig. 120 due to the armature alone, may be quite easily calculated as follows: Consider the magnetic circuit CC, Fig. 124, which encircles all of the conductors under a pole face. The number of these conductors is $Z \times \beta / 360$, and the current in each conductor is I_a/p', so that the magnetomotive force around CC is $Z \times \beta / 360 \times I_a/p'$ † ampere-turns, or $4\pi/10 \times Z \times \beta/360 \times I_a/p'$ c.g.s. units.‡ Now the iron of the pole face and of the armature core may be considered to have zero magnetic reluctance as compared with the air in the gap space, so that approximately the whole of the magnetomotive force around the circuit CC is brought into action at the two points a and d where the circuit crosses the gap space. Therefore one half of the whole magnetomotive force is brought into action at a, and one half at d, Fig. 124, so that, dividing the magnetomotive force across the gap at a or d by the distance l in centimeters across the gap, we have the desired value of $\mathcal{H}'$, namely,

$$\mathcal{H}' = \frac{\pi\beta}{1{,}800} \cdot \frac{ZI_a}{p'l} \tag{42}$$

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* If a wire encircles an iron rod which constitutes a magnetic circuit, then the iron rod encircles the wire. In reckoning an ampere-turn, one must be sure either that the wire encircles the iron rod which constitutes the magnetic circuit, or that the magnetic circuit encircles the wire.

† This gives the cross-magnetizing ampere-turns (per magnet pole) of a dynamo armature.

‡ See appendix A of Volume I of Franklin and Esty's *Elements of Electrical Engineering* for a full discussion of magnetomotive force and of the magnetization of iron.

The various devices and schemes for reducing the effect of armature reaction are chiefly of interest to the dynamo builder, but they are of some importance to the operating engineer also, and they are briefly as follows:

In the first place the demagnetizing action of the armature can be reduced to a very small value by making the brush lead small, a condition which is realized, without excessive sparking, in most modern machines. In the second place, the disturbing effect of cross-magnetizing action depends upon the ratio $\mathcal{H}'/\mathcal{H}$, where $\mathcal{H}'$ is the field intensity under the pole tips due to the armature alone, and $\mathcal{H}$ is the field intensity in the gap space due to the field winding alone. The disturbing effect of armature reaction is never troublesome when the value of this ratio is small. (*a*) A small value of the ratio $\mathcal{H}'/\mathcal{H}$ may be obtained by using a very large value of $\mathcal{H}$ or, in other words, by using a very strong field excitation. (*b*) A small value of the ratio $\mathcal{H}'/\mathcal{H}$ may be realized by keeping $\mathcal{H}'$ small; and to keep $\mathcal{H}'$ small for a given current in each path in the armature and for a given number of armature conductors, requires, according to equation (42), the use of narrow pole faces (small value of β), and of a wide air gap (large value of l).

The angular breadth β of the pole faces is never reduced below about 65 per cent. of $360/p$ degrees, where p is the number of field magnet poles.*

The value of $\mathcal{H}'$ may be reduced below the value given by equation (42) by constructing the pole pieces as shown in Fig. 125, so that the magnetic circuit *CC* includes, not only the air gaps at *a* and *d*, but also the very considerable air space at *r*, which does not interfere with the flow of the useful flux Φ.

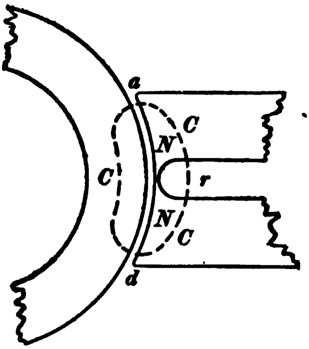

Fig. 125.

The effect of a long gap space may be produced by using narrow teeth and wide slots on the armature. These narrow teeth are then highly saturated by the flux Φ, so that their magnetic reluctance is great, and this high reluctance is the exact equivalent of a long air gap.

A high relucance in the path *CC*, Fig. 125, of the cross-flux, but not in the path of the useful flux Φ, may be produced by lengthening the gap space under the pole tips, leaving a short gap space under the middle of the pole face. This effect may be accomplished most advantageously in a laminated pole piece by cutting away the tips of half of the laminations, thus greatly reducing the sectional area of the iron in the pole tips. Under these conditions the least overcrowding of the flux under a pole tip, as shown in Fig. 122, tends to push the iron of the pole tip beyond magnetic saturation, thus greatly increasing its reluctance. The nearly sparkless operation under all loads, of dynamos of recent design, is largely due to the use of pole pieces of the kind here described.

The effect of armature reaction may be to some extent overcome by the use of what are called inter-poles as described in Art. 97.

The best method of all for reducing armature reaction is by the use of the so-called

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* The ratio $\beta \div 360/p$ (= pole span/pole pitch) is usually made about 0.7 in multipolar dynamos.

compensating winding, which consists of a layer of wires, parallel to the armature shaft, embedded in the pole faces, as shown in Fig. 126, and so connected that the total current, or a definite fraction of the current, from the brushes of the machine flows through each wire, as indicated by the crosses and dots in Fig. 126. The use of this compensating winding is not ordinarily warranted on account of its cost. It is, however, an essential feature of the ordinary commutator motor when driven by alternating current, as exemplified in the recent development of the single-phase alternating-current motor for railways. See Chapter XVI.

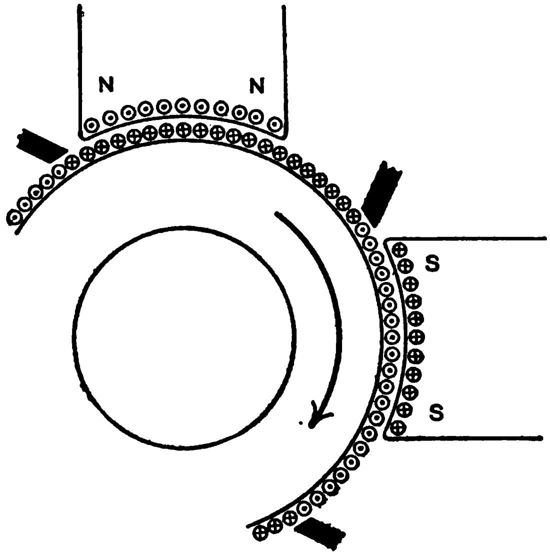

Fig. 126.

96. The phenomena of commutation. Non-sparking condition.—The cause of sparking, and a clear idea of the means available for reducing sparking, may be obtained by a careful consideration of what takes place in a section of the armature winding and at the two commutator bars to which it is connected, when the two bars pass under a brush.

Figure 127 shows the positive brush of a generator, a portion of the commutator with its segments or bars *c*, *d*, *e*, *f* and *g*, a portion of the ring winding of the armature with its sections *T*, *U*, *V*, *W* and *X*, and the leading tip of one of the pole pieces. The line *nn'* represents the neutral axis (see Art. 68).

Before a given section of the armature winding reaches the positive brush, current is flowing through it towards the brush, as indicated by the small arrows in Fig. 127, and after the given section of the armature winding has passed the brush, the current in the section is reversed in direction, still, of course, flowing towards the brush.

The figure shows the instant when the section *V* is short-circuited by the contact of the brush with the two commutator bars *e* and *f*. This state of affairs lasts for a very short time, and *if an action could be brought about whereby the previously existing current in this section could be reduced to zero and an equal and opposite current established in the section during this short time*, then there would not be the least tendency for a spark to form between the brush and the bar *e* at the instant they separate.

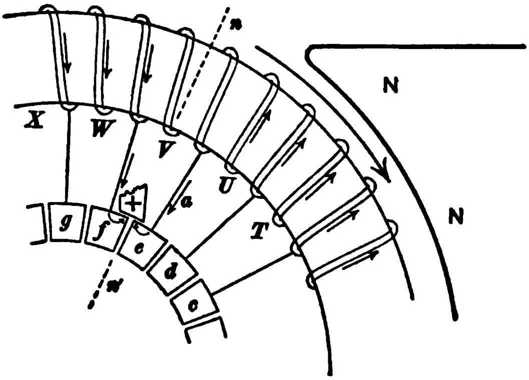

Fig. 127.

(*a*) As to the establishing of a reversed current in the section *V* while it is short-circuited, it is to be noted that electromotive forces are induced in the armature windings everywhere except exactly in the neutral axis, as explained in Art. 68. Therefore, if we give the brush shown in Fig. 127 a forward lead from the neutral axis, the sections as they are short-circuited by the brush will be in a region where they have electromotive forces induced in them in the proper direction to establish the reversed current above mentioned; and the greater the forward lead, the greater these electromotive forces will be, so that an angle of lead may be found by trial for which the condition of sparkless commutation above stated, is completely realized.

(*b*) The use of high resistance brushes of carbon tends not only to cause a decrease to zero of the current existing in the sections before they reach a brush, but also to establish the necessary reversed current during the time that a section is short-circuited by the brush. The decrease to zero is due mainly to the resistance of the brush material which forms a part of the short circuit. The establishment of the reversed current in the sections is due mainly to the contact resistance between the brush and the commutator as follows: When the commutator has moved a little beyond the position shown in Fig. 127, the increased resistance between bar *e* and the brush, due to decreased area of contact, diverts the current *a* through the section *V* as a reversed current in *V*.

It is impracticable to depend solely upon the action described above under (*a*) for producing sparkless commutation, inasmuch as to do so would require the lead of the brushes to be readjusted with every change of load. Therefore the action described above under (*b*) must be depended upon to a greater or less extent.

In the case of generators and of motors which are driven always in the same direction, the brushes are adjusted to give the best commutation at, say, half load, and at greater (or smaller) loads the action (*a*) is deficient (or excessive) and the deficiency (or excess) must be counteracted by action (*b*).

In motors for cranes and electric cars, the direction of running is repeatedly reversed and the brushes are fixed permanently in the diagonal axes *AB*, Fig. 123. In this case the cross-magnetizing action of the armature current shifts the neutral axis so that the action (*a*) tends to *maintain* rather than to *reverse* the current in the short-circuited sections. In this case the action described above under (*b*) is not only the sole cause of the reversal of the current in the short-circuited sections, but in producing this reversal the action (*b*) has to counteract the opposing action (*a*). Motors which are repeatedly reversed would not operate satisfactorily with low-resistance brushes of metal.

A current in a circuit of large inductance has great electrical momentum, and the quick reversal of a current in such a circuit requires a very considerable electromotive force. Therefore the reversal of current in the short-circuited sections of an armature during the brief time that the short-circuit continues, is vitally dependent upon a small value of the inductance of the sections. Small inductance is especially important, inasmuch as the action (*b*) above described is chiefly relied upon, and this action is equivalent to a very small electromotive force. The inductance of an armature section, wound in slots of a given size, is proportional to the square of the number of turns of wire in the section. Therefore a small inductance necessitates few turns of wire in each armature section.

97. The use of commutating poles. The inter-pole motor.— In order to produce sparkless reversal of current in the armature sections of a dynamo at commutation by the action described under (*a*) of Art. 96, the field in which the armature sections are moving at the instant of commutation would have to be intense when the armature current is large, and weak when the armature

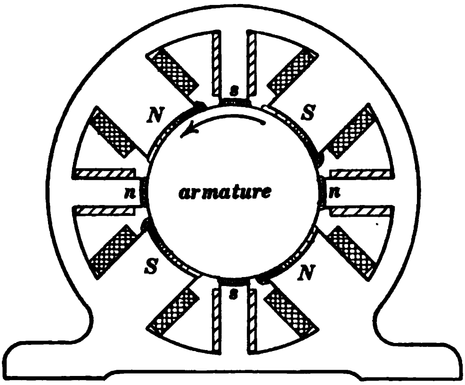

Fig. 128a.

current is small, but the field under the leading pole tips of a generator or the field under the trailing pole tips of a motor (this is the field which must be depended upon in each case for the production of sparkless reversal) becomes weaker with increasing armature current, as shown in Figs. 115 and 116 and as explained in connection with Figs. 120, 121 and 122. It is evident, therefore, that the action described under (*a*) in Art. 96 cannot be depended upon to any great extent for producing sparkless commutation in an ordinary dynamo. Figure 128*a* shows a four-pole dynamo (motor) provided with auxiliary field-poles *nsns* for creating a magnetic field suitable for the reversal of current in the armature sections at commutation. These auxiliary poles are excited by windings of coarse wire which are connected in series with the armature of the machine (in series with the outside receiving circuit in the case of a generator) so that with increase of armature current the field strength under these auxiliary poles increases and is thus able at all loads to produce approximately sparkless reversal of current in the armature sections at commutation. These auxiliary poles are called *commutating poles* and a dynamo which is provided with them is sometimes called an *inter-pole dynamo.*

The use of series-excited commutating poles is especially advantageous in a shunt motor of which the speed is to be controlled through a wide range by means of a field rheostat; the use of the commutating poles obviates to a great extent the tendency of the machine to spark when the shunt field excitation is reduced to a low value and permits therefore of a greatly increased range of speed control. Thus, the inter-pole motors of the Electro-Dynamic Company, Bayonne, New Jersey, have a practicable range of speed control from *n* revolutions per minute at greatest field excitation to 4*n* revolutions per minute at the weakest permissible field excitation, whereas the practicable range of speed control of an ordinary shunt motor is from *n* to 2*n*, or from *n* to 3*n*, at the utmost. Figure 128*b* shows the field structure of the inter-pole motor of the Electro-Dynamic

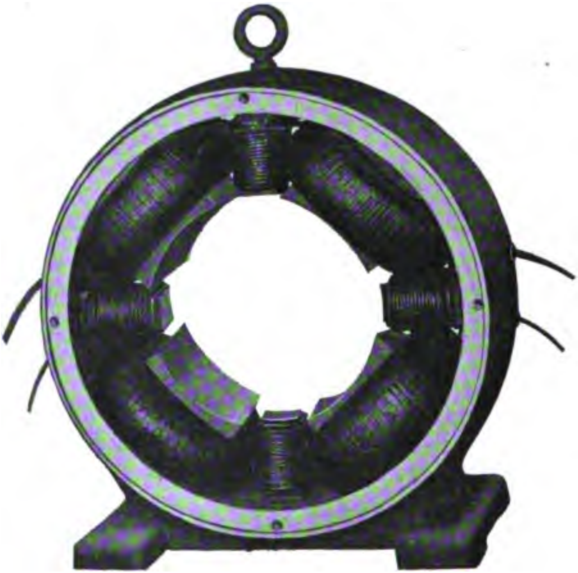

Fig. 128b.

Company. The commutating poles are wound with coarse wire as above explained and these poles are small in area, being narrower than the main field poles and also shorter in the direction parallel to the shaft of the machine.

98. Standard ratings and guarantees. — In the early days of electrical engineering, different manufacturers followed widely divergent rules for fixing the ratings of their machines with the result that ratings were almost meaningless. To remedy this state of affairs and to bring about uniformity in practice in regard to voltages and frequencies, a Committee on Standardization was appointed by the American Institute of Electrical Engineers in 1898. The first report of this committee was laid before the Institute in June, 1899. The revised report is dated May, 1902, and a second revision was made in 1907.*

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* See *Proceedings of American Institute of Electrical Engineers*, July, 1907, pages 1076 to 1106. Copies of this report may be obtained for ten cents from the Secretary of the American Institute of Electrical Engineers, 33 West Thirty-ninth St., New York City. Every student of electrical engineering should have a copy of this report.

PART III.

ELEMENTARY THEORY OF ALTERNATING CURRENTS.

CHAPTER VIII.

HARMONIC ELECTROMOTIVE FORCE AND CURRENT.

99. Definition of harmonic electromotive force and current. — A harmonic electromotive force (or current) is an electromotive force (or current) which is at each instant proportional to the sine or cosine of an angle which increases at a constant rate. Thus, ωt is an angle which increases at a constant rate, t being elapsed time reckoned from some chosen instant, and ω being a constant angular velocity, so that a harmonic electromotive force e may be represented by the equation

$$e = \mathbf{E} \sin \omega t \tag{43}$$

A harmonic electromotive force (or current) is represented, in rectangular coördinates, by a curve of sines of which the abscissas represent elapsed times, and the ordinates represent successive values of electromotive force (or current).

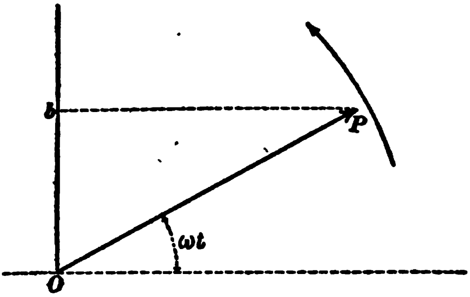

Fig. 129.

The clock diagram. — A line OP, Fig. 129, rotates at a uniform angular velocity of f revolutions per second, or ω ($= 2\pi f$) radians per second, about the point O in the direction of the curved arrow, and the vertical projection of OP is at each instant equal to OP times the sine of the angle ωt. Therefore, if the length of OP represents the maximum value **E** (or **I**) of a harmonic electromotive force (or current), then the vertical projection Ob will represent at each instant the actual value of the harmonic electromotive force e (or current i).

Consider, for example, the two lines **E** and **I**, Fig. 130, which are imagined to be rotating f revolutions per second in the direction of the curved arrow, so that their vertical projections may represent successive instantaneous values of a harmonic electromotive force and of a harmonic current, respectively. Under these conditions the lines **E** and **I** are said to *represent* the harmonic electromotive force and current, and the diagram, Fig. 130, is called a *clock diagram.*

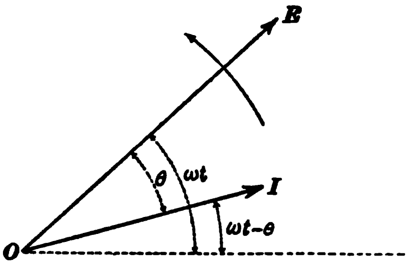

Fig. 130.

The definitions of cycle, period and frequency given in Art. 29 apply also to harmonic electromotive forces and currents. In case of harmonic electromotive force and current, however, the frequency f can be thought of as the number of revolutions per second of the lines **E** and **I** in Fig. 130, or f can be expressed in *radians per second*, ω; and, since there are 2π radians in a revolution, we have

$$\omega = 2\pi f \tag{44}$$

100. Phase difference. — Consider a harmonic electromotive force e and a harmonic current i of the same frequency. It often happens that the current reaches its maximum value *after* the electromotive force has passed its maximum value, as shown in Fig. 131. In this case the current is said to *lag behind* the electromotive force in phase. In some cases the current reaches its maximum value *before* the electromotive force reaches its maximum value; and in such a case the current is said to be *ahead of* the electromotive force in phase. When a current is behind an electromotive force in phase, it is called a *lagging current;* and when a current is ahead of an electromotive force in phase, it is called a *leading current.*

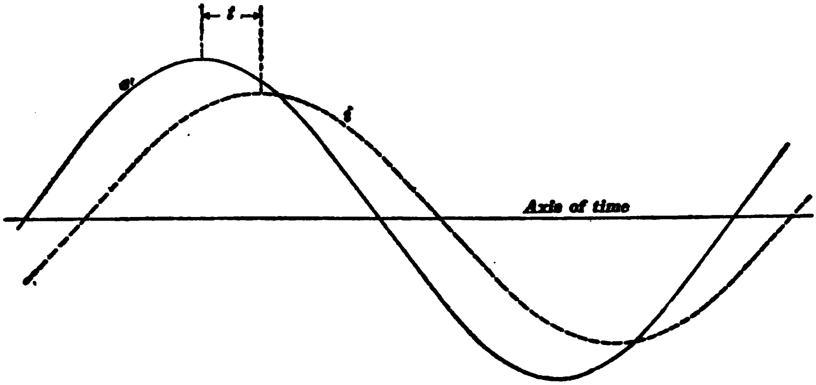

FIG. 131.

The time interval t, Fig. 131, between the instant of maximum e and the instant of maximum i is called the *phase difference* of e and i; it is, however, most convenient to express phase difference as an angle. Thus, Fig. 132 is a clock diagram in which the lines **E** and **I** represent the harmonic electromotive force e and the harmonic current i which are shown in Fig. 131, and the phase difference is represented by the angle θ.

The electromotive force and current represented in Fig. 130 differ in phase, and the algebraic expressions for the vertical projections of the lines **E** and **I** are

$$e = \mathbf{E} \sin \omega t$$

and

$$i = \mathbf{I} \sin (\omega t - \theta)$$

When the angle θ is zero, e and i are said to be *in phase* with each other. In this case e and i reach their maximum values together, they pass through zero together, and so on.

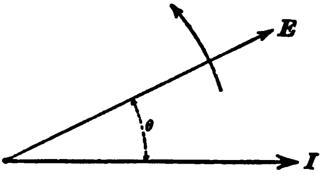

FIG. 132.

When the angle θ is 90°, e and i are said to be in *in quad-*

rature with each other. In this case e is zero when i is at its maximum value, and *vice versa.*

When the angle θ is 180°, e and i are said to be *in opposition.* In this case e and i pass through zero together, but when one is positive the other is negative.

101. Composition and resolution of harmonic electromotive forces and currents. — It often happens that two or more harmonic electromotive forces (or currents) are added together. Thus, when two alternators are connected in series their electromotive forces are added, and when two alternators are connected in parallel their currents are added. Again, it often happens that a given harmonic electromotive force (or current) is divided into parts. Thus, when two coils are connected in series between supply mains, the supply voltage is divided between them, and when two coils are connected in parallel between supply mains, the total current is divided between them. The adding of harmonic electromotive forces (or currents) is called *composition,* and the dividing of a harmonic electromotive force (or current) into parts is called *resolution.*

Composition. — Consider two harmonic electromotive forces e_1 and e_2 of the same frequency. Let e_1 and e_2 be represented by the lines E_1 and E_2 in the clock diagram, Fig. 133. The sum $e_1 + e_2$ is a harmonic electromotive force of the same frequency as e_1 and e_2, and this sum is represented by the line E, Fig. 133, which is the geometric or vector sum of the lines E_1 and E_2. This is evident when we consider that *the projection, on any line, of the diagonal of a parallelogram is equal to the sum of the projections of the two adjacent sides of the parallelogram.*

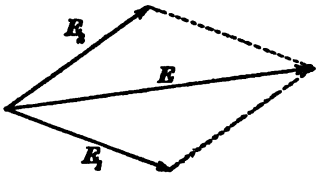

Fig. 133.

Resolution. — A given harmonic electromotive force (or current) may be resolved into a number of parts each of which is a harmonic electromotive force (or current) of the same frequency as the given harmonic electromotive force (or current). Thus the line **E**, Fig. 133, represents a given harmonic electromotive force, and the lines $\mathbf{E}_1$ and $\mathbf{E}_2$ represent two parts into which the given electromotive force may be resolved.

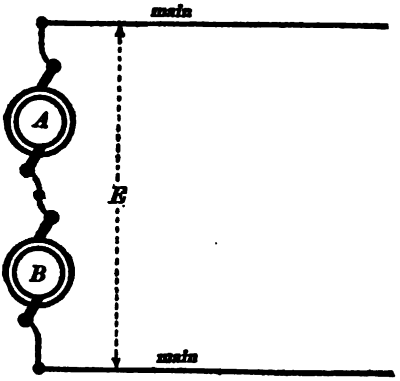

Fig. 134a.

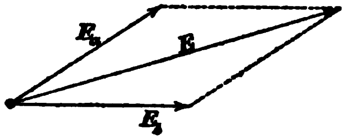

Fig. 134b.

Examples of composition and resolution. — (*a*) Two alternating-current generators *A* and *B* running in synchronism, are connected in series between mains as shown in Fig. 134*a*, and the total electromotive force between the mains is related to the electromotive forces of the respective machines as shown in Fig. 134*b*. The phase difference between $\mathbf{E}_a$ and $\mathbf{E}_b$ may have any value whatever.

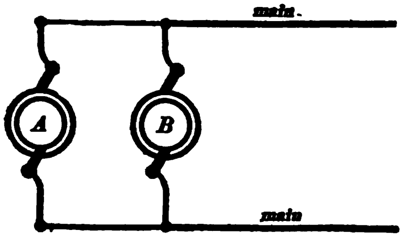

Fig. 135a.

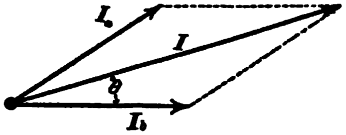

Fig. 135b.

(*b*) Two alternating-current generators *A* and *B* running in synchronism are connected in parallel between mains as shown in Fig. 135*a*, and the total current delivered to the mains is related to the currents delivered by the respective machines as shown in Fig. 135*b*. The phase difference between $\mathbf{I}_a$ and $\mathbf{I}_b$ may have any value whatever.

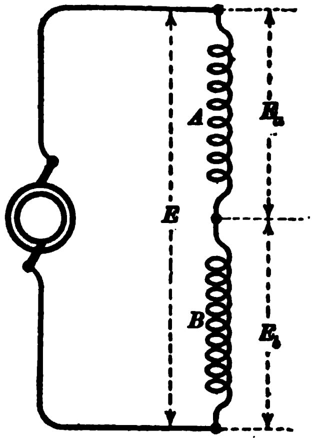

Fig. 136a.

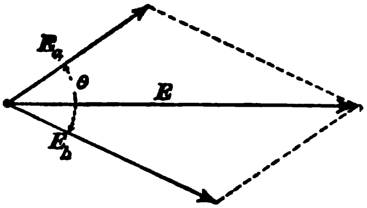

Fig. 136b.

(*c*) Figure 136*a* shows two coils (or receiving units of any kind) *A* and *B*, connected in series between alternating-current supply mains. The voltage between the mains is represented by the line **E** in Fig. 136*b*, and the voltages across *A* and across *B* are represented by the lines $\mathbf{E}_a$ and $\mathbf{E}_b$ respectively. The phase difference θ may have any value between zero and 180°.

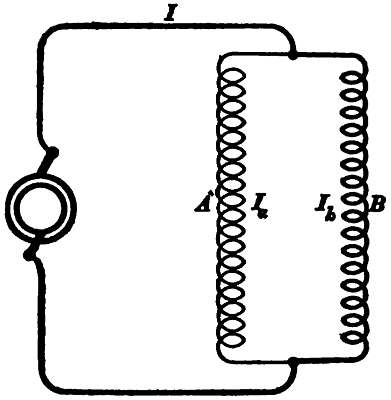

Fig. 137a.

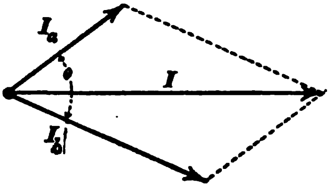

Fig. 137b.

(*d*) Figure 137*a* shows two coils (or receiving units of any kind) *A* and *B*, connected in parallel between alternating-current supply mains. The total current delivered to *A* and *B* is represented by the line I in Fig. 137*b*, and the currents in *A* and *B* are represented by the lines I_a and I_b respectively. The phase difference θ may have any value between zero and 180°.

102. Rate of change of harmonic current. — When the current in a circuit changes, a portion of the electromotive force which acts on the circuit is used to cause the current to change. The portion of the electromotive force so used is equal to the product of the inductance of the circuit and the rate of change of the current. Therefore a discussion of the relation between current and voltage in an alternating-current circuit depends upon a knowledge of the rate of change of the current.

The rate of change di/dt *of a harmonic current* i *is a harmonically varying quantity, that is,* di/dt *is proportional at each instant to the sine or cosine of a uniformly increasing angle* ωt. *The maximum value of* di/dt *is equal to* ω *times the maximum value of* i, *and* di/dt *is* 90° *ahead of* i *in phase.*

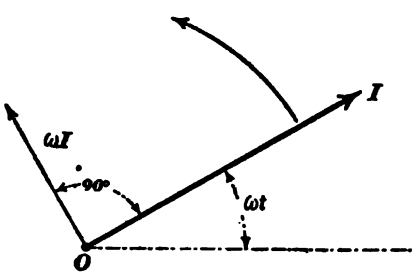

Fig. 138.

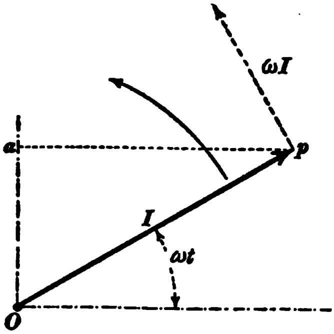

Fig. 139.

The meaning of this proposition may be made clear by a clock diagram. Thus the vertical projection of the rotating line I, Fig. 138, represents the value of i at each instant, and the vertical projection of the line ωI represents the value of di/dt at each instant.

The truth of this proposition is evident from the following considerations: The length of the rotating line Op, Fig. 139, is I, its angular velocity is ω, and the actual velocity of the end p of the line is ωI, as shown by the dotted arrow in Fig. 139. Furthermore, the vertical projection of this velocity is the velocity of the point a, the velocity of a is the rate of change of Oa, and Oa is the value of i.

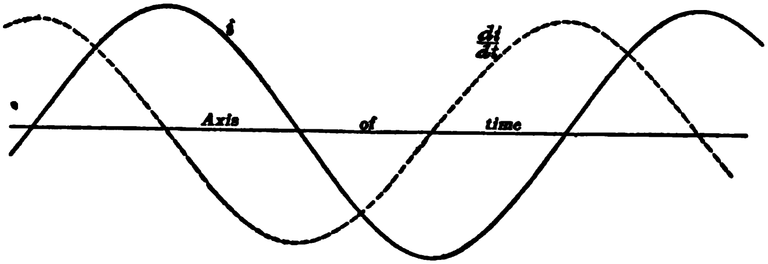

Fig. 140.

The truth of the above proposition may also be shown by differentiating the expression for i, namely,

$$i = I \sin \omega t$$

which gives

$$\frac{di}{dt} = \omega I \cos \omega t$$

or

$$\frac{di}{dt} = \omega I \sin (\omega t + 90^\circ)$$

The relation between a harmonic alternating current i and its rate of change, di/dt, is so important in the discussion of the relation of voltage and current, that it is worth while to show this relation in every possible way. Thus, the ordinates of the full-line curve in Fig. 140 represent the successive values of an alternating current i, and the steepness of this curve represents the value of di/dt at each instant. To show the relation between i and di/dt still more clearly, a curve may be drawn, the dotted curve in Fig. 140, of which the ordinates at each point represent the steepness of the current curve. This dotted curve has its maximum value where the current curve crosses the axis; that is, the maximum positive value of di/dt occurs one quarter of a cycle (= 90° of angle) before the maximum positive value of i.

Example. — A harmonic alternating current has a frequency of 60 cycles per second or $2\pi \times 60$ radians per second $(=\omega)$, and its maximum value is 100 amperes. The maximum rate of change of this current occurs at the instant that the current passes through zero, and it is equal to $2\pi \times 60 \times 100$ amperes per second, or 37,699 amperes per second. To force this current through a circuit having an inductance of 0.1 henry, but having no resistance at all, would require a harmonic electromotive force whose maximum value would be $0.1 \times 37{,}699$ volts, or 3769.9 volts, and this electromotive force would be 90° ahead of the current in phase.

103. Average and effective values of harmonic electromotive forces and currents. (*a*) *Proposition concerning average value. — The average value of a harmonic electromotive force (or current) during a half cycle is equal to two times the maximum value* **E** (*or* **I**) *divided by* π.*

The average value during one or more complete cycles is of course equal to zero.

(*b*) *Proposition concerning effective value. — The square root of the average square of a harmonic electromotive force (or current) during one or more whole cycles is equal to the maximum value* **E** (*or* **I**) *divided by the square root of two.*† That is,

$$E = \frac{\mathbf{E}}{\sqrt{2}} \tag{45}$$

$$I = \frac{\mathbf{I}}{\sqrt{2}} \tag{46}$$

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* See page 61, Vol. 2, Franklin and Esty's *Elements of Electrical Engineering.*

† See page 62, Vol. 2, Franklin and Esty's *Elements of Electrical Engineering.*

The use of effective values of electromotive force and current in the clock diagram.— According to Art. 99 the lines in a clock diagram are supposed to represent *maximum values* of electromotive force and current, but, since the ratio of maximum values to effective values is a constant according to equations (45) and (46), it is permissible and indeed it is customary to consider that the lines in a clock diagram represent *effective values.*

104. Expression for power in the case of harmonic electromotive force and current.— The power developed by an alternating electromotive force pulsates as explained in Art. 54, and the average power (average value of $\overline{ei}$) is the important consideration.

The average power P developed by a harmonic electromotive force e $(= \mathrm{E}\sin \omega t)$ *in maintaining a harmonic current i* $[= \mathrm{I}\sin(\omega t - \theta)]$ *is*

$$P = \frac{\mathrm{EI}}{2}\cos\theta$$

or

$$P = EI\cos\theta \qquad (47)^*$$

in which E and I are the effective values of the electromotive force and current respectively, and θ is their phase difference, as shown in Fig. 141.

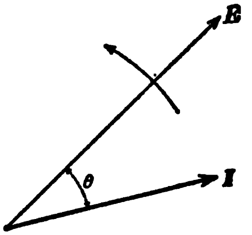

Fig. 141.

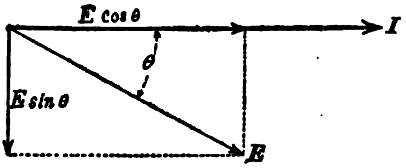

Fig. 142.

Power factor. — By comparing equation (47) with equation (27), it is evident that the power factor of a receiving circuit is equal to the cosine of the angle of phase difference between the harmonic electromotive force which acts on the circuit and the harmonic current which is maintained in the circuit.

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* See page 63, Vol. 2, Franklin and Esty's *Elements of Electrical Engineering.*

Power-component and wattless-component of electromotive force. — Let the clock diagram, Fig. 142, represent a given harmonic electromotive force E and the harmonic current I which is maintained by E, let the line I be chosen as the reference axis, and let us consider the two components of E, namely, $E \cos \theta$ and $E \sin \theta$, as shown. The component, $E \cos \theta$, which is parallel to I is called the *power-component* of E, and the component, $E \sin \theta$, which is at right angles to I is called the *wattless-component* of E.

So-called power-component and wattless-component of current. — Let the clock diagram, Fig. 143, represent a given harmonic electromotive force E and the harmonic current I which is maintained by E, let the line E be chosen as the reference axis, and let us consider the two components of I, namely, $I \cos \theta$ and $I \sin \theta$, as shown. The component $I \cos \theta$ which is parallel to E is sometimes called the *power-component* of the current, and the component $I \sin \theta$ which is at right angles to E is sometimes called the *wattless-component* of the current.

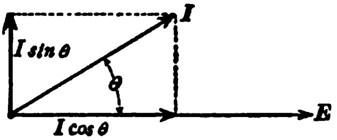

Fig. 143.

It is more legitimate, physically, to speak of a wattless electromotive force than it is to speak of a wattless current. Thus, the rate at which heat is generated in a circuit by a harmonic current is RI^2, and this is equal to $R \times$ (power component of $I)^2$ *plus* $R \times$ (wattless component of $I)^2$. That is, there is no such thing, physically, as a wattless current in a circuit which has resistance, although a given current may be resolved into two components as shown in Fig. 143, and the component $I \sin \theta$ *does not receive power from the generator.*

CHAPTER IX.

FUNDAMENTAL PROBLEMS.

105. The current-voltage relation. — *To determine the electromotive force required to maintain a given harmonic current in a circuit having resistance and inductance and containing a condenser.*

A coil of resistance R and inductance L, and a condenser of capacity C are connected in series across alternating-current mains as shown in Fig. 144. An alternating current flows back and forth through the coil and charges the condenser first in one direction and then in the reverse direction repeatedly. The problem of finding the relation between the current in the coil and the electromotive force between the mains is reduced to its simplest form as follows:

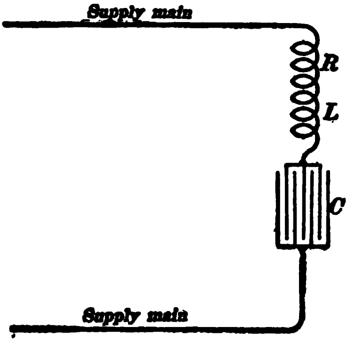

Fig. 144.

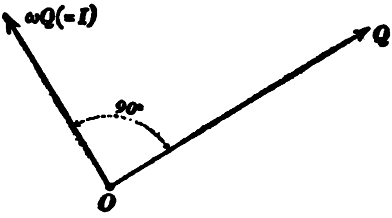

Fig. 145.

To determine the electromotive force necessary to make the charge q on the condenser vary harmonically, so that

$$q = Q \sin \omega t \tag{a}$$

in which t is the elapsed time, ωt is an angle increasing at a constant rate, and Q is the maximum charge on the condenser.

This varying charge may be represented by the rotating line Q in the clock diagram, Fig. 145. The current in the circuit is the rate of change of q, that is,

$$i = \frac{dq}{dt} \tag{b}$$

Now, in Art. 102 it is shown that the *rate of change of a harmonically varying quantity* is another harmonically varying quantity of the same frequency, that its maximum value is ω times the maximum value of the given harmonically varying quantity, and that it is 90° ahead of the given harmonically varying quantity in phase. That is, $dq/dt\,(=i)$ is 90° ahead of q in phase, and its maximum value is $\omega\mathrm{Q}\,(=\mathrm{I})$ as shown in Fig. 145.

The total electromotive force required to cause the charge on the condenser to vary according to equation (*a*) under the conditions shown in Fig. 144, may be considered in three parts:

1. The part required to overcome the resistance R. This part is equal at each instant to Ri, and it is therefore a harmonic electromotive force in phase with i and its maximum value is $R\mathrm{I}$.

2. The part required to cause the current i to increase and decrease, or, in other words, the part required to overcome the reaction of the inductance L. This part is equal to $L\,(di/dt)$ at each instant, and it is therefore (see Art. 102) a harmonic electromotive force which is 90° ahead of i in phase, and its maximum value is $\omega L\mathrm{I}$.

3. The part required at each instant to hold the charge q on the condenser, or, in other words, the part that is required to overcome the reaction of the condenser. Now, the charge q on a condenser is equal to eC, where e is the electromotive force across the condenser terminals, and C is the capacity of the condenser, so that $e = q/C$. Therefore the electromotive force required to overcome the reaction of the condenser is equal to q/C, that is, it is in phase with q (or 90° behind i), and its maximum value is Q/C (or $\mathrm{I}/\omega C$, since $\mathrm{I} = \omega\mathrm{Q}$).

Solution of the given problem by the clock diagram. — Let the line I, Fig. 146, represent the given harmonic current (the same as the line ωQ in Fig. 145), then the three parts of the total electromotive force are represented by the lines *R*I, ω*L*I and I/ω*C*, respectively, as shown, and the total electromotive force is represented by the line E which is the vector sum of *R*I, ω*L*I and I/ω*C*. Therefore, from the right triangle whose sides are (*R*I), (ω*L*I − I/ω*C*) and E we have

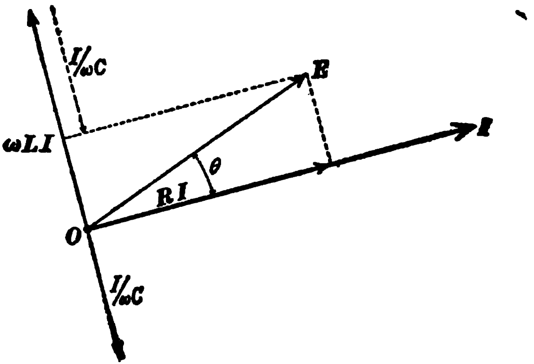

Fig. 146.

$$\mathbf{E} = \mathbf{I}\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

or, using equations (45) and (46) of Art. (103), we have

$$E = I\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \tag{48}$$

and

$$\tan\theta = \frac{\omega L - \dfrac{1}{\omega C}}{R} \tag{49}$$

These two equations determine the effective value and phase of the harmonic electromotive force required to maintain a given harmonic current, or they determine the effective value and phase of the harmonic current that is maintained by a given harmonic electromotive force under the conditions represented in Fig. 144.

106. Resistance, reactance and impedance. — By inspection of Fig. 146 it is seen that the component of E parallel to I is equal to RI.[*] *The* **resistance** *of an alternating-current circuit is in general defined as that factor which multiplied by I gives the component of E parallel to I.* In the case discussed above, the resistance of the circuit is the ordinary resistance R of the coil shown in Fig. 144. This simple case is the only one that need now be kept in mind, although the definition here given applies to cases to be discussed later, in which the so-called *equivalent resistance* of an alternating-current circuit is very different from the resistance of the circuit as ordinarily defined.[†]

By inspection of Fig. 146 it is seen that the component of E, which is 90° ahead of I in phase, is equal to $(\omega L - 1/\omega C)I$. *The factor X $(= \omega L - 1/\omega C)$, by which an alternating current is to be multiplied to give the component of E which is 90° ahead of I in phase, is called the* **reactance** *of the alternating-current circuit.* According to this definition the reactance of the circuit is to be considered as negative if E is behind I in phase. In the case discussed above, the reactance is equal to $(\omega L - 1/\omega C)$, and this simple case is the only one that need now be kept in mind, although the definition here given applies to cases to be discussed later, in which the so called *equivalent reactance* of an alternating-current circuit is not equal, simply, to $(\omega L - 1/\omega C)$.[‡]

By inspecting equation (48) it is seen that the effective value of an alternating electromotive force is equal to the effective value of the current multiplied by the square root of the sum of the squares of the resistance and the reactance of the circuit. The square root of $(R^2 + X^2)$ is called the *impedance* of the circuit.

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[*] See Art. 103, for a statement as to the representation of effective values in the clock diagram.

[†] See the topic "equivalent resistance" in the index.

[‡] See the topic "equivalent reactance" in the index.

By substituting the value

$$X = \omega L - \frac{1}{\omega C} \tag{50}$$

in equations (48) and (49) we have

$$\left.\begin{aligned} E &= I\sqrt{R^2 + X^2} \\ \text{or} \qquad I &= \frac{E}{\sqrt{R^2 + X^2}} \end{aligned}\right\} \tag{51}$$

and

$$\tan\theta = \frac{X}{R} \tag{52}$$

Resistance, reactance and impedance are all expressed in ohms. It is to be remembered, however, that reactance, and of course impedance also, depend upon frequency as well as upon the conditions actually existent in the circuit.

107. Conductance, susceptance and admittance. — *The factor g by which the electromotive force E is multiplied to give the component of I parallel to E is called the* **conductance** *of an alternating-current circuit.*

The factor b by which the electromotive force E is multiplied to give the component of I, which is 90° behind E, is called the **susceptance** *of an alternating-current circuit*, and

The factor $\sqrt{g^2 + b^2}$ *by which the value of the electromotive force must be multiplied to give the value of I is called the* **admittance** *of an alternating-current circuit.*

The quantities conductance, susceptance and admittance are related to resistance, reactance and impedance as follows:

$$g = \frac{R}{R^2 + X^2} \tag{i}$$

$$b = \frac{X}{R^2 + X^2} \tag{ii}$$

$$\text{admittance} = \frac{1}{\text{impedance}} \tag{iii}$$

108. Special cases of the problem of Art. 105. (*a*) *Non-inductive circuit.* — In this case the electromotive force must overcome resistance only, and therefore the electromotive force is equal to Ri at each instant, that is, the electromotive force is in phase with i and its effective value is equal to RI.

(*b*) *Circuit containing resistance and inductance.*— In this case the component of E parallel to I is equal to RI, the reactance of the circuit is equal to ωL, and the component of E which is 90° ahead of I is equal to ωLI, as shown in Fig. 147. In this case the current lags behind E in phase by the angle whose tangent is $\omega L/R$, an angle which approaches 90° when ωL is very large compared with R.

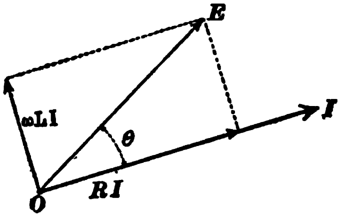

Fig. 147.

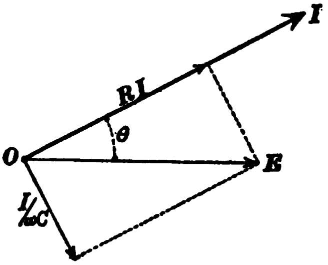

Fig. 148.

(*c*) *Circuit containing resistance and a condenser.* — In this case the component of E parallel to I is equal to RI, the reactance of the circuit is equal to *minus* $1/\omega C$, and the component of E which is 90° *behind* I is equal to $I \times 1/\omega C$ as shown in Fig. 148. In this case the current is ahead of E in phase by the angle whose tangent is $1/\omega C$ divided by R, an angle which approaches 90° when $1/\omega C$ is very large compared with R.

109. Electric resonance. — A harmonic electromotive force of given effective value will produce the greatest possible value of alternating current in a given circuit (given resistance, given inductance and given capacity arranged in a series as shown in Fig. 144) if the frequency of the electromotive force is such as to make

$$\omega L - \frac{1}{\omega C} = 0 \tag{1}$$

This is at once evident from equation (48) of Art. 105. *This production of a maximum current in a given circuit by an electromotive force of a given effective value* **at a certain critical fre-**

quency *is called* **resonance.** The critical frequency in radians per second (ω) is determined by equation (i), namely,

$$\omega = \sqrt{\frac{1}{LC}} \tag{ii}$$

or since the frequency f in cycles per second is equal to $\omega/2\pi$, we have

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \tag{iii}$$

At the critical frequency the reactance $(\omega L - 1/\omega C)$ is equal to zero, equation (48) of Art. 105 reduces to $I = E/R$, and equation (49) of Art. 105 shows that θ is zero. That is, at the critical frequency the reaction of the condenser annuls the reaction of the inductance, and, *after the current becomes established*, the circuit behaves as if it contained resistance only. The annulling of inductive reaction by capacity reaction means simply that the electromotive force ωLI, Fig. 146, which overcomes inductance is equal and opposite to the electromotive force $I/\omega C$, which overcomes the reaction of the condenser, so that the electromotive force which acts on the circuit has to overcome resistance only.

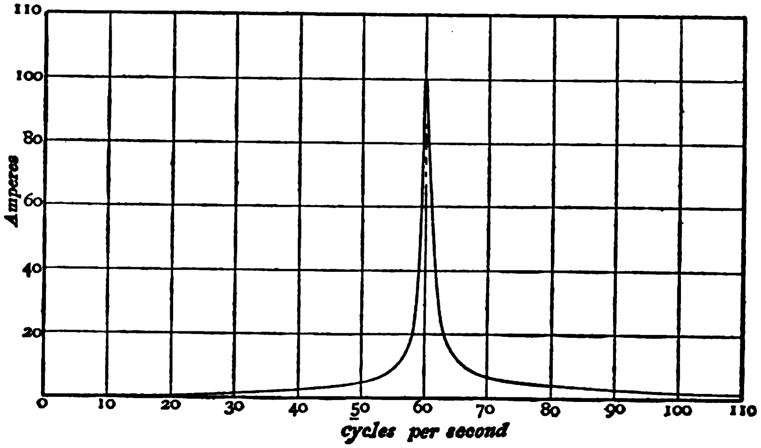

Fig. 149.

The very sharply defined frequency at which resonance occurs is shown in Fig. 149. The ordinates of the curve represent the effective values of the current produced by a harmonic electromotive force of 200 volts (effective) in a circuit like Fig. 144 in which $R = 2$ ohms, $L = 0.352$ henry, and $C = 20 \times 10^{-6}$ farad; and the abscissas represent various frequencies. At very low frequencies the value of $1/\omega C$ is very great, and the current is limited by the condenser reaction; at the critical frequency of 60 cycles per second, the current is equal to E/R (= 100 amperes effective); and at very high frequencies the value of ωL is very great, and the current is limited by the inductance reaction. It is interesting to note that at the critical frequency of 60 cycles per second, with a current of 100 amperes, the electromotive force across the condenser terminals is 13,270 volts effective ($= I/\omega C$) although the total electromotive force acting on the circuit is only 200 volts.

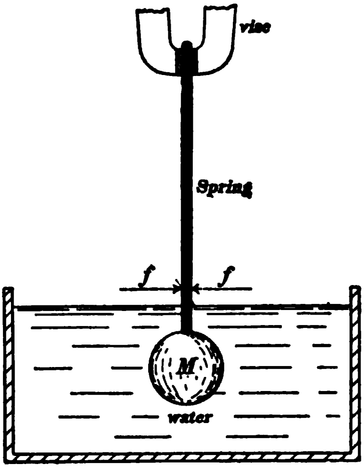

Fig. 150.

Mechanical resonance, analog of electric resonance. — A weight M, Fig. 150, is attached to the end of a flat spring which is clamped in a vise, and the weight M is caused to oscillate back and forth in a vessel of water by an alternating force f of definite frequency. In general the force f has to do three things, namely: (*a*) Overcome the resistance with which the water opposes the to-and-fro motion of M; (*b*) overcome the inertia of the weight M as it gains and loses velocity; and (*c*) overcome the elastic reaction of the spring. These three parts of f are related to each other and to the changing velocity v of M in exactly the same way that the three parts, Ri, $L(di/dt)$ and q/C (see Art. 105), of an impressed electromotive force are related to each other and to the changing current i in the circuit shown in Fig. 144. The weight M when mounted as shown in Fig. 150 has a definite frequency of free oscillation, and at this frequency the elasticity of the spring takes care of the inertia reaction of M, and the inertia reaction of M provides the force necessary to bend the spring. Therefore, if the frequency of the force f, Fig. 150, is the same as the frequency of the free oscillations of M, then, after the oscillatory motion of M is fully established, the force f is used only to overcome resistance.

A careful consideration of this mechanical analog will serve to elucidate one of the most important aspects of electrical resonance. Suppose that the mechanical resistance of the water in Fig. 150 is not very large, then a very small alternating force f applied to the system shown in Fig. 150 will, if it continues to act for some time, build up a very violent oscillatory motion, and the spring will be subjected to bending forces vastly greater than the maximum value of the force f. In fact the spring may be broken by the continued application of a comparatively weak alternating force of the proper frequency. Similarly a weak alternating electromotive force of the critical frequency applied to the circuit shown in Fig. 144 will, if it continues to act for some time, build up a very large alternating current, and the electromotive force across the condenser terminals ($= I/\omega C$) may become so large as to break through the insulation of the condenser.

110. Multiplication of electromotive force by resonance. — When resonance exists in a circuit containing a condenser and an inductance in series, the effective electromotive force ωLI across the terminals of the inductance and the effective electromotive force $I/\omega C$ across the terminals of the condenser may both be much greater than the effective electromotive force RI which acts on the circuit. This fact is easily understood by referring to the mechanical analog. Thus, if a periodic force acts on a weight which is suspended by a helical spring, the weight will be set into very violent oscillation even if the periodic force is weak, provided the frequency of the force is the same as the frequency of free oscillation of the weight. Under these conditions the force which alternately stretches and compresses the spring, and the force which accelerates the weight up and down may both be enormously greater than the periodic force which acts on the system from outside.

Example. — Under the conditions specified in connection with Fig. 149, namely, $R = 2$ ohms, $L = 0.352$ henry, and $C = 20 \times 10^{-6}$ farad, the electromotive force of 200 volts at the critical frequency produces 100 amperes, and ωLI and $I/\omega C$ are both equal to 13,270 volts.

111. Multiplication of current by resonance. — When an inductance and a condenser are connected in parallel,* as shown in Fig. 151, the total current delivered to the two divides, and the current in each may greatly exceed the total current. The current relations are shown in the clock diagram, Fig. 152. In this figure the line OE represents the electromotive force between the branch points a and b, the line OI_1 represents the current

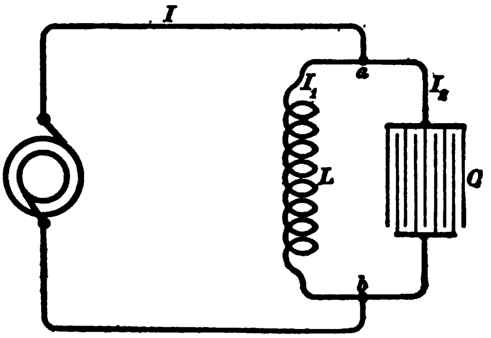

Fig. 151.

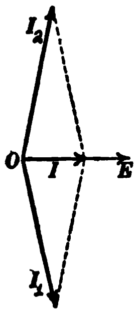

Fig. 152.

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* Although the inductance and the condenser in Fig. 151 are in *parallel* with each other with reference to the alternator, they are in *series* with each other in the short circuit *LaCb*, and *multiplication of current by resonance is due to the surging of current to and fro around this short circuit* so that the physical action of the inductance and condenser during resonance is exactly the same in Fig. 151 as it is in Fig. 144. In both cases the condition of resonance is given by equation (i), Art. 109.

that flows through the inductance, the line OI_2 represents the current that flows through the condenser, and the line OI represents the total current delivered by the alternator.

The multiplication of current by resonance may be easily understood by referring to the mechanical analog. A lever *ll*, Fig. 153, is suspended at its center by a bar *b* which oscillates up and down through a small amplitude. One end of the lever carries a weight *L*, and the other end is held down by a helical spring *C*. If the up-and-down oscillations of the bar *b* take place at the proper frequency, the lever will be set into a violent see-saw motion, and the velocities I_1 and I_2 of the ends of the lever will greatly exceed the velocity I of the center of the lever, although (half) the sum of the velocities of the ends of the lever is at each instant equal to the velocity of the center of the lever, just as the sum of the currents in *L* and *C*, Fig. 151, is at each instant equal to the current delivered by the alternator.

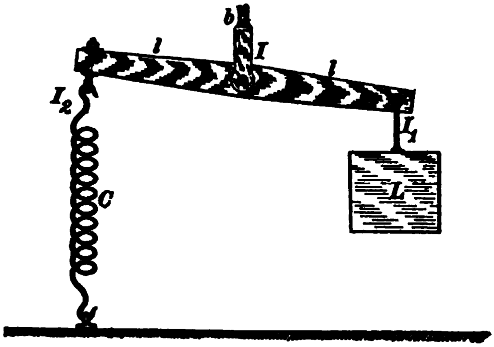

Fig. 153.

112. Circuits in series. Voltage drop in transmission lines. — A transmission line is of course in series with the circuit to which it delivers current, and a consideration of the problem of voltage drop in a transmission line will furnish an example of the general problem of circuits in series.

(*a*) *Voltage drop in a transmission line which delivers current to a non-inductive receiving circuit.* — Let OI, Fig. 154, represent the current flowing through the transmission line and the receiving circuit. The voltage E_1 at the terminals of the receiving circuit is in phase with I, and the generator voltage E_0 is the vector sum of E_1, rI and xI; where rI is the electromotive force used to overcome the line resistance, and xI is the electromotive force used to overcome the line reactance. An examination of the figure shows that when the receiving circuit is non-inductive the line resistance produces a difference in *value* between E_0 and E_1, whereas the line reactance produces a difference in *phase* between E_0 and E_1.*

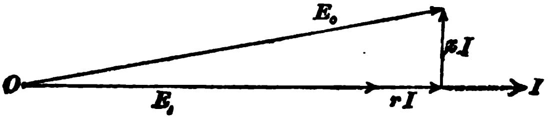

Fig. 154.

(*b*) *Voltage drop in a transmission line which delivers current to a highly inductive receiving circuit of small resistance.* — Let OI, Fig. 155, represent the current flowing through the transmission line and the receiving circuit. The voltage E_1 at the terminals of the receiving circuit is nearly 90° ahead of I in phase as shown, and the generator voltage E_0 is the vector sum of E_1, rI and xI as before. In the present case, however, the line resistance produces a difference in *phase* between E_0 and E_1, and the line reactance produces a difference in *value* between E_0 and E_1.†

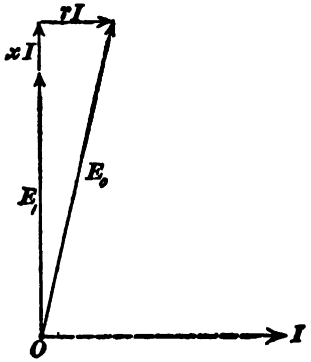

Fig. 155.

(*c*) *Voltage drop in a transmission line which delivers current to a condenser or any receiving circuit having negative reactance, but having small resistance.* — This case is shown in Fig. 156, and E_0 is the vector sum of E_1, rI and xI as before; but, E_1 and xI are nearly opposite in direction, inasmuch as E_1 is nearly 90° behind I under the assumed conditions. Therefore in this case the line resistance produces a difference in *phase* between E_0 and E_1 and the line reactance produces a difference in *value* between E_0 and E_1, making E_1 greater than E_0.

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* This statement is made on the assumption that xI is small as compared with E_1.

† This statement is made on the assumption that rI is small as compared with E_1.

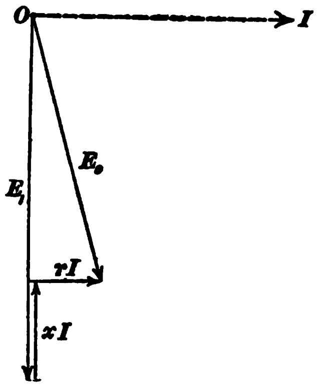

Fig. 156.

(*d*) *Voltage drop in a transmission line which delivers current to a receiving circuit whose power factor* (*cos* θ) *has any given value.* — Let the line OI, Fig. 157, represent the current flowing through the transmission line and the receiving circuit. Let the line E_1 represent the voltage across the receiving circuit, θ being the phase difference between I and E_1 as shown. Let r be the resistance and x the reactance of the transmission line, and let E_0 be the voltage at the generator. Lay off rI parallel to OI, and xI at right angles to OI.

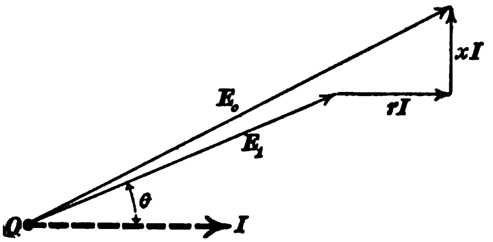

Fig. 157.

Then the difference between the values of E_1 and E_0 is the required transmission line drop.

The value of $\sqrt{r^2I^2 + x^2I^2}$ is called the *impedance drop*, and the difference between the numerical values of E_0 and E_1 is called simply the (*voltage*) *drop* in the line.

113. Circuits in parallel. — The general problem of circuits in parallel is somewhat complicated if one attempts to find algebraic expressions for the combined resistance and reactance of two or more circuits in parallel. Two simple cases of this problem are, however, of considerable interest, namely, (*a*) the problem of finding the power factor of two receiving circuits in parallel, the power factor of each and the current delivered to each being given, and (*b*) the problem of compensating for lagging currents.

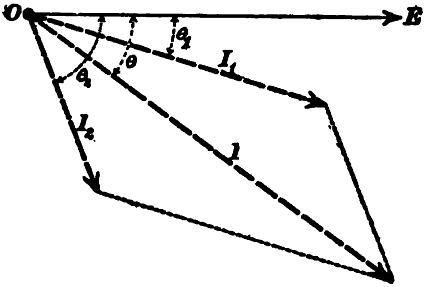

Fig. 158.

(*a*) *Power factor of receiving circuits in parallel.* — Let the line *OE*, Fig. 158, represent the voltage across the two receiving circuits. Let I_1 be the current delivered to receiving circuit number one and cos θ_1 its power factor, and let I_2 be the current delivered to receiving circuit number two and cos θ_2 its power factor. Lay off the clock diagram in Fig. 158 carefully to scale, and draw the diagonal I as shown. The angle θ between E and I is then the angle whose cosine is the required power factor of the two receiving circuits in parallel.

(*b*) *Compensation for lagging currents.* — When a transmission line delivers current to an inductive receiving circuit, power is delivered over the line to the receiving circuit during the time that the product *ei* is positive, and power is delivered back over the line from the receiving circuit to the generator during the time that the product *ei* is negative (compare Art. 54). This backward flow of energy from receiving circuit to generator represents an essentially unnecessary service of the transmission line, and it is desirable, and in some cases feasible, to reduce this backward flow of energy by connecting a condenser or something which is equivalent to a condenser (such as an over-excited synchronous motor) in parallel with the receiving circuit.

Let OE, Fig. 159, represent the voltage at the terminals of the receiving circuit, and let OI represent the current delivered to the receiving circuit. Let R be the resistance of the receiving circuit and X its reactance. Then

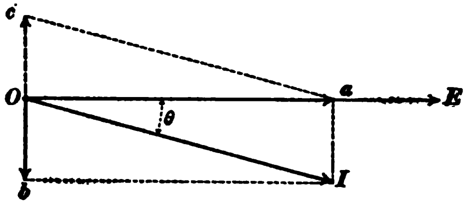

Fig. 159.

$$\tan\theta = \frac{X}{R}$$

and

$$\sin\theta = \frac{X}{\sqrt{R^2+X^2}}$$

The component Ob of the current I is equal to $I\sin\theta$ or $IX/\sqrt{R^2+X^2}$. But I is equal to $E/\sqrt{R^2+X^2}$, so that the component Ob is equal to $EX/(R^2+X^2)$. If a condenser of capacity C is connected across the transmission line at the receiver end, then the current flowing into the condenser will be 90° ahead of E, or parallel to Oc, Fig. 159, and its value will be equal to E divided by the condenser reactance $1/\omega C$, or to $E\omega C$; and if this current is numerically equal to Ob, the sum of I and Oc will be in phase with E. That is, the receiving circuit and the condenser together will take current in phase with E, and the instantaneous value of power delivered over the transmission line will never be negative. The capacity of the condenser required to produce this result is given by the relation above mentioned, namely, $E\omega C = EX/(R^2+X^2)$, which gives

$$C = \frac{X}{\omega(R^2+X^2)}$$

CHAPTER X.

THE POLYPHASE SYSTEM.

114. The single-phase system and the polyphase system. — A brief reference was made in Chapter II to the distinction between the single-phase alternator and the polyphase alternator, and, although the importance of the polyphase system cannot be fully appreciated until the induction motor and the rotary converter are discussed, still it is necessary to outline here some of the general features of the polyphase system inasmuch as this system enters into every division of the subject of alternating currents.

The single-phase system consists essentially of a simple alternator delivering an alternating current to a single circuit.

The polyphase system consists essentially of two or more electrically distinct (but not mechanically distinct) alternators delivering distinct alternating currents over distinct transmission lines to electrically distinct receiving circuits.

115. The two-phase alternator. — A two-phase alternator is an alternator upon whose armature two distinct windings are placed each with its own pair of collector rings, the two windings being arranged so that the electromotive force generated in one winding is in quadrature with the electromotive force generated in the other winding. A clear idea of the essential features of the two-phase alternator may be obtained from Fig. 160 which represents two distinct armatures *A* and *B* mounted on one shaft and revolving inside of the same crown of field magnet poles. The armature *B* is shown smaller than the armature *A* for the sake of clearness. The two armatures are arranged so that the groups of winding slots in one armature are midway between the field magnet poles when the groups of winding slots in the other armature are midway under the field magnet poles. This brings the electromotive forces of the two armature windings into quadrature with each other. In practice a two-phase alternator is made

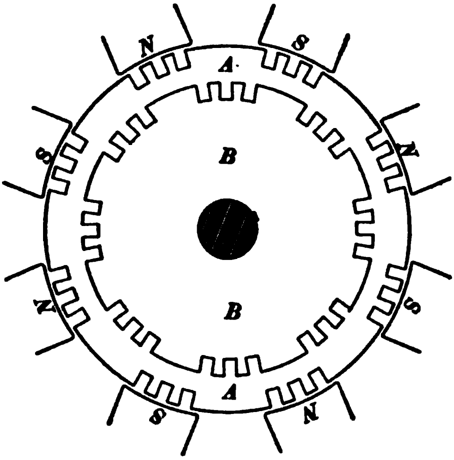

Fig. 160.

by placing all the slots shown in Fig. 160 on one armature body, the two windings being however kept as distinct as if they were on separate armature bodies.

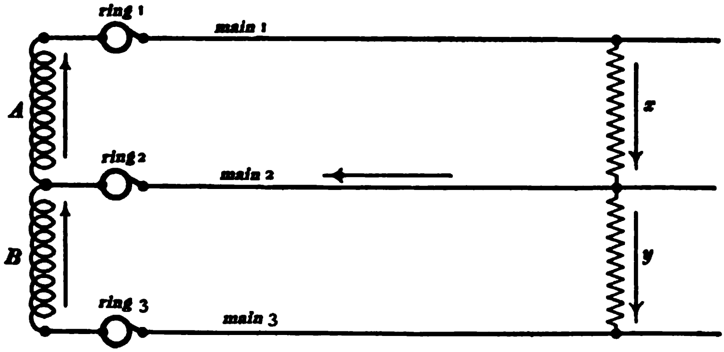

Fig. 161.

A two-phase alternator is usually provided with four collector rings, two for each armature winding; one ring may, however, be made to serve as a common terminal for the two windings *A* and B as shown in Fig. 161. In this case three transmission wires are used, and the two receiving circuits x and y are connected as shown in the figure. The objections to the arrangement shown in Fig. 161 are: (a) that the voltage drop in the middle main may cause the voltages across x and y to become unequal in value and more or less than 90° apart in phase, and (b) that a two-phase (4-ring) rotary converter cannot be connected to three-wire mains.

Two-phase electromotive forces and currents. — The two lines A and B in the clock diagram, Fig. 162, represent the electromotive forces of the A and B windings respectively of a two-phase alternator. If the circuit which receives current from the A winding has the same resistance and reactance as the circuit which receives current from the B winding, the system is said to be *balanced*. In this case the currents delivered by A and B are equal to each other in value, and they lag in phase equally behind A and B respectively, as shown by the two lines a and b.

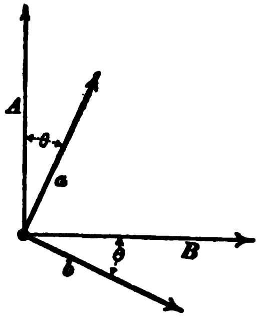

Fig. 162.

It is worthy of note that it is not possible to say whether A is 90° behind B, or B is 90° behind A, even when the physical facts concerning a two-phase alternator are completely established. Complete definiteness of phase relation depends upon an arbitrary choice of the direction in which the electromotive force and current of each phase is to be considered as positive. Conversely, a clock diagram like Fig. 162 cannot completely represent the physical facts concerning a two-phase alternator until positive directions are chosen. Let the arrows in Fig. 161 represent the arbitrarily chosen *positive directions* in the windings, in the middle main, and in the receiving circuits, and suppose that, on the basis of this choice of signs, the clock diagram, Fig. 162, represents the facts. Then the vector sum $A+B$ represents the electromotive force from main one across to main two, and the vector difference, $a-b$, of the currents in x and y represents the current in the middle main. If, however, the positive directions in winding B and receiving circuit y be chosen opposite to the directions shown in Fig. 161, then, *if the physical facts remain unchanged*, the lines B and b in the clock diagram must be reversed, the electromotive force from main one to main two will be represented by the vector difference $A-B$, and the current in the middle main will be represented by the vector sum $a+b$. If in the first case A is 90° ahead of B, then in the second case A must be considered as 90° behind B because of the change in the choice of signs.[*]

116. The three-phase alternator. — A three-phase alternator is an alternator upon whose armature three distinct windings A, C and B are placed, each having, in the most general case, a separate pair of collector rings, and the windings are arranged so that the electromotive forces A, C and B are 60° apart in phase.[†]

The four-ring arrangement of the three-phase alternator. — A number of batteries can supply currents to a number of distinct bell circuits, and all of the bell circuits can have a common return wire as shown in Fig. 163. In the same way the three armature windings A, B and C of a three-phase alternator can deliver currents to three receiving circuits x, y and z, a common return wire being used as indicated in Fig. 164. In this case one of the collector rings, No. 4 in Fig. 164, forms a common terminal for all three armature windings.

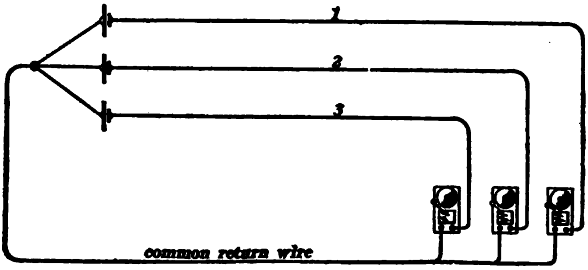

Fig. 163.

The essential features of the windings of a three-phase armature are shown in Fig. 165. The figure shows each winding distributed in two slots per field pole (the A winding only is shown complete, windings B and C are exactly like A), and it represents the simple case in which each slot contains but one conductor. The straight radial lines represent the conductors lying in the slots, the curved lines outside the circular row of radial lines represent the end-connections at one end of the armature, and the curved lines inside of the circular row of radial lines represent the end-connections at the other end of the armature. One terminal of each winding is connected to ring 4 (or to a neutral connection *N*, Fig. 164, if ring 4 is dispensed with), and the other terminals of windings of *A*, *B* and *C* are connected to rings 1, 2 and 3 respectively.

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[*] See A Discussion of Some Points in Alternating-current Theory [Section (*b*) On the Necessity of Choosing Signs in Complicated Networks], by W. S. Franklin, *Transactions of the American Institute of Electrical Engineers*, Vol. 21, page 589, 1903.

[†] It will be found convenient later to consider the three electromotive forces A, B and C to be 120° apart in phase, the change being merely a matter of the choice of signs. See Art. 118.

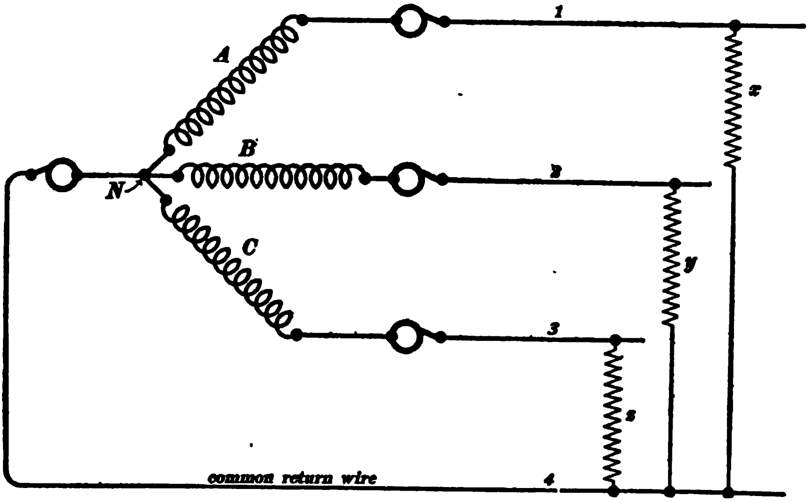

Fig. 164.

117. The three-ring three-wire arrangement of the three-phase alternator. (*a*) *The Y-connection.* — Let the three lines *a*, *b* and *c*, Fig. 166, represent the three equal currents, 60° apart in phase, which are delivered to *three similar receiving circuits* *x*, *y* and *z* connected as shown in Fig. 164. It is evident that the current *c* is equal to the vector sum of *a* and *b*, or, in other words, the current *c* is at each instant equal to the sum of the two currents *a* and *b*. Therefore, if the connections of the winding *C*, Fig. 164, are arranged so that the current *c* flows *towards* the generator in main No. 3 when *a* and *b* flow *away from* the generator in mains No. 1 and No. 2, then the current in main No. 4 will be zero, since $a + b = c$ at each instant. Therefore, when the receiving circuits x, y and z are similar, main No. 4 may be dispensed with if the windings are properly connected. The proper connections are shown in Fig. 165. This arrangement of the windings of a three-phase armature is called the *Y-connection.*

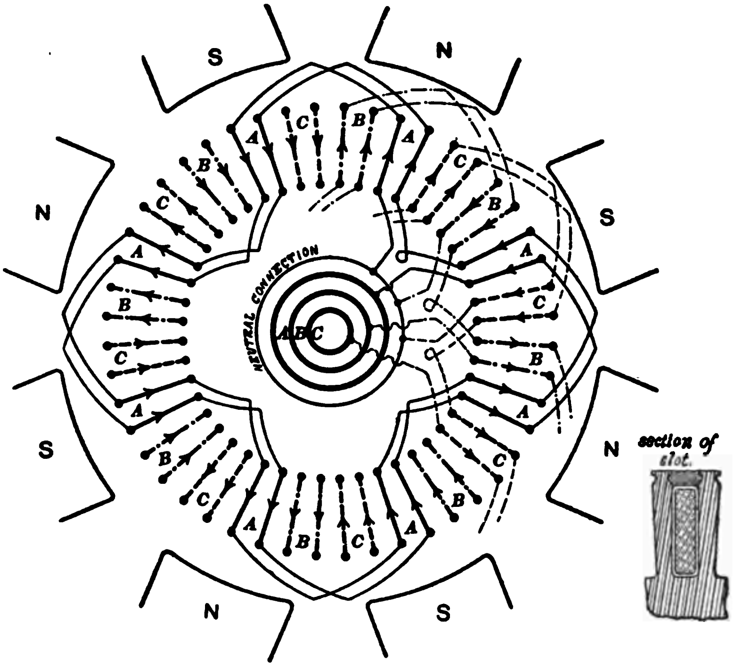

Fig. 165.

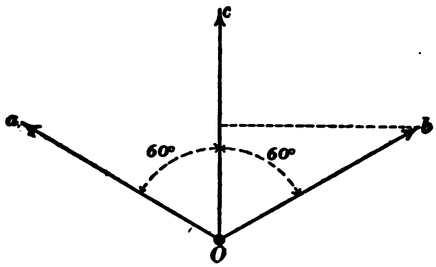

Fig. 166.

(*b*) *The* Δ-*connection.* — Suppose the three armature windings of a three-phase alternator to be connected so as to form a closed circuit, and let the three lines A, C and B in the clock diagram, Fig. 167, represent the electromotive forces of the respective windings. The electromotive force C is equal to $A + B$, and therefore, if the windings are properly connected, the electromotive force around the closed circuit of the three windings will be zero. Each junction of two windings may then be connected to a collector ring so that the three windings can deliver current to three-wire mains. This arrangement of the windings of a three-phase armature is called the Δ-*connection.*

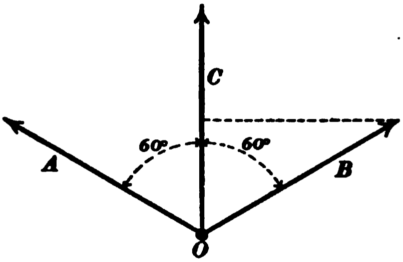

Fig. 167.

118. Conventional diagrams. Choice of signs to give symmetrical clock diagrams. (*a*) *Y-connection.* — The three armature windings of a three-phase armature branch out from a common junction, the so-called neutral point N, Fig. 164, when the windings are Y-connected. The essential features of this scheme of connections are best shown by a symmetrical diagram like Fig. 168.

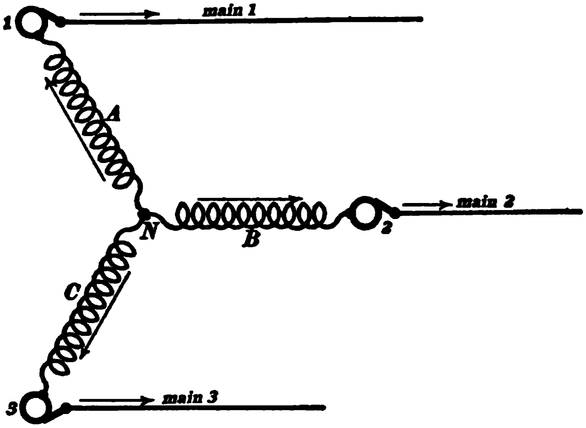

Fig. 168.

*It is desirable to consider the current (and voltage) in each winding as positive when it flows away from the neutral point and the arrows in Fig. 168 show these positive directions.** On the basis of this choice of signs, the clock diagram which shows the currents *a*, *b* and *c* and the electromotive forces *A*, *B* and *C* in the respective windings, becomes symmetrical as shown in Fig. 169.

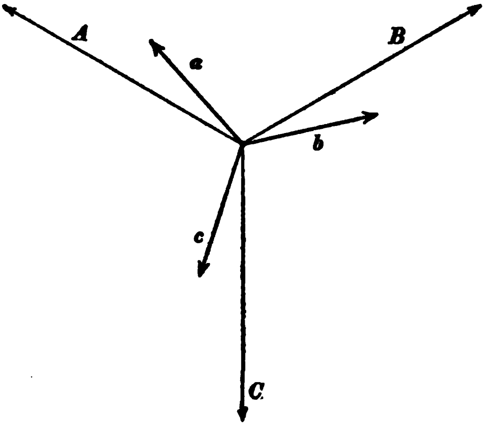

Fig. 169.

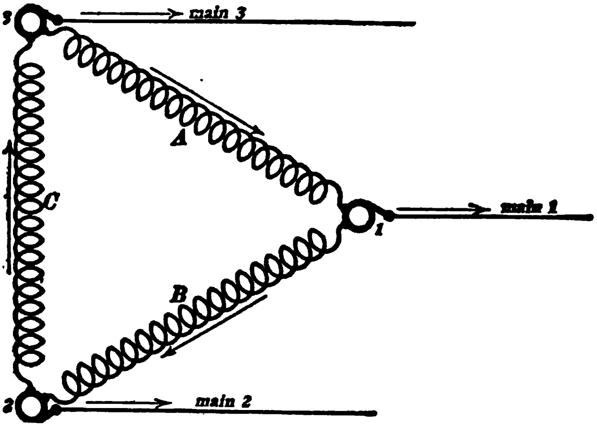

Fig. 170.

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* It must not be imagined that the arrows in Figs. 168 and 170 represent the actual directions of the currents at any given instant.

(*b*) *Δ-connection.* The three armature windings of a three-phase alternator, when Δ-connected, form a complete circuit or mesh, and the essential features of this scheme of connections are best shown by a symmetrical diagram like Fig. 170. *It is desirable to consider the current (and voltage) in each winding as positive when it flows in a given direction around the mesh, and the arrows in Fig. 170 show these positive directions.* On the basis of this choice of signs, the clock diagram of the currents and electromotive forces in the respective armature windings becomes symmetrical. The diagram of Fig. 169 represents the electromotive forces and currents in the armature windings, whether the windings be Y-connected or Δ-connected.

119. Manner of connecting receiving circuits to the three-wire mains of a three-phase system.

(*a*) *For dissimilar receiving circuits (unbalanced system).* — In this case the four-wire system should be employed as shown in Fig. 164. This requires of course four collector rings on the generator. A polyphase generator is, however, used primarily for supplying currents to rotary converters and induction motors, such machines always involve similar receiving circuits, one receiving circuit for each phase, and when current is delivered

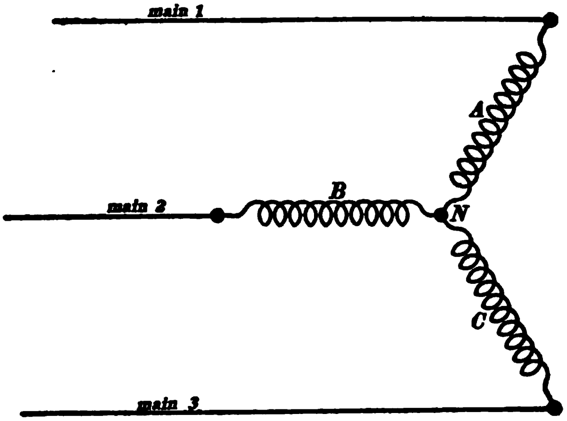

Fig. 171.

from a polyphase system to individual (single-phase) receiving circuits, such as lamps it is generally possible to distribute these individual receiving circuits among the several phases so as to give an approximately balanced system. Polyphase systems are in fact nearly always arranged with the idea of delivering currents to balanced receiving circuits.

(*b*) *For similar receiving circuits* (*balanced system*). — In this case the receiving circuits may be connected to three-wire three-phase mains according to the Y-scheme as shown by Fig. 171, or according to the Δ-scheme as shown by Fig. 172, and either scheme may be employed irrespective of the manner in which the armature windings of the generator are connected.

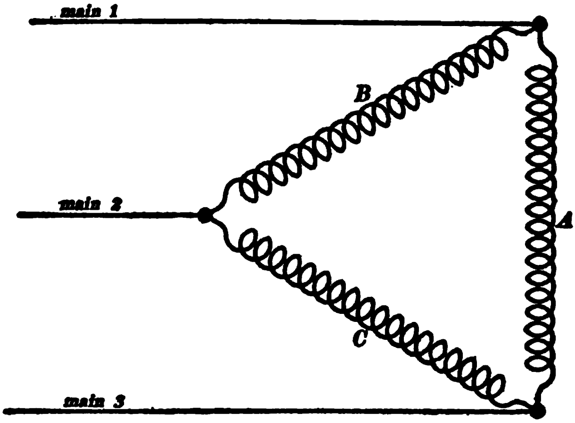

FIG. 172.

120. Electromotive force and current relations in the Y-scheme of connections.

(1) **Referring to the generator.** (*a*) *Electromotive force relations.* — Let E_1, E_2 and E_3 be the electromotive forces from main 1 to main 2, from main 2 to main 3, and from main 3 to main 1, respectively, see Fig. 168. From Fig. 168 it is evident that a positive electromotive force in winding A produces an electromotive force from main 1 to main 2 (positive), and that a positive electromotive force in winding B produces an electromotive force

from main 2 to main 1 (negative). Therefore E_1 is equal to $A - B$ (vector difference). Similarly E_2 is equal to $B - C$, and E_3 is equal to $C - A$. These relations are shown in Fig. 173, and from this figure we see that *the value of the electromotive force between any pair of mains is* $\sqrt{3}$ *times the electromotive force in each Y-connected armature winding.*

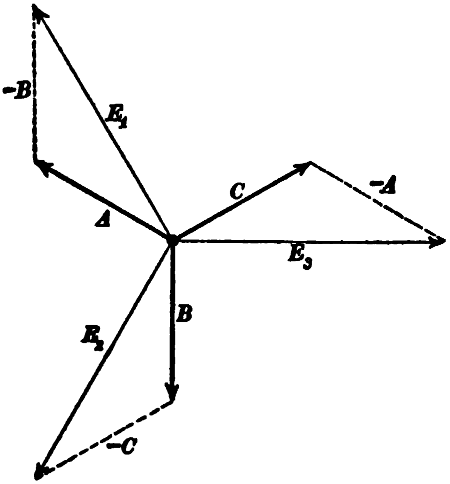

Fig. 173.

(*b*) *Current relations.* — In the Y-scheme of connections the currents in the mains are identical to the currents in the respective windings. This is evident from Fig. 168.

(2) **Referring to the receiving circuits.** (*a*) *Electromotive force relations.* — Given the electromotive forces, E_1, E_2 and E_3 between the mains as shown in the clock diagram, Fig. 173, then the electromotive forces acting on the respective Y-connected receiving circuits are represented by the lines A, B and C. Therefore *the value of the electromotive force acting on each Y-connected receiving circuit is equal to the electromotive force between mains divided by* $\sqrt{3}$.

(*b*) *Current relations.* — In the Y-scheme of connections the current in each main is identical to the current in each receiving circuit.

121. Electromotive force and current relations in the Δ-scheme of connections.

(1) **Referring to the generator.** (*a*) *Electromotive force relations.* — In the Δ-scheme of connections the electromotive forces between mains are identical to the electromotive forces developed by the respective armature windings.

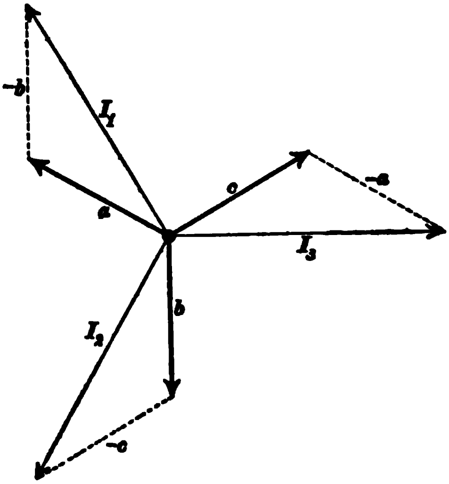

Fig. 174.

(*b*) *Current relations.* — Let I_1, I_2 and I_3 be the currents in mains 1, 2 and 3 respectively, each being considered positive when it flows away from the generator as shown by the arrows in Fig. 170. From this figure it is evident that a positive current in winding A produces a positive current in main 1, and that a positive current in winding B produces a negative current in main 1. Therefore I_1 is equal to $a - b$ (vector difference). Similarly, I_2 is equal to $b - c$, and I_3 is equal to $c - a$. These relations are shown in Fig. 174, and from this figure we see that *the value of the current in each main is* $\sqrt{3}$ *times the current in each* Δ*-connected armature winding.*

(2) **Referring to the receiving circuits.** (*a*) *Electromotive force relations.* — In the Δ-scheme of connections the electromotive

forces acting on the receiving circuits are identical to the electromotive forces between the respective pairs of mains.

(*b*) *Current relations.* — Given the currents a, b and c in the respective receiving circuits (see Fig. 174), then the currents in the respective mains are represented by the lines I_1, I_2 and I_3. Therefore, *the value of the current in each* Δ*-connected receiving circuit is equal to the current in each main divided by* $\sqrt{3}$.

122. Power in polyphase systems. — When the receiving circuits are unlike (system unbalanced) a different amount of power is, in general, delivered to each, and the system must be treated as two or three independent single-phase systems. The only case in which the power relations in a polyphase system can be treated on a simpler basis than that corresponding to independent single-phase systems, is the case in which the system is balanced. In this case there are two important matters to be considered: (*a*) The constancy of power, and (*b*) the value of the power.

(*a*) *Constancy of power in balanced polyphase system.* — In the single-phase alternator the instantaneous value ei of the power pulsates as explained in Art. 54, whereas a polyphase generator delivers a perfectly steady flow of energy, that is, a steady power, to balanced receiving circuits. In this case the power delivered to each receiving circuit of course pulsates, but the pulsations are so related that the sum does not pulsate. An interesting consequence of this steady flow of power in a balanced polyphase system is that, whereas, the driving torque of a single-phase alternator must pulsate except in so far as the pulsations of torque are averaged out by the fly-wheel effect of the rotating armature, the driving torque of a polyphase generator is perfectly steady when it delivers current to balanced receiving circuits. Moreover a single-phase motor of any kind is driven by a pulsating torque, whereas a polyphase motor of any kind is driven by a steady torque.

(*b*) *Value of power in balanced polyphase system.* — If A is the value of the electromotive force across one of the receiving circuits of an n-phase system, a the value of the current in the receiving circuit, and $\cos\ \theta$ the power factor, then $Aa\ \cos\ \theta$ is the power delivered to the receiving circuit, and the total power delivered to the n similar receiving circuits is

$$P = nAa \cos \theta \tag{a}$$

If E is the electromotive force between mains of a three-wire three-phase system, and if I is the current in each main, then the total power delivered by the mains is

$$P = \sqrt{3}EI \cos \theta \tag{b}$$

in which $\cos \theta$ is the power factor of each receiving circuit. This form of the equation is confusing for the reason that the voltage E, the current I, and the power factor do not all refer to a given circuit. The form (a) of the equation is therefore to be preferred.

123. Measurement of power in polyphase systems. — When a polyphase system is unbalanced, the power delivered by each phase must be measured exactly as in the case of an ordinary single-phase system (see Arts. 48 and 50). When a polyphase system is balanced, the measured power delivered by one phase may be multiplied by the number of phases to give the total power, but exact similarity of the various receiving circuits is never realized in practice and therefore a considerable error may be made if the total power is assumed to be nP, where P is the measured power delivered by one phase and n is the number of phases. Therefore it is usually considered necessary to measure the actual total power.

Best method for measuring power approximately by means of a single wattmeter. — It is usually very inconvenient to disconnect the current coil of a wattmeter and transfer it quickly from one main to another, but it is always very easy to change the connections of the voltage coil. The best method therefore for using a single wattmeter for measuring the power delivered to approximately balanced receiving circuits by a three-wire system (three-phase) is to connect the current coil in a given main and take the wattmeter readings with the voltage coil connected from the given main first to one and then to the other of the two other mains. The sum of these two wattmeter readings gives a fairly accurate value for the total power delivered.

Use of two wattmeters for measuring accurately the power delivered by any three-wire system. — The total power delivered by any three-wire system may be measured by two wattmeters with their current coils placed in two of the mains, and their voltage coils connected from each of these two mains to the third main. Thus Fig. 175 shows two wattmeters connected for measuring the power delivered by a three-wire three-phase system. The receiving circuits may be either Δ-connected or Y-connected.

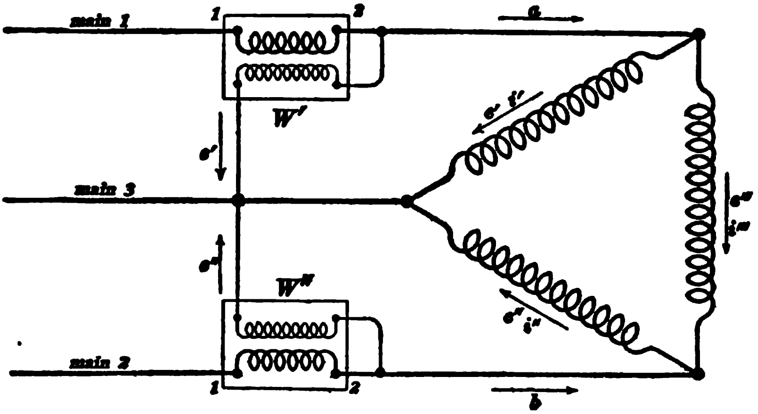

Fig. 175.

In some cases the two wattmeter readings are to be added, and in other cases the lesser reading must be subtracted from the greater to give the total power delivered. To determine whether the readings are to be added or subtracted, disconnect receiving circuit $e'''i'''$ in Fig. 175 for a moment. Under these conditions both wattmeters must indicate delivered power, so that if either wattmeter deflection is reversed by disconnecting

$e'''i'''$, then the original reading of that wattmeter must be considered negative.

124. Relative copper economy of two-phase and three-phase transmission lines. — A given amount of power can be transmitted over a certain distance with an indefinitely small amount of copper if no limit is set to the voltage that can be used. There is, however, a limit to the voltage that can be used satisfactorily, and it is customary therefore to compare the copper economy of two transmission systems on the basis of equal voltages. This basis of comparison is slightly misleading, however, as between the three-phase system and the three-wire two-phase system, as explained later.

As a basis of comparison of the copper economy let us consider a four-wire two-phase transmission line and a three-wire three-phase transmission line of the same length with the same voltages between mains in each case. Let E be the common value of the voltages at the receiving end, and *let it be required to find the relative amounts of copper required to transmit the same amount of power (unity power factor) with the same loss of power in the mains in each case.* Let R_2 be the resistance of each of the four wires of the two phase line, let R_3 be the resistance of each of the three wires of the three-phase line, let I_2 be the current in each of the four wires of the two-phase line, and let I_3 be the current in each of the three wires of the three-phase line. Then $2EI_2$ is the power delivered by the two-phase line and $\sqrt{3}EI_3$ is the power delivered by the three-phase line, and, since the total power is assumed to be the same in each case, we have $2EI_2 = \sqrt{3}EI_3$ or

$$I_2 = \frac{\sqrt{3}}{2} \cdot I_3 \qquad (a)$$

Furthermore, $4R_2I_2^2$ is the power lost in the two-phase line, and $3R_3I_3^2$ is the power lost in the three-phase line, so that

$$4R_2I_2^2 = 3R_3I_3^2 \qquad (b)$$

Therefore, using equation (*a*), we have

$$R_2 = R_3 \qquad (c)$$

That is, the wires have the same resistance and therefore the same weight, and, since four wires are used in the two-phase line and only three wires in the three-phase line, it follows that the total weight of copper in the two-phase line is $\frac{4}{3}$ times as great as in the three-phase line. The same result is reached on the basis of equal percentage drops of electromotive forces in the two systems. The result is independent of the power factor of the receiving circuits provided the power factor is the same for both systems.

The three-wire two-phase transmission line requires less copper than the three-wire three-phase line in the ratio 2.914 to 3, but the three-wire two-phase arrangement is not desirable for reasons explained in Art. 115, and, furthermore, a three-wire two-phase line with E-volts between the middle wire and each outside wire has $\sqrt{2}E$-volts between the outside wires, whereas, in a three-wire three-phase line with E-volts between any two wires, there is no higher voltage than E anywhere in the system. Therefore, it is unfair to compare the three-phase system with the three-wire two-phase system on the basis of equal voltages E.

On account of the greater copper economy and on account of the use of three wires instead of four, the three-phase system is nearly always used for long distance transmission.

PART IV.

ALTERNATING-CURRENT GENERATORS AND MOTORS.

CHAPTER XI.

THE ALTERNATOR AS A GENERATOR.

VOLTAGE REGULATION.

125. Electromotive force of the alternator, its dependence upon speed and flux. — The electromotive force of a direct-current generator depends in a simple manner upon the magnetic flux Φ per pole, upon the number of armature conductors Z,[*] and upon the armature speed n. The electromotive force (effective) of an alternator, however, depends not only upon Φ, Z and n but also upon the width and shape[†] of the pole pieces of the field magnet, and upon the extent to which the windings are spread out over the surface of the armature.

The average value[‡] of the electromotive force induced in one armature conductor is *always* equal to $p\Phi n$ abvolts, where p is the number of field magnet poles, Φ is the magnetic flux from one pole, and n is the armature speed in revolutions per second. This is evident when we consider that a given armature conductor cuts Φ lines of force in $1/p$ of a revolution or in $1/pn$ of a second, so that the average rate of cutting lines of force is Φ divided by $1/pn$ which is equal to $p\Phi n$.

Concentrated versus distributed armature windings. — When the armature conductors are grouped in p equidistant slots, p being the number of field magnet poles, the armature winding is said to be *concentrated;* when the armature conductors are spread out in p broad bands or in p groups of slots, the winding is said to be *distributed.*

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[*] Throughout this chapter Z stands for the number of armature conductors per path.

[†] This is intended to include everything which affects the distribution of flux around the armature.

[‡] Whenever the average value of an alternating electromotive force or current is spoken of, the average value during a half-cycle is always meant.

In a concentrated winding the same electromotive force e is induced in every armature conductor at any given instant; therefore the electromotive force between the collector rings at the given instant is equal to Ze, Z being the number of armature conductors in series between the collector rings; the average electromotive force between the collector rings is equal to Z times the average electromotive force $p\Phi n$ in a single conductor; and the effective electromotive force between the collector rings is equal to Z times the effective electromotive force in a single conductor.

In a distributed winding, on the other hand, the same value of electromotive force is *not* induced in all of the armature conductors at any given instant, because the conductors cannot be all similarly situated under the respective field magnet poles. Or, in other words, there is a phase difference between the electromotive forces induced in the different conductors. On account of this phase difference the average electromotive force between the collector rings is *not* equal to Z times the average electromotive force in a single conductor, and the effective electromotive force between the collector rings is *not* equal to Z times the effective electromotive force in a single conductor.

Discussion of the electromotive force of an alternator for the case in which the flux is harmonically distributed around the armature. — Figure 176 shows an armature between two nearly flat pole-pieces, so that the flux density in the gap space is greatest at the

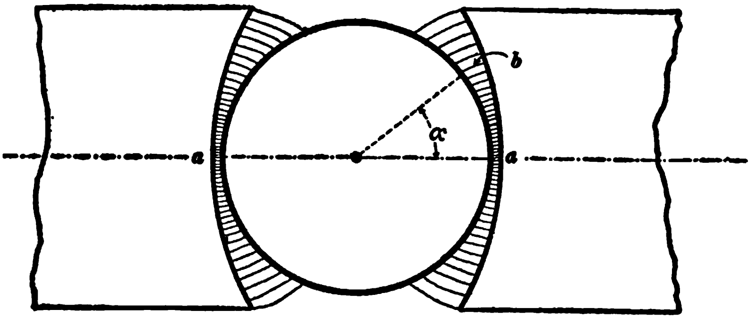

Fig. 176.

points *aa*. If the pole faces were so shaped that the flux density at any point *b* in the gap space would be proportional to the cosine of the angle α, we would have what is called a ***harmonic distribution of flux***, and the electromotive force induced in each armature conductor would be a harmonic electromotive force capable of being represented by a line in a clock diagram.

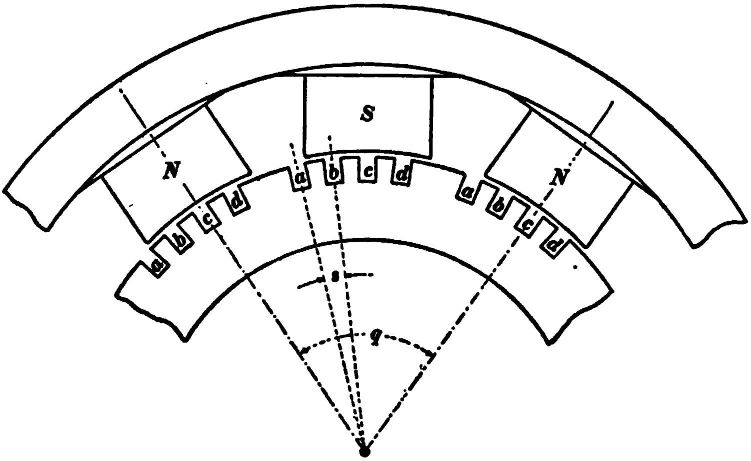

Fig. 177.

(*a*) *Concentrated armature winding.* — In the case of a concentrated winding, the average value (during half-a-cycle) of the electromotive force of the alternator is equal to $p\Phi Zn$ abvolts as stated above, and if the electromotive force is harmonic (harmonic distribution of flux) the ratio, effective value divided by average value, is equal to $\pi/(2\sqrt{2})$ according to Art. 103. Therefore the effective value of the electromotive force of an alternator having a concentrated armature winding, and having a harmonically distributed flux, is equal to

$$\pi/(2\sqrt{2}) \times p\Phi Zn \text{ abvolts}$$

(*b*) *Distributed armature winding.* — Consider an armature winding in which the conductors are grouped in $4p$ slots, that is, in p groups of slots, each group having four slots, as shown in

Fig. 177. Let q be the angular distance between centers of adjacent north poles of the field magnet, and let s be the angular distance from center to center of adjacent slots.

All of the conductors in the a slots constitute a concentrated winding, all of the conductors in the b slots constitute a concentrated winding, all of the conductors in the c slots constitute a concentrated winding, and so on. These concentrated windings are all alike, the respective electromotive forces A, B, C and D which they develop are all the same in value, but they differ in phase, and the phase difference A to B, B to C, and C to D is $s/q \times 360°$. Therefore the four electromotive forces A, B, C and D may be represented by the four equidistant vectors A, B, C and D in the clock diagram, Fig. 178.

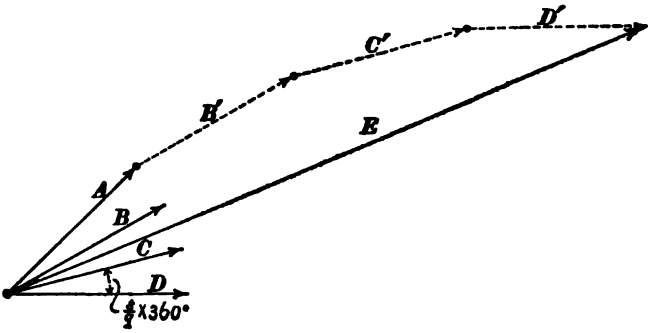

Fig. 178.

If the windings in all of the slots in Fig. 177 are connected in series as a single distributed winding, the effective value of the electromotive force produced by the combined winding will be the vector sum of A, B, C and D, as shown by the line E in Fig. 178 in which the lines A, B', C' and D' form a portion of a regular polygon of which E is the chord. If the angular distance between A and B, B and C and so on, is small, the lines A, B', C' and D' form sensibly the arc of a circle of which E is the chord, and the angle subtended by the arc is the phase difference between A and D.

If the electromotive forces A, B, C and D were in phase with each other, that is, if all of the armature conductors were concentrated in p slots, the effective value of E could be calculated from the formula $\pi/(2\sqrt{2}) \times p\Phi Zn$ abvolts, and it would be equal to the arithmetical sum (not the vector sum) of A, B, C and D; that is, the value of E would be represented by the length of the *arc* $AB'C'D'$ in Fig. 178. In fact, however, E is represented by the length of the *chord*, so that $\pi/(2\sqrt{2}) \times p\Phi Zn$ must be multiplied by the ratio *chord/arc* to give the correct value of E, the chord and arc being such as to subtend an angle equal to the phase difference between the electromotive forces induced in those armature conductors which lie in the most distant slots of a group. This ratio chord/arc, as here defined, is sometimes called the *phase constant* of a distributed winding. The following table gives the values of this phase constant based not upon the length of circular arc but upon the actual length of the proper portion of the periphery of the electromotive force polygon, see Fig. 178.

VALUES OF PHASE CONSTANTS FOR DISTRIBUTED ARMATURE WINDINGS.

Number of slots in each group.	Widths of groups of slots in fractional parts of N to S.				
	¼	⅓	½	¾	Whole.
2	.980	.966	.924	.831	.707
3	.977	.960	.912	.805	.666
4	.976	.958	.908	.795	.653
Infinity.	.975	.955	.901	.784	.637

The slots for a given winding are always arranged in p similar groups, p being the number of field magnet poles. The slots in a group are always equidistant, and the width of a group is understood to mean ns, where n is the number of slots in a group and s is the distance from center to center of adjacent slots. It is evident that if the width of the groups of slots is ⅓ of the distance from N to S, center to center, then three such windings can be placed on the armature giving a three-phase alternator, and if the width of the groups of slots is ½ of N to S, center to center, then two such windings can be placed on the armature giving a two-phase alternator. Figure 179 shows the outline of an 8-pole alternator with its armature slotted for a winding distributed in $3p$ slots, the width ns of each group of slots being ⅓ of the distance N to S, center to center, so that there is room on this armature for two additional windings of the same type.

It is undesirable to spread the winding of a single-phase alternator over the whole armature surface. Thus the phase constant of a winding spread over the whole surface is 0.637 and the phase constant of a winding spread over ¾ of the armature surface is 0.784. The number of armature conductors in the first case being taken as 100, the number in the second case would be 75, and the electromotive forces would be in the ratio of 100 × 0.637 to 75 × 0.784, that is, in the ratio of 63.7 to 58.8. It is therefore evident that only a slight increase of electromotive force would be obtained by filling up the whole armature surface of a single-phase alternator with windings, whereas the armature resistance, the loss of power due to armature resistance, and the cost of copper would all be increased in the ratio of 75 to 100 in

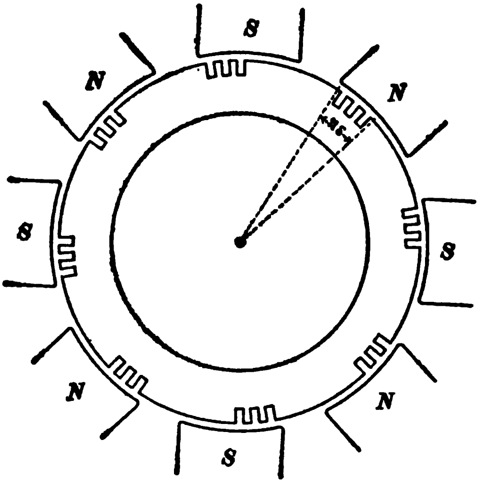

Fig. 179.

the case cited. A single-phase alternator giving, say, 1,000 volts and having an armature wound with wire large enough to carry 100 amperes, and therefore able to deliver 100 kilowatts to a non-inductive receiving circuit, must be larger and more costly than a two-phase or three-phase alternator capable of delivering the same total amount of power.

126. Voltage regulation of an alternator. — An alternator, driven at a constant speed and having a constant field excitation, gives a certain terminal voltage when its current output is zero, and its terminal voltage generally *falls off* in value with increase of current output; or, if the machine is adjusted to give a certain terminal voltage at full load, its terminal voltage generally *rises* with decrease of current output. The increase of terminal voltage from full load to zero load expressed in per cent. of normal full-load voltage, field excitation and speed both being constant, is called the *voltage regulation* of the machine.

The voltage regulation of an alternator varies greatly with the power factor of the receiving circuit. For example, a certain alternator delivers its full-load current to a non-inductive receiving circuit (power factor equal to unity) at its normal full-load voltage of 1,100 volts, and the terminal voltage rises to 1,188 volts when the load is reduced to zero by opening the main switch. The same alternator has its field excitation adjusted so that it gives its normal full-load voltage of 1,100 volts when it delivers its full-load current to an inductive receiving circuit of which the power factor is 0.80, and the terminal voltage rises to 1,386 volts when the load is reduced to zero. That is, the given alternator has a regulation of 8 per cent. when it delivers current to a receiving circuit having unity power factor, and it has a regulation of 26 per cent. when it delivers current to an inductive receiving circuit having a power factor equal to 0.80. When an alternator delivers current to a receiving circuit like a condenser, in which the current is ahead of the electromotive force in phase, the regulation of the alternator is better (smaller) than when the receiving circuit is non-inductive.

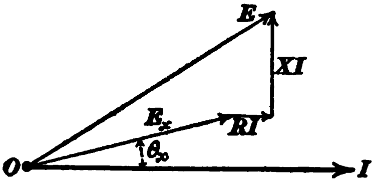

Fig. 180.

127. Resistance drop and reactance drop of voltage in an alternator armature. — Let the line OI, Fig. 180, represent the current delivered by an alternator to a receiving circuit and let cos θ_x be the power factor of the receiving circuit, that is to say, θ_x is the phase difference between the voltage E_x across the terminals of the receiving circuit (terminal voltage of alternator) and the current. The total electromotive force OE of the alternator may be considered in three parts, namely, (*a*) the part RI which overcomes the resistance of the armature, (*b*) the part XI which overcomes the reactance of the armature, and (*c*) the part E_x which is left over as terminal voltage, so that the total induced electromotive force OE is the vector sum of E_x, RI and XI. The electromotive force RI which is used to overcome the resistance of the armature is in phase with the current and it is called the *resistance drop* of voltage in the armature. The electromotive force XI which is used to overcome the reactance of the armature is 90° ahead of the current in phase and it is called the *reactance drop* of voltage in the armature. The reactance of the armature is equal to $2\pi fL$, where L is the inductance of the armature winding and f is the frequency of the electromotive force developed by the alternator.

The inductance of any coil of wire is the factor by which the current in that coil must be multiplied to give the magnetic flux through a mean turn of the coil due to the current.* That is to say, the inductance of a coil is a factor which measures the magnetizing action of a coil in producing flux through itself. Therefore the reactance drop of voltage in an alternator armature may be looked upon in either of two ways, as follows: (*a*) it may be considered as the electromotive force required to overcome the inductance of the armature, or (*b*) it may be looked upon as equal and opposite to the electromotive force which is induced in the armature windings by that part of the actual flux which is due to the magnetizing action of the armature current. This latter point of view is the one which is followed in the theory of voltage regulation in the next article.

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* To be exact, the inductance is the factor by which the current in a coil must be multiplied to give $Z\Phi$, where Z is the number of turns of wire in the coil and Φ is the flux through a mean turn.

Figures

✓ **128. Theory of voltage regulation of the alternator.** — The decrease of terminal voltage of an alternator with increase of current output is due to two effects, namely, (*a*) the resistance of the armature windings, and (*b*) the magnetizing action of the armature current or currents. In discussing the effect of magnetizing action of the armature current, or currents, it is convenient to think of the field windings as producing a certain constant flux Φ, and of the armature current or currents as producing a certain additional flux Φ'; for brevity we will call Φ the "field flux" and Φ' the "armature flux," although of course both Φ and Φ' pass through the armature.

The field flux Φ may be thought of as producing a certain "total induced electromotive force" in each armature winding; a portion of this total induced electromotive force is lost in balancing the electromotive force induced by the armature flux Φ', another portion is lost in overcoming the resistance of the armature winding, and the remainder appears as the terminal voltage of the machine. Therefore, to calculate the voltage regulation of an alternator it is necessary to consider the value and phase of the electromotive force induced in the armature windings by the flux Φ'.

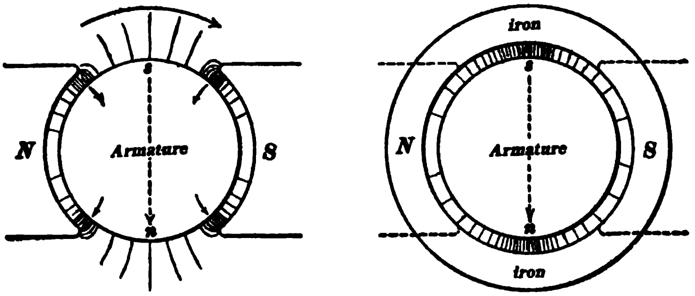

Fig. 181a. Fig. 181b.

Actual and assumed distributions of Φ', unity power factor.

In the polyphase alternator with a balanced load the armature flux Φ' is constant in value (for a given value of the armature currents) and fixed in space, and electromotive forces are induced in the moving armature conductors as they cut this flux. In the single-phase alternator, on the other hand, the armature flux Φ' is neither constant in value nor fixed in space, and the electromotive forces induced in the armature conductors by Φ' are therefore very complicated in the single-phase alternator. The following discussion applies primarily to the polyphase alternator with a balanced load, and the discussion is made to refer explicitly to a two-pole machine for the sake of simplicity.

When a polyphase alternator delivers current to balanced non-inductive circuits, the axis of the armature-magnetizing-action (axis of Φ') is at right angles to the axis of the field as shown in Fig. 181*a*, in which *n* and *s* represent the magnetic poles on the armature core due to the armature magnetizing action. The fine lines in Fig. 181*a* represent the actual distribution of the armature flux Φ' and *the fine lines in Fig. 181b represent the distribution of* Φ' *on the assumption that the iron of the pole pieces completely and uniformly surrounds the armature.*

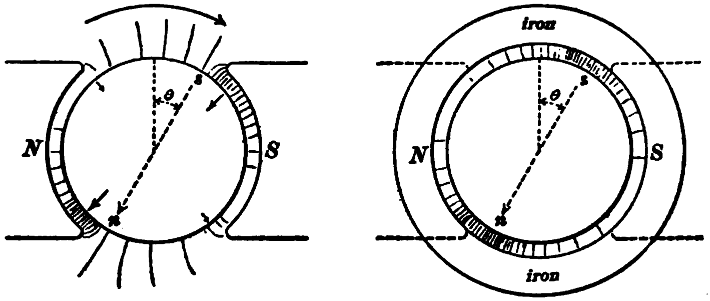

Fig. 182a. Fig. 182b.

Actual and assumed distributions of Φ', lagging currents.

When a polyphase alternator delivers current to balanced inductive receiving circuits, the axis of armature-magnetizing-action (axis of Φ') is shifted forwards θ° as shown in Figs. 182*a* and 182*b*, where θ° is the phase angle between voltage and current in each receiving circuit. When the currents in the receiving circuits lead the electromotive forces in phase by the angle θ, then the axis of Φ' is shifted backwards as shown in Figs. 183*a* and 183*b*.

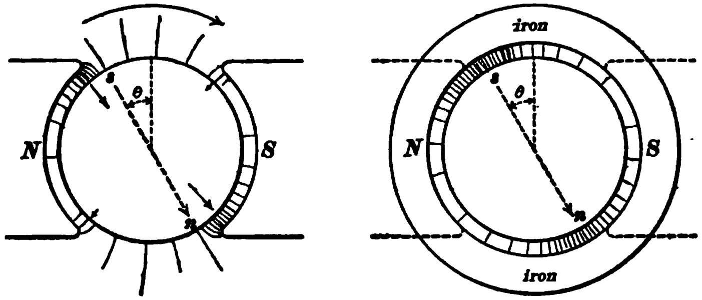

Fig. 183a. Fig. 184b.

Actual and assumed distributions of Φ', leading currents.

Several important things are at once evident from Figs. 181, 182 and 183 as follows: (*a*) The armature flux Φ' is to be a certain extent opposed to the field flux Φ in

Fig. 182. (*b*) The armature flux Φ' is to a certain extent in the direction of the field flux Φ in Fig. 183. (*c*) The armature flux Φ' neither helps nor opposes the field flux Φ in Fig. 181. This matter is discussed in Art. 130.

(*d*) The distribution of Φ' is far from being harmonic (like Fig. 176) and therefore the electromotive forces induced by Φ' in the moving armature conductors are far from being harmonic. This is the most serious obstacle to the formulation of a simple method for calculating the voltage regulation of an alternator, and indeed any simple method must be based on the assumption that Φ' is harmonically distributed as shown in Figs. 181*b*, 182*b* and 183*b*, and this means that the field magnet is assumed not to have polar projections but to surround the armature uniformly on all sides as indicated.

(*e*) It is evident from a careful consideration of Fig. 181*b* that the electromotive force induced in each armature winding by the flux Φ' is 90° *behind* the electromotive force induced by the field flux Φ, so that the portion of the "total induced electromotive force" (due to Φ) which is used to overcome the electromotive force produced by Φ' is 90° *ahead* of the total induced electromotive force, or 90° ahead of the current in the given armature winding, since, in Fig. 181, current and voltage are supposed to be in phase with each other. A careful consideration of Figs. 182*b* and 183*b* will make it evident that *in every case the portion of the total induced electromotive force which is lost in overcoming the electromotive force produced by* Φ' *is 90° ahead of the current in phase.* This lost electromotive force is, furthermore, proportional to the armature current I so that it may be expressed as XI where X is a reactance exactly like an inductance-reactance. This quantity X is called the "*synchronous reactance*" of the armature.

The general relation between the total induced voltage (due to field flux Φ) and the terminal voltage of an alternator is shown in Fig. 180. The line OI represents the current in one of the armature windings, the line OE represents the total induced electromotive force, the line E_z represents the terminal voltage, RI represents the electromotive force lost in overcoming armature resistance, and the line XI represents the electromotive force which is lost on account of armature magnetizing action.

0 **129. Calculation of voltage regulation of an alternator.** — Nearly all alternating-current generating stations deliver current at an approximately constant voltage, and the various lamps and motors are connected in parallel across the mains. It would be desirable therefore if an alternating-current generator could be made to give an exactly constant voltage, however its current output might vary, and in practice alternators are constructed to give as small a variation of voltage as possible from zero load to full load. Purchasers generally require of a manufacturer a certain guarantee as to the voltage regulation of an alternator. In view of this guarantee, it is always necessary for the manufacturer to determine

the voltage regulation of an alternator by test, so as to be sure that it comes within the guarantee. In the case of a small machine, this test can be made directly by driving the machine, adjusting its field excitation to give normal voltage at full load, and observing the rise of voltage when the load is thrown off, speed and field excitation being unchanged. In the case of large machines, however, this direct method is not usually feasible, and it is therefore customary to calculate the voltage regulation of large machines in terms of certain easily obtained data. The data which are ordinarily used for this purpose are included in what are called the *saturation curve* and the *synchronous reactance curve* of the machine.

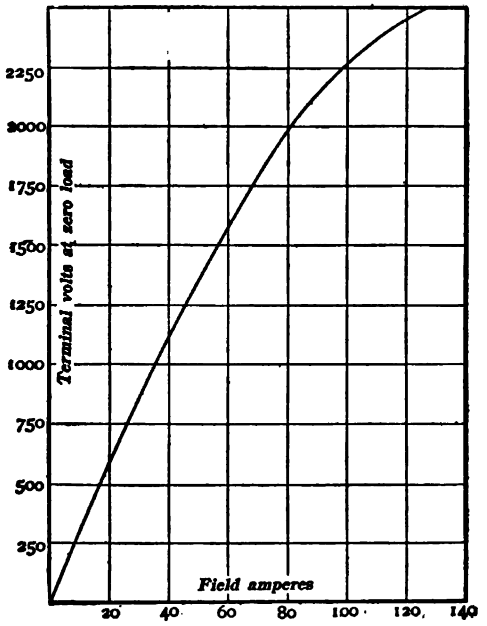

Fig. 184.

The saturation curve. — The saturation curve of an alternator is a curve of which the abscissas represent the values of the field current and the ordinates represent the corresponding observed values of the terminal voltage of the machine at zero load. Thus, Fig. 184 shows the saturation curve of a certain 2,000 kilowatt, Δ-connected, 3-phase alternator. The ordinates of this curve represent the observed voltages across any two collector rings of the machine, and the abscissas represent the corresponding values of field current in amperes.

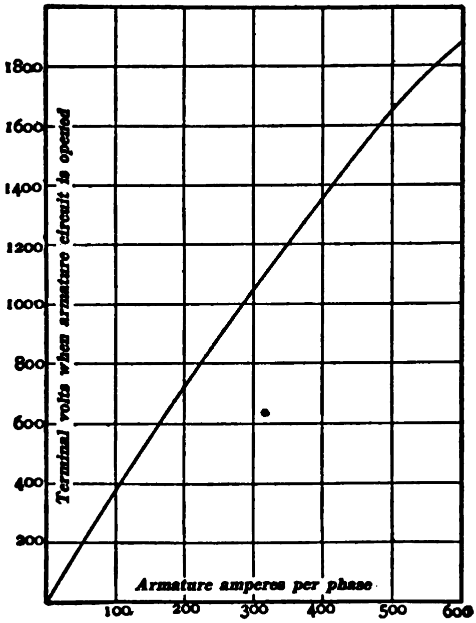

Fig. 185.

The synchronous reactance curve is determined as follows: The armature is short-circuited * through an ammeter, the machine is driven at normal speed, the field excitation is adjusted until full-load current (or any desired fraction of full-load current) flows through the ammeter and the ammeter reading is taken. The short-circuit is then opened, and, with unchanged speed and field excitation, the voltage across the alternator terminals is measured. The ammeter readings are then plotted as abscissas and the corresponding open-circuit voltages are plotted as ordinates. Figure 185 is the synchronous reactance curve of the 2,000-kilowatt,

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* In a polyphase armature all of the phases must be simultaneously short-circuited.

3-phase alternator which is referred to in connection with Fig. 184 above. In Fig. 185 the abscissas represent amperes in each armature winding (ammeter readings divided by $\sqrt{3}$).

The reason for calling this curve the synchronous reactance curve is that the voltage corresponding to any given abscissa (current) is the "total induced electromotive force" required to force the given current through the short-circuited armature, and, since the resistance of the short-circuited armature is negligibly small, it is evident that the only thing which opposes the flow of this short-circuited current is the armature inductance or magnetizing action, whichever one may wish to call it. The reactance value of this inductance depends of course upon the frequency, and the term synchronous reactance refers to the reactance value of the armature inductance or magnetizing action at the normal speed of the machine.

Electromotive force method of calculating voltage regulation. — This method may be best explained by giving numerical examples. Let it be required to calculate the voltage regulation (full load to zero load) of the 2,000-volt, 2,000-kilowatt, 3-phase alternator referred to in connection with Figs. 184 and 185; (*a*) for receiving circuits of unity power factor, and (*b*) for receiving circuits of 0.85 power factor, the resistance of one of the armature windings (one phase) being 0.07 ohm.

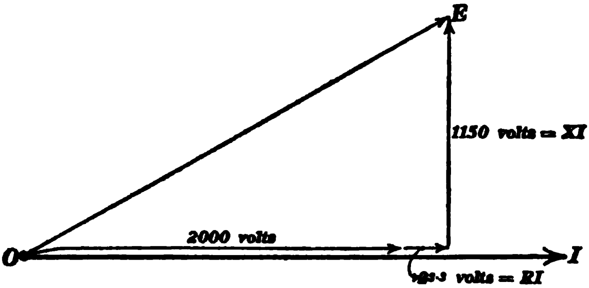

Fig. 186.

(*a*) *For unity power factor.* — The full-load current in one armature winding is 333 amperes, so that the RI drop in the armature at full load is 23.3 volts. This RI drop is in phase with the current and therefore in phase with the terminal voltage, since the power factor of the receiving circuit is assumed to be unity, so that the sum of the RI drop and the terminal voltage is 2,023.3 volts. From Fig. 185 we find that 1,150 volts correspond to 333 amperes, that is to say, the value of XI in Fig. 186 is equal to 1,150 volts. Therefore the "total induced electromotive force," that is, the electromotive force which is induced by the "field flux" Φ, is equal to $\sqrt{(2{,}023.3)^2 + (1{,}150)^2}$ or 2,327 volts, which is of course the value to which the terminal voltage would rise if the current output were reduced to zero. Therefore the regulation of the machine for unity power factor is $(2{,}327 - 2{,}000)/2{,}000 = 0.163$ or 16.3 per cent.

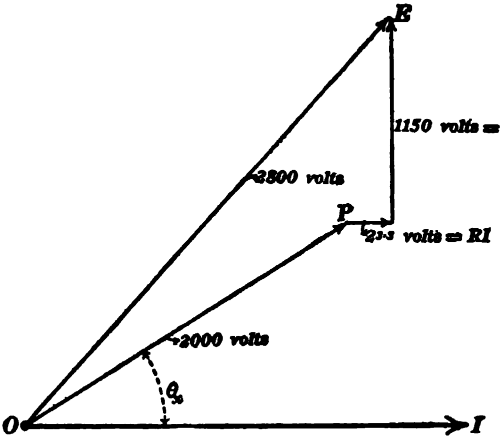

Fig. 187.

(*b*) *For a power factor equal to 0.85.* — Choose the current vector OI, Fig. 187, as the reference axis, lay off the full-load terminal voltage of 2,000 volts in its proper phase relation to I ($\cos\theta_x = 0.85$), and lay off RI (23.3 volts) and XI (1,150 volts) as shown; the line OE will then represent the total induced electromotive force, and it is, of course, the value to which the terminal voltage of the machine would rise if the current output were reduced to zero. The value of the zero-load voltage (corresponding to 2,000 volts terminal voltage at full load and 0.85 power factor) is thus found to be 2,800 volts, so that the regulation is (2,800 − 2,000)/2,000 = 0.40 or 40 per cent.

The magnetomotive force method.* — If the magnetizing action of the field winding and of the armature were not complicated by the polar projections of the field magnet, that is, if the armature were completely and uniformly surrounded with iron as shown in Figs. 181*b*, 182*b* and 183*b*, then the combined effect of two magnetizing actions A and B would be as follows: Consider the flux Φ across the armature due to A and the flux Φ' due to B. Draw lines in the directions of Φ and Φ', and let the lengths of these lines represent the magnetomotive force values (ampere-turns) of A and B respectively. Then the vector sum of these lines will represent the direction and value (in ampere-turns) of the single magnetizing action which is equivalent to the combined action of A and B. This is the fundamental idea of the magnetomotive force method for calculating voltage regulation, and the method is best explained by considering numerical examples. Let it be required to calculate the voltage regulation (full load to zero load) of the 2,000-volt, 2,000-kilowatt, 3-phase alternator referred to in connection with Figs. 184 and 185, (*a*) for receiving circuits of unity power factor and (*b*) for receiving circuits of 0.85 power factor, the resistance of one of the armature windings (one phase) being 0.07 ohms.

(*a*) *Unity power factor.* — When the alternator delivers full-load current of 333 amperes from each of its armature windings to non-inductive receiving circuits, the two components of the total induced voltage parallel to and at right angles to the current are 2,023.3 volts and 1,150 volts as explained above under the electromotive force method. From Fig. 184 we find that a field current of 84 amperes is required to give 2,023 volts total induced electromotive force; and a field current of 42 amperes is required to give a total induced electromotive force of 1,150 volts. To give these two quadrature electromotive forces simultaneously, a field current equal to 94 amperes ($=\sqrt{84^2+42^2}$) would be required; and, according to Fig. 184, this field current of 94 amperes would give a total induced electromotive force of 2,200 volts, which is the value to which the terminal voltage of the machine would rise at zero load. Therefore the regulation of the machine is $(2{,}200 - 2{,}000)/2{,}000 = 0.10$ or 10 per cent.

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* This is called the American Institute Method, inasmuch as it is recommended by the Institute Committee on Standardization.

(*b*) *For power factor equal to 0.85.* — From Fig. 187 the two components of the total induced electromotive force, parallel to and perpendicular to the current, are 1,723.3 volts and 2,207 volts respectively. The field currents required to produce total induced voltages of 1,723.3 and 2,207 are found from Fig. 184 to be 68 amperes and 94 amperes respectively, and to give these two quadrature electromotive forces simultaneously would require 116 amperes ($=\sqrt{68^2+94^2}$). The zero-load voltage corresponding to 116 amperes of field current is found from Fig. 184 to be 2,400 volts. Therefore the regulation of the alternator is $(2{,}400 - 2{,}000)/2{,}000 = 0.20$ or 20 per cent.

Note. — The American Institute rule for calculating the voltage regulation of an alternator is ambiguous. Under the rule, one can calculate the field excitations (ampere-turns) corresponding separately to the two *components* of *OE*, Fig. 187, as above; or one can calculate the field excitation which corresponds to *OP*, and the field excitation which corresponds to *PE*, Fig. 187, and add these two field excitations as vectors parallel to *OP* and *PE*, respectively, to give the total field excitation required at full load.

Comparison of the two methods for calculating voltage regulation. — The two methods for calculating voltage regulation are based on assumptions that depart very widely indeed from the conditions which actually exist in an alternator (compare Figs. 181*a*, 182*a* and 183*a*, with Figs. 181*b*, 182*b* and 183*b*), and a theoretical comparison of the two methods would depend upon a complete analysis of the actual conditions represented in Figs. 181*a*, 182*a* and 183*a*. The only satisfactory way to compare the two methods is to compare the results of calculations with actually observed values of voltage regulation. The electromotive force method generally gives a larger (poorer) regulation than is found by actual test, and the magnetomotive force method generally gives a smaller (better) regulation than is found by actual test. The former method has therefore been called the "pessimistic method" and the latter the "optimistic method."[*]

130. Armature demaguetizing action.—The discussion of the magnetizing action of the armature current, or currents, of an alternator which is given in Arts. 128 and 129 covers the ground as completely as it can be covered without entering into the theory of non-harmonic electromotive forces, and to do this would lead to impractical mathematical complications. There are, however, two fairly simple aspects of armature magnetizing action[†] which are of some practical importance as follows: (*a*) the magnetic flux under the pole pieces is crowded to one side (in the direction of rotation of the armature), and (*b*) the armature currents *oppose* the action of the field winding in forcing flux through the armature when the receiving circuit is inductive, and they *help* the field winding in forcing flux through the armature when the receiving circuit has negative reactance like a condenser.

Discussion of demagnetizing action.—Consider the conductors TT, Fig. 188, which constitute the armature winding of a 2-pole single-phase alternator armature. The electromotive force of the machine may be written

$$e = \mathbf{E} \sin \omega t \tag{i}$$

where ωt is the angle shown in Fig. 188, and the current delivered by the alternator may be expressed thus

$$i = \mathbf{I} \sin (\omega t - \theta) \tag{ii}$$

where θ is the angle of lag of the current behind the electromotive force, which angle of lag depends of course upon the character of the receiving circuit.

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[*] See "The Experimental Basis for the Theory of Regulation of Alternators," by B. A. Behrend, *Transactions of the American Institute of Electrical Engineers*, Vol. 21, pages 497–517, 1903; and "The Regulation of Alternators," by D. B. Rushmore, *Transactions of the International Electrical Congress*, Vol. I, pages 729–761, St. Louis, 1904.

[†] These two aspects of armature magnetizing action are involved by implication in the discussion which is given in Arts. 128 and 129.

Let T be the number of turns of wire on the armature (four turns are shown in Fig. 188), then iT is the ampere-turns due to the winding at a given instant, and this action is in the direction of the arrow iT, Fig. 188. The component of iT in the direction of the line NS is equal to $iT\cos\omega t$, and this is a measure, in ampere-turns, of the amount that the armature current is helping the field winding at the given instant. Let this be represented by m, so that

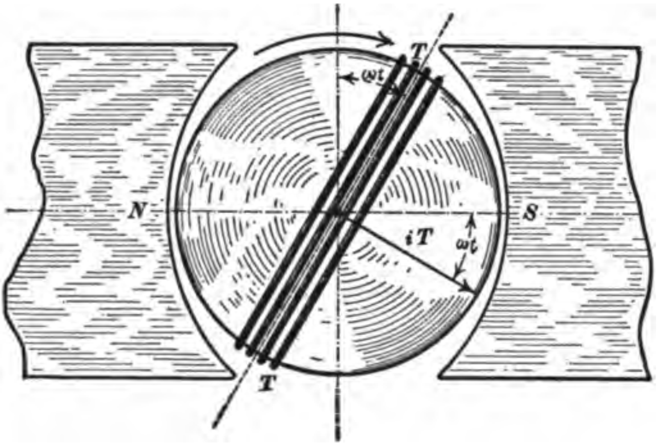

Fig. 188.

$$m = iT\cos\omega t \tag{iii}$$

or, substituting the value of i from equation (ii) we have

$$m = \mathrm{I}T\cos\omega t\sin(\omega t - \theta)$$

or

$$m = \mathrm{I}T\cos\omega t\sin\omega t\cos\theta - \mathrm{I}T\cos^2\omega t\sin\theta \tag{iv}$$

The average value of m is the average amount in ampere-turns that the field winding is helped by the armature current, and the average value of m is equal to the sum of the average values of the two terms of the right-hand member of equation (iv), but the average value of the first term is zero and the average value of the second term is $-\frac{1}{2}\mathrm{I}T\sin\theta$, inasmuch as the average value of $\cos^2\omega t$ is equal to $\frac{1}{2}$. Therefore

$$\text{average value of } m = -\tfrac{1}{2}\mathrm{I}T\sin\theta$$

or, substituting the effective value $I = \mathrm{I}/\sqrt{2}$, we have

$$\text{average value of } m = -\frac{1}{\sqrt{2}}IT\sin\theta \tag{v}$$

The meaning of this equation is as follows: (*a*) When θ is zero the armature current neither helps nor opposes the passage of flux, on the average; (*b*) when θ is positive, that is, when the current lags behind the voltage in phase, the armature current opposes the passage of flux, on the average; and (*c*) when θ is negative, that is, when the current is ahead of the voltage in phase, the armature current helps the passage of flux, on the average.

The constancy of the magnetizing action of the armature currents in a two-phase armature, when the receiving circuits are balanced, is easily shown. Let m, equation (iv), be the instantaneous value of the magnetizing action of phase A of a two-phase armature. The instantaneous value of the magnetizing action m' of phase B is obtained by substituting $\omega t \pm 90^\circ$ for ωt in equation (iv) which gives

$$m' = -\mathrm{I}T \sin \omega t \cos \omega t \cos \theta - \mathrm{I}T \sin^2 \omega t \sin \theta. \qquad \text{(vi)}$$

Therefore the total magnetizing action of the two phases is

$$m + m' = -\mathrm{I}T(\sin^2 \omega t + \cos^2 \omega t) \sin \theta$$

or

$$m + m' = -\mathrm{I}T \sin \theta$$

which is constant.

131. Power rating of alternators.* — The current delivered by an alternator generates heat in the armature, which, together with the heat generated by eddy currents and hysteresis, causes the temperature of the armature to rise until it gives off heat as fast as heat is generated in it. Therefore, to increase the current output of an alternator causes an increased rise of temperature of the machine. This heating effect usually determines the current rating of an alternator, and the power rating of an alternator is always understood to be the power that it can deliver steadily to a non-inductive receiving circuit (power factor unity) without undue rise of temperature. That is, the power rating of an alternator is understood to be the product of its rated voltage and its rated current. This power rating of an alternator is sometimes specified in *volt-amperes* or *kilovolt-amperes*, to distinguish it from the actual power the alternator can deliver, for the actual power depends upon the power-factor of the receiving circuit. Thus an alternator rated at 2,000 volts and 100 amperes is called a 200 kilowatt or a 200 kilovolt-ampere alternator. Such an alternator could, in fact, deliver 200 kilowatts to a non-inductive receiving circuit, but with the same rise of temperature it could deliver only 150 kilowatts to a receiving circuit of 0.75 power factor.

* See reference to Report of Standardization Committee of A. I. E. E., on page 190.

132. Frequencies. — The standard frequencies employed in American practice are 25 cycles per second, 60 cycles per second, and in older installations 125 or 133 cycles per second.

Very low frequencies are not suitable for lighting on account of the tendency of low frequency to produce flickering of the lights. A frequency of about 60 cycles per second seems to be the lower limit for satisfactory lighting.

High frequencies are not well adapted for the operation of induction motors, synchronous motors, or rotary converters, because high frequencies necessitate either great speed or a great number of field poles, and in single-phase commutator motors high frequencies involve excessive sparking and low power factors. For driving motors and rotary converters a frequency of 25 cycles per second is the standard in American practice.

When an alternator is to supply current for lighting and also for driving motors a frequency of 60 cycles per second is generally used in American practice.

CHAPTER XII.

THE SYNCHRONOUS MOTOR.

133. The synchronous motor. The operation of alternators in parallel. — Figure 189 shows the essential connections for operating two alternators in parallel so that the combined output of the two alternators may be delivered to a single receiving circuit; and Fig. 190 shows the essential connections for operating one alternator, say *A*, as a generator and the other, *B*, as a synchronous motor. In the operation of an alternator as a synchronous motor it is usually at a distance from the alternating-current generator which drives it, so that the generator *A* and the motor *B*, Fig. 190, require separate sources of direct current for field excitation.

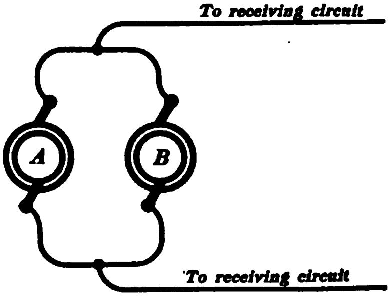

Fig. 189.

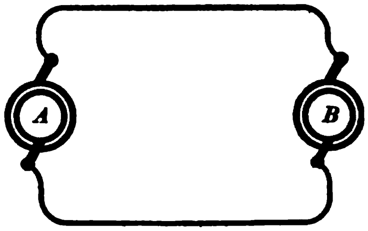

Fig. 190.

In Fig. 189 the two alternators *A* and *B* form a closed circuit independently of the receiving circuit exactly as in Fig. 190. In fact, the interaction of two alternators which are connected in parallel as generators is in many respects the same as the interaction of two alternators, one of which drives the other as a synchronous motor. Therefore the two cases are here treated under one general heading; in fact, the parallel operation of alternators as generators is best treated as a special case of synchronous motor operation.

The two machines *A* and *B*, in Figs. 189 and 190, are in series with each other in regard to the circuit formed by the two machines, and this circuit formed by the two machines is the only circuit at present to be considered. The electromotive force which tends to produce current in this circuit is at each instant equal to the algebraic sum * of the electromotive forces of the respective machines. Therefore, if the machines are in synchronism, that is, if their electromotive forces have the same frequency, the total electromotive force which tends to produce current in the circuit may be represented in the clock diagram as the resultant *E* of the two lines *A* and *B* which represent the electromotive forces of the respective machines, as shown in Fig. 191. In this diagram ϕ is the phase difference between *A* and *B*, and this angle remains constant in value so long as the two machines *A* and *B* run exactly in synchronism. If, however, one machine runs momentarily faster than the other, the value of ϕ will change. It is evident that ϕ can have any value whatever, except in so far as it may be kept within certain limits by the action of the two machines on each other. For the present we will consider that the angle ϕ can have any value from zero to 360°.

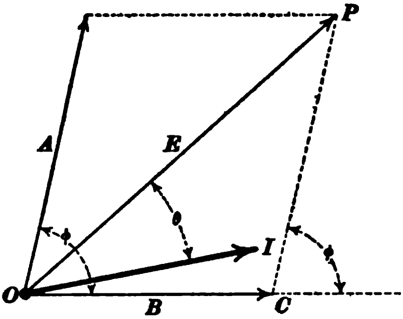

Fig. 191.

The resultant electromotive force *E* produces in the circuit of the two machines a current *I* of which the value is

$$I = \frac{E}{\sqrt{R^2 + X^2}} \qquad \text{(i)}$$

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* The electromotive force of each machine is here considered to be positive when it acts in a chosen direction around the circuit formed by the two machines.

and which lags θ degrees behind E in phase where θ is the angle whose tangent is X/R, that is,

$$\tan\theta = \frac{X}{R} \tag{ii}$$

R being the resistance and $X(=\omega L)$ the reactance of the circuit. The power P' delivered to the circuit by machine A is

$$P' = AI\cos(AI) \tag{iii}$$

where (AI) is the angle between A and I in Fig. 191, and the power P'' delivered to the circuit by machine B is

$$P'' = BI\cos(BI) \tag{iv}$$

where (BI) is the angle between B and I in Fig. 191.

(*a*) For certain values of ϕ the angle (AI) may be less than 90° and the angle (BI) greater than 90°, and if this condition occurs, then P' will be *positive* and P'' will be *negative*. That is, machine A will give out power and machine B will take in power, or in other words, machine A will act as a generator and machine B will act as a motor.

(*b*) For certain values of ϕ the angle (AI) may be greater than 90° and the angle (BI) less than 90°, and if this condition occurs, machine A will act as a motor and machine B will act as a generator.

134. Variation of P′ and of P″ with the phase angle ϕ.— Before the physical aspects of either of the above conditions *a* or *b* can be discussed, it is necessary to examine carefully into the geometrical relations involved in Fig. 191 on the assumption that ϕ may have any value whatever. It is sufficient for our purpose however to consider the power output (or intake) of one machine, say machine B. For this purpose it is convenient to take the line B in Fig. 191 as a fixed reference axis. Then, as the angle ϕ changes, the point P will describe a circle about the point C, for the line CP is of constant length, being equal to A [the electromotive forces (effective) of machines A and B are of course supposed to be constant and their phase difference

ϕ, only, is supposed to change]. The triangle OCP in Fig. 192 is the triangle OCP of Fig. 191, and the circle in Fig. 192 is the above-mentioned locus of the point P.

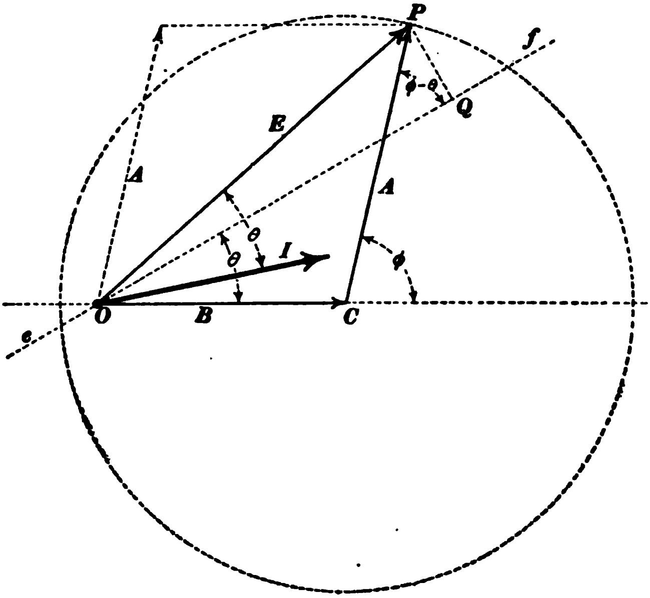

Fig. 192.

Draw the line ef, Fig. 192, through the point O making with OC the angle θ as shown, then OQ, *the projection of* OP *on the line* ef, *represents the power output* P' *of machine* B. This is shown as follows: The angle between B and E is equal to $\theta + (BI)$ and it is also equal to θ plus the angle POf, so that the angle POf is the same as the angle (BI). Therefore

$$\overline{OQ} = \overline{OP} \cos (BI) = E \cos (BI) = \sqrt{R^2 + X^2} \cdot I \cos (BI)$$

or

$$I \cos (BI) = \frac{\overline{OQ}}{\sqrt{R^2 + X^2}}$$

whence, substituting this value of I cos (BI) in equation (iv), we have

$$P'' = \frac{B \cdot \overline{OQ}}{\sqrt{R^2 + X^2}} \tag{v}$$

That is, the power output of machine B is proportional to $\overline{OQ}$ and the proportionality factor is the constant $B/\sqrt{R^2 + X^2}$.

Equation (v) may be put into a more convenient form as follows: The projection of A, Fig. 192, on the line ef is equal to $A \cos(\phi - \theta)$; the projection of B on ef is $B \cos\theta$; and $\overline{OQ}$ is the sum of these two projections. Therefore, substituting $A \cos(\phi - \theta) + B \cos\theta$ for $\overline{OQ}$ in equation (v), we have

$$P'' = \frac{AB}{\sqrt{R^2 + X^2}} \cdot \cos(\phi - \theta) + \frac{B^2}{\sqrt{R^2 + X^2}} \cdot \cos\theta \tag{53}$$

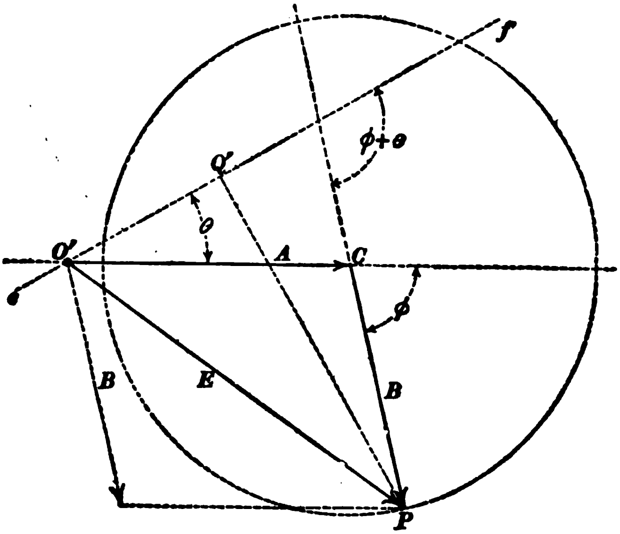

Fig. 193.

Figure 193 is a construction essentially similar to Fig. 192, but with the electromotive force A taken as the fixed reference axis; both Figs. 192 and 193 being constructed for the same value of ϕ. From Fig. 193 it may be shown that

$$P' = \frac{A . \overline{O'Q'}}{\sqrt{R^2 + X^2}} \tag{vi}$$

from which an expression for P' may be derived corresponding to equation (53), namely,

$$P' = \frac{AB}{\sqrt{R^2+X^2}} \cdot \cos(\phi+\theta) + \frac{A^2}{\sqrt{R^2+X^2}} \cdot \cos\theta \qquad (54)$$

These equations, (53) and (54), are the fundamental equations of the synchronous motor. Of course the algebraic sum of P' and P'' is equal to RI^2.

A very clear idea of the variation of P'' with ϕ is given by the variation of $\overline{OQ}$, Fig. 192, and a clear idea of the variation of P' with ϕ is given by the variation of $\overline{O'Q'}$, Fig. 193. This variation of P' and P'' with ϕ is extremely important in the following physical discussion, and a clearer insight into this variation may be obtained with the help of Figs. 194 and 195,

$A = 1,100$ volts. $R = 1.00$ ohm.
$B = 1,000$ volts. $\omega L = 0.58$ ohm.

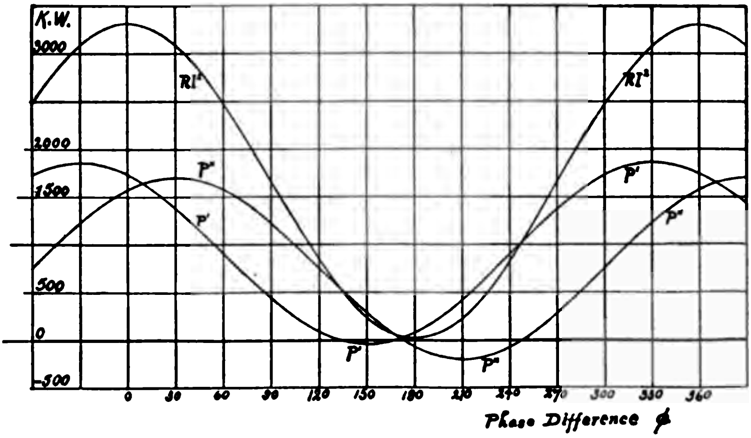

Fig. 194.

in which the values of ϕ are represented as abscissas, and the values of P', P'' and $RI^2 (= P' + P'')$ as ordinates. Figure 195 is an enlargement of the middle portion of Fig. 194, and the ordinates of the curve η represent the values of the ratio P''/P', disregarding sign. This ratio, when P'' is an intake of power by machine B (P'' negative) and P' an output of power by machine A (P' positive), is the efficiency, ignoring all losses except the RI^2 loss in the circuit.

The variation of P'' with ϕ may be shown in greater detail as follows: Draw a line ss' through O, Fig. 192, at right angles to *ef*, as shown in Fig. 196. Consider that if machine A runs momentarily faster than B or, in other words, if machine B runs momentarily slower than A, then the angle ϕ increases, and the point P, Fig. 196, moves around the circle in the direction of the curved arrow. So long as $\overline{OQ}$ is towards f the value of P'' is positive and machine B gives out power; when the point P, Fig. 196, is at s or s', however, the value of $\overline{OQ}$ is zero and machine B neither gives out nor takes in power; and when the point P is anywhere in the region sMs' the value of $\overline{OQ}$ is negative, and machine B takes in power. This region in which $\overline{OQ}$, Fig. 196, is negative is the region in which P'' is negative in Figs. 194 and 195.

$A = 1,100$ volts. $R = 1.00$ ohm.
$B = 1,000$ volts. $\omega L = 0.58$ ohm.

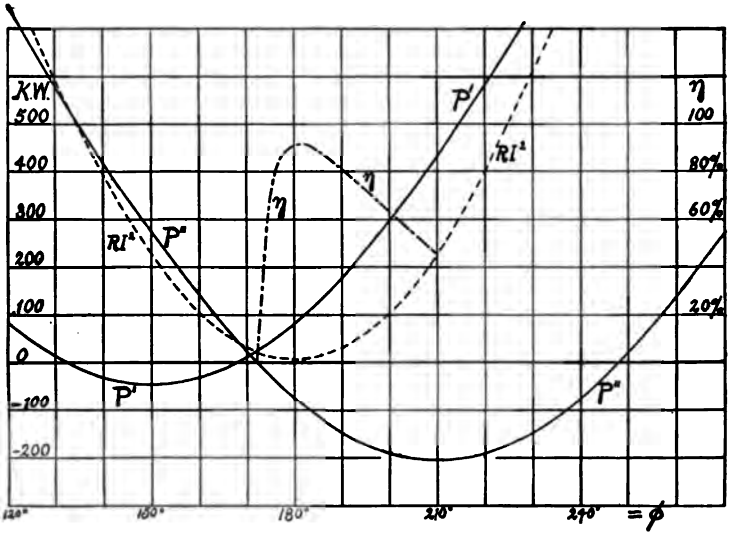

Fig. 195.

It is to be particularly noted that when the value of ϕ is nearly zero, that is, when the electromotive forces A and B are nearly in phase with each other, the power output of both machines is excessively large. Thus, in the neighborhood of $\phi = 0°$ or $360°$ in Fig. 194, P' and P'' are both positive and each is more than 1,500 kilowatts, although the machines to which Figs. 194 and 195 refer, are each of about 100 kilowatts rating. The excessive values of P' and P'' in the neighborhood of $\phi = 0°$ or $360°$ is due to the fact that when ϕ is zero each machine forms a short-circuit for the other, whereas in the neighborhood of $\phi = 180°$ the two electromotive forces A and B oppose each other and the current is small.

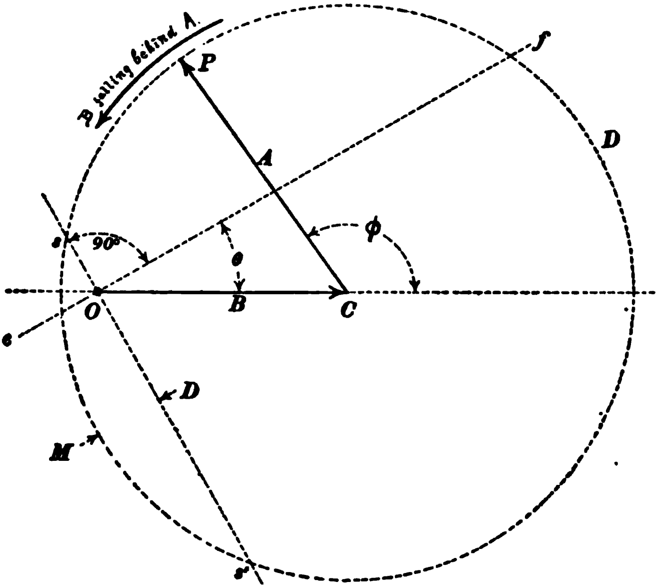

Fig. 196.

135. Necessity of exact synchronism for the operation of an alternator as a motor. — Suppose that machine B runs continuously a little slower or a little faster than machine A, then the value of the angle ϕ will change continuously, and the point P, Figs, 192 and 193, will move steadily around the circle in one direction or the other. Under these conditions the power outputs P' and P'' of the two machines pass repeatedly through all of the values shown in Fig. 194, and, on the average, both P' and P'' will be positive, that is, both machines will on the average give out power.

If, however, machines A and B are in exact synchronism the value of ϕ remains constant, and either P' or P'' may be negative, that is, either machine A or machine B may take in power steadily * (motor action), according to the value of the angle ϕ.

Therefore, if an alternator (motor) is to take power steadily from another alternator (generator), the two machines must run in exact synchronism, and it is for this reason that an alternator when used as a motor is called a synchronous motor.

136. Stability of running of a synchronous motor. — The necessity of exact synchronism of running of an alternator which is to be used as a motor may seem to be an impracticable condition; but if an alternator is once started to operate as a motor, exact synchronism is automatically maintained, unless the load on the motor becomes excessive. This stability of operation may be shown by the help of Fig. 196, remembering that any kind of a motor whatever, steam or electric, will show stability of running if a momentary decrease in its speed causes it to take more power from boiler or electric generator, whereas the running will be unstable if a momentary decrease in its speed causes it to take less power from boiler, or electric generator.

Suppose that machine A is driven at constant speed by a governed engine or water wheel, and suppose that *machine B is in synchronism with A with the point P, Fig. 196, at any given position on the circle.* If the point P is anywhere between s and M* a momentary decrease of speed of B will cause the point P to move towards M (B falling behind A in phase), and the power intake of machine B will increase, and thus counteract the momentary decrease of speed due for example to a sudden increase of motor load on machine B. If, on the other hand, machine B runs momentarily faster than A, then the point P, Fig. 196, will move away from M and towards s (B getting ahead of A in phase), and the power intake of machine B

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* Of course the power intake pulsates as explained in Art. 54.

will decrease and thus counteract the momentary increase of speed due for example to a sudden decrease of motor load on machine *B*. That is to say, the region between *s* and *M* is the region of stable running of machine *B* as a synchronous motor.

The region between *M* and *s'*, Fig. 196, is a region of unstable running, for, when the point *P* is in this region, a momentary decrease of speed of machine *B*, due for example to a sudden increase of its motor load, causes machine *B* to take in a decreased amount of power so that the decrease of speed will not only persist but it will become more and more pronounced until the point *P*, Fig. 196, is carried beyond *s'* ; then machine *B* not only has a belt load as a motor but it actually gives out power as a generator, so that its speed will fall very rapidly indeed and carry the point *P* round and round the circle until the momentum of the rotating armature of machine *B* is exhausted and the machine comes to a dead stop.

If machine *B* is operating steadily with the point *P* in the stable region *sM*, then any increase of load which carries the point *P* beyond *M* will cause the machine to come to a dead stop as above described.

From Fig. 196 it is evident that there are two positions of the point *P* for a given value of motor intake, one position is in the region *sM* of stable running and the other position is in the region *Ms'* of unstable running. The position for stable running has the smaller value of *E* and therefore the smaller value of *I*.

If machine *B* is operating steadily with the point *P* in the stable region *sM*, then if the motor load on *B* were reduced to zero, the speed of *B* would be momentarily increased and the point *P* would move to *s*. When *P* is at *s*, $\overline{Os}$ is the resultant electromotive force *E*, the current vector *I*, which is $\theta°$ behind *E* in phase, is vertically upwards, the angle (BI) is 90°, and P' is zero according to equation (iv), Art. 133.

137. Hunting action of the synchronous motor. — When the load on a synchronous motor is suddenly increased, the motor speed decreases momentarily and the motor falls behind the generator in phase. By the time the motor has fallen behind the generator sufficiently to take in power enough to enable it to carry its load, its speed has become a minimum because up to this instant it has been taking less than enough power to carry its load. Therefore the motor continues to fall behind the generator, taking in more than enough power to carry its load, and this excess of power quickly brings the motor up to the generator speed. At this instant the motor is farthest behind the generator in phase because up to this instant its speed has been less than the speed of the generator. Then, the motor as it continues to take in an excess of power begins to exceed the generator in speed and gains on the generator until it takes in less than enough power to carry its load, when its speed again falls below that of generator and so on. This oscillation of a synchronous motor backwards and forwards through a certain mean position is called ***hunting*** and it is similar to the hunting of a badly governed steam engine.

The hunting oscillations of a synchronous motor are always accompanied by oscillations of the driving generator, the motor and generator oscillations being of the same rhythm and related to each other like the oscillations of the two prongs of a tuning fork.

The hunting of a synchronous motor (or rotary converter) is often very troublesome, making it impossible to operate the motor even with a fairly light load, on account of the carrying of the point P, Fig. 196, so far into the region Ms' of unstable running that it cannot return (motor intake being less than motor load), but goes on beyond s', and the motor comes to a dead stop as described in Art. 136.

In general the hunting oscillations of a synchronous motor when once started tend to go on increasing in amplitude, and the prevention of troublesome hunting depends upon some action which tends to decrease the amplitude of the oscillations. Two such actions are available, namely, (*a*) The use of a fly-wheel loosely attached to the synchronous motor shaft so as to slip as the speed of the synchronous motor increases and decreases. This device sometimes takes the form of a hollow fly-wheel rigidly fixed to the motor shaft and filled with liquid, and (*b*) The use of damping frames which consist of copper bars set into the pole faces of the synchronous motor parallel to the armature shaft, the ends of these bars being short-circuited by end-bars of copper.

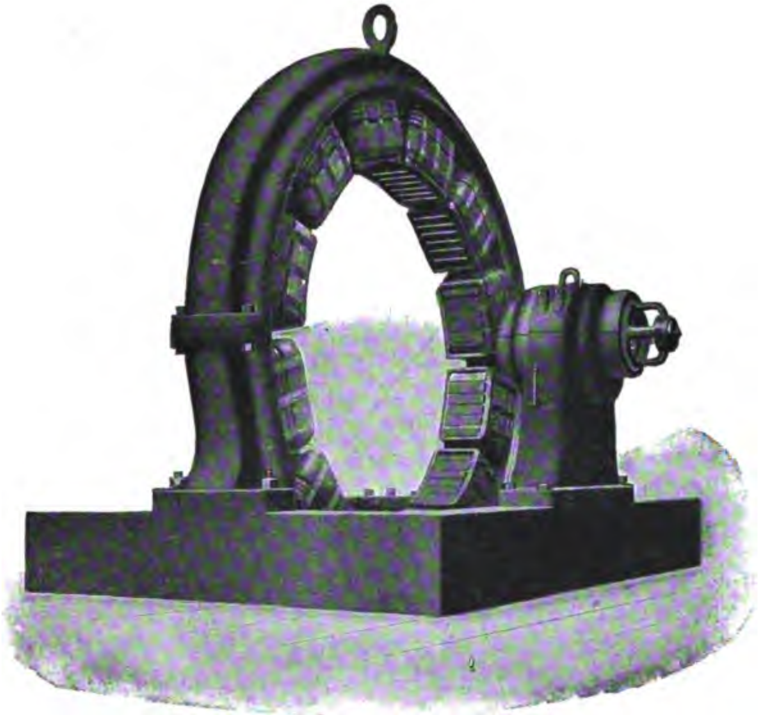

Fig. 197.

Figure 197 shows the field magnet of a synchronous motor, or rotary converter, the pole faces of which are provided with copper damping frames or grids. This arrangement is seldom used on belted synchronous motors, it is however very frequently used on rotary converters and upon synchronous motors which are coupled to direct-current generators. The action of these damping frames is as follows: As the speed of a hunting synchronous motor increases and decreases, the torque exerted on the armature by the field magnet increases and decreases, this increase and decrease of torque is accompanied by a shifting of the magnetic flux backwards and forwards over the pole faces, and this shifting flux induces current in the short-circuited bars of the damping frame, which current opposes the shifting of the flux and thereby dampens the hunting oscillations.

A synchronous motor with a belt load is much less prone to hunting oscillations than a rotary converter, or a synchronous motor directly connected to a generator, for the reason that the slipping of the belt dampens the hunting oscillations like a loose fly-wheel.

When the generator which supplies current to a synchronous motor is driven by an engine the slight pulsations of speed of the engine tend to set up hunting oscillations, especially if the frequency of the hunting oscillations is the same as the frequency of pulsations of the engine speed. In such a case troublesome hunting may be avoided by increasing the amount of inertia of the armature of the synchronous motor by attaching a fly-wheel to it.

138. Starting of the synchronous motor. — Let the machine A which is to operate as a generator be running with its field fully excited; then to put the machine B into operation as a synchronous motor it has to be driven by some independent source of power, its field has to be properly excited, and its speed has to be carefully adjusted until (*a*) *it is exactly in synchronism with machine* A, *and* (*b*) *its electromotive force is nearly opposite in phase to the electromotive force of* A*;* and when these two conditions are realized the circuit of the two machines A and B may be closed, and the independent source of power which was used to start B may be disconnected.

In bringing about the two conditions (*a*) and (*b*) previous to the closing of the circuit of the two machines, it is necessary to use some sort of a device for indicating when the two machines are in synchronism and for indicating their phase relation. Such a device is called a *synchronizing device* or a *synchronizer.*

The simplest synchronizer is an ordinary incandescent lamp connected in the circuit of the two machines as shown in Fig. 198.

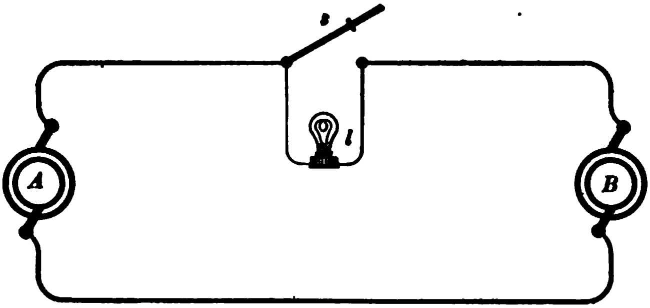

Fig. 198.

This lamp pulsates in brightness as machine B is speeded up, and the pulsations of brightness become slower and slower as the frequency of B approaches the frequency of A. When the lamp is at its maximum of brightness the phase difference ϕ of A and B, see Fig. 192, is zero, and when the lamp is at its minimum of brightness the phase difference of A and B is 180°. When the pulsations of brightness become very slow the machines A and B are practically in synchronism [condition (a)] and the switch s, Fig. 198, is closed when the lamp is at its minimum of brightness [condition (b)].

In practice the voltages of machines A and B are much too large to permit of the use of a lamp as shown in Fig. 198, and two step-down transformers T and T' are used as shown in Fig. 199, the lamp l being connected in circuit with the two secondary coils of the two transformers. With this arrangement the connections of one of the transformers may be reversed so that the lamp may be at its *maximum brightness* when the electromotive forces of machines A and B are opposed, that is, when condition (b) is right for closing the double-pole switch ss'.

The dial synchronizer is a device in which a pointer is caused to rotate at a speed which is equal to the difference of the frequencies of machines A and B, Fig. 199. The dial synchronizer, which is now extensively used, indicates approximate synchronism [condition (*a*)] by slowness of speed of the pointer, and it indicates the proper phase relation [condition (*b*)] by the position of the pointer on the dial. The chief advantage of the dial synchronizer is that it shows whether the speed of machine B is too great or too small, whereas the synchronizing lamp behaves in exactly the same way whether the speed of machine B be too great or too small.

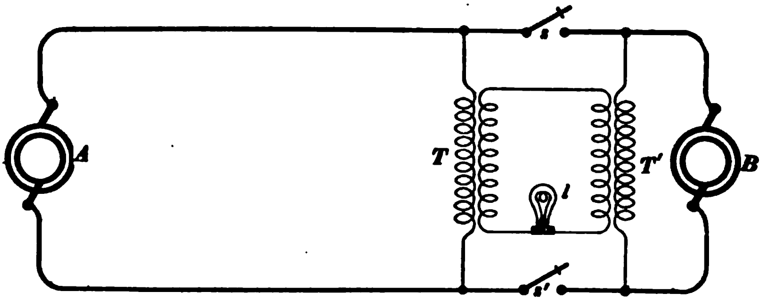

Fig. 199.

✓ **139. The polyphase synchronous motor.** — The foregoing articles (and also the remainder of this chapter) refer explicitly to the single-phase synchronous motor, that is, to a single-phase alternator taking power as a motor from a single-phase generator. The entire discussion applies equally well, however, to the polyphase synchronous motor, that is, to a polyphase alternator taking power as a motor from a polyphase generator. In such a case each armature winding of the motor takes current from one armature winding of the generator and the total power intake is nP'', where P'' is the intake of each armature winding or phase and n is the number of phases. To apply any portion of the discussion given in this chapter to a polyphase synchronous motor it is only necessary to remember that the quantities A, B, R, X, P', P'', E and I refer to one phase only of the system.

The single-phase synchronous motor will run in either direction, but the polyphase synchronous motor with given connections to the polyphase supply mains will run in a certain direction only.

140. The rotary converter. — The rotary converter, or, as it is frequently called, the synchronous converter, is described in the next chapter. This machine, as ordinarily used, is at once a synchronous alternating-current motor and a direct-current generator, it is usually polyphase, and it exhibits all of the features of the synchronous motor as to stability of running, as to hunting and as to starting. The next chapter is devoted chiefly to those aspects of the rotary converter in which it differs from a synchronous motor developing mechanical power, such, for example, as the relation between its voltages, its armature reaction, and its armature heating.

141. Use of the over-excited synchronous motor. — A synchronous motor of which the electromotive force (effective) is greater than the electromotive force (effective) of the generator (or supply mains) from which it takes current * is called an *over-excited synchronous motor*. The current taken from the supply mains by such a synchronous motor is ahead of the supply voltage in phase, and therefore an over-excited synchronous motor can be used instead of a condenser for supplying the wattless component of the current output of a station. †

In all of the foregoing discussion the machine B which acts as a synchronous motor is supposed to have a smaller voltage than the machine A which acts as the generator. It is evident, however, from Figs. 194 and 195 that the higher-voltage machine can be operated as a motor and take power from the lower-voltage machine acting as a generator, and two things concerning the motor action of the higher-voltage machine are at once apparent from Figs. 194 and 195, namely, (*a*) the larger-voltage machine cannot take in as a motor as much power as the lower-voltage machine, and (*b*) the RI^2 loss in the circuit is very large when the higher-voltage machine acts as a motor. It is therefore undesirable to operate an over-excited synchronous motor when the only object in view is to develop mechanical power.

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* To say that the field magnet of a synchronous motor is over-excited means that the excitation is such as to give a terminal voltage exceeding the supply voltage when the machine is disconnected from the supply mains. When an over-excited machine is connected to the supply mains it takes leading current which, according to Art. 130, weakens the field of the machine and brings the terminal voltage down to the value of the supply voltage. See Art. 150.

The quantity B in this chapter means the "total induced electromotive force" in machine B. See Art. 128.

† See Art. 113.

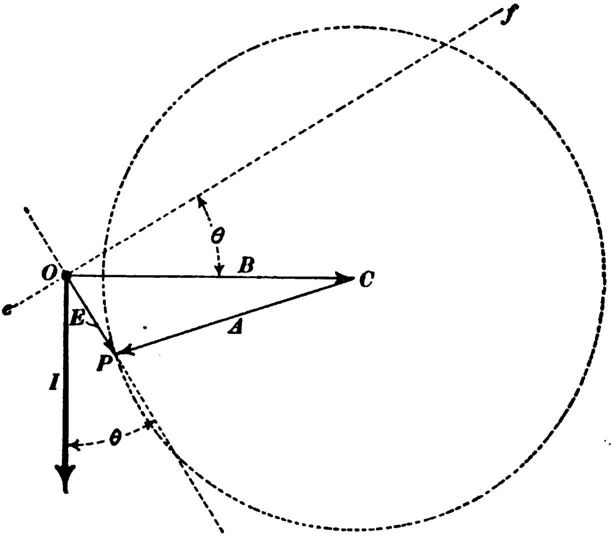


FIG. 200.

The clock diagram, Fig. 200, which is exactly similar to Fig. 196, represents the operating conditions of a synchronous motor B (at zero load) of which the electromotive force exceeds the supply voltage A.

In this clock diagram A represents the electromotive force between the supply mains, the angle θ is the angle whose tangent is equal to $\omega L/R$, and the current I is equal to $E/\sqrt{R^2 + \omega^2 L^2}$, where R is the resistance of the synchronous motor armature and its connecting wires (not including any resistance beyond the points to which the supply voltage *A* refers), and *L* is the inductance of the synchronous motor armature and connecting wires (not including any inductance beyond the points to which the supply voltage *A* refers). In the previous articles of this chapter, *R* is the resistance of the entire circuit including generator and motor, *L* is the inductance of the entire circuit including generator and motor, *A* is the "total induced electromotive force" in the generator armature, and *B* is the "total induced electromotive force" in the synchronous motor armature. A careful study of Fig. 200 will show that the current *I* is ahead of *A* in phase.

The most satisfactory conditions under which to use an over-excited synchronous motor for supplying the wattless component of the current output of a station is to make *double* use of the motor, that is, to use the motor for delivering a certain amount of mechanical power at the same time that it is being used to deliver wattless current. In fact the field excitation of the motor should be adjusted until its power factor is 0.707, in which case, the motor is capable of delivering 70.7 per cent. of its full rating as mechanical output, and at the same time it is capable of doing 70.7 per cent. of the full service of which it is capable in the matter of supplying wattless current.

✓**142. The operation of alternating-current generators in parallel.** — Consider an alternator *A*, Fig. 201, driven by an engine and delivering current to a receiving circuit, and consider what must be done to start up another alternator *B* and connect it in parallel with *A* so that the receiving circuit may be supplied from both generators. *The procedure is exactly the same as would be followed in getting machine B into operation as a synchronous motor.* Machine *A* being in operation, machine *B* is started up by starting its driving engine, its field is excited, and its speed is adjusted until the synchronizing lamp shows (1) that *B* is in synchronism with *A* and (2) that the electromotive force of *B* is opposite (opposite, that is, with reference to the circuit formed by the two machines) to the electromotive force of *A*, and then the double-pole switch *ss'* is closed. If machine *B* is to be operated as a synchronous motor its voltage should be somewhat less than the voltage of *A*, but if *B* is to be operated as a generator in parallel with *A* its voltage should be nearly the same as the voltage of *A*.

A number of alternating-current generators operating in parallel must run at absolutely the same speed, or rather, at absolutely the same frequency, and the possibility of operating a number of alternating-current generators in parallel depends upon the fact that when one generator runs momentarily slower than the others its consequent change of phase relative to the other machines causes its share of the station load to be greatly reduced thus counteracting its tendency to run at a reduced speed.

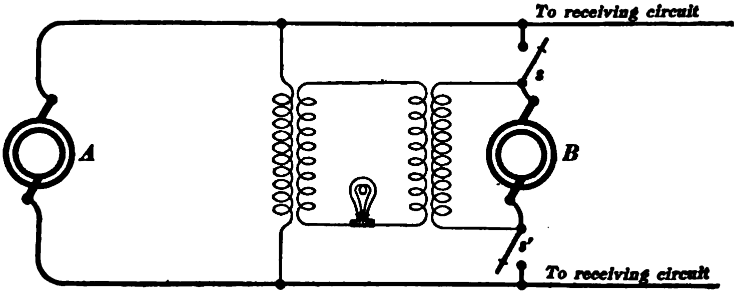


Fig. 201.

The necessity of absolute equality of speed of a number of alternating-current generators operating in parallel makes it impossible for the governors of the several engines to operate independently of each other, inasmuch as the independent operation of the governors requires the speed of each engine to vary with its load.

143. The excitation characteristic of the synchronous motor. Conditions which give maximum efficiency — A peculiarity of the synchronous motor (*B* less than *A* as in Fig. 196) is that, starting at zero load, *the current intake of the motor decreases with increase of load* until the point *P*, in Fig. 196, reaches the line *CO*. This is evident when we consider that the resultant electromotive force OP is a minimum when P is in the line CO or, in other words, when A and B are in exact opposition in phase. This peculiarity of the synchronous motor may be stated in another way, namely, *when the load on the motor is fixed in value there is a certain value of B (certain degree of field excitation of machine B) for which the current I is a minimum and for which the RI^2 loss is a minimum, generator voltage A being fixed in value.* This value of B which enables the synchronous motor to carry its given load with minimum value of I gives the maximum efficiency for given load and given value of A, and therefore the field excitation of a synchronous motor should always be adjusted (a different adjustment is necessary for each motor load) to give minimum I.

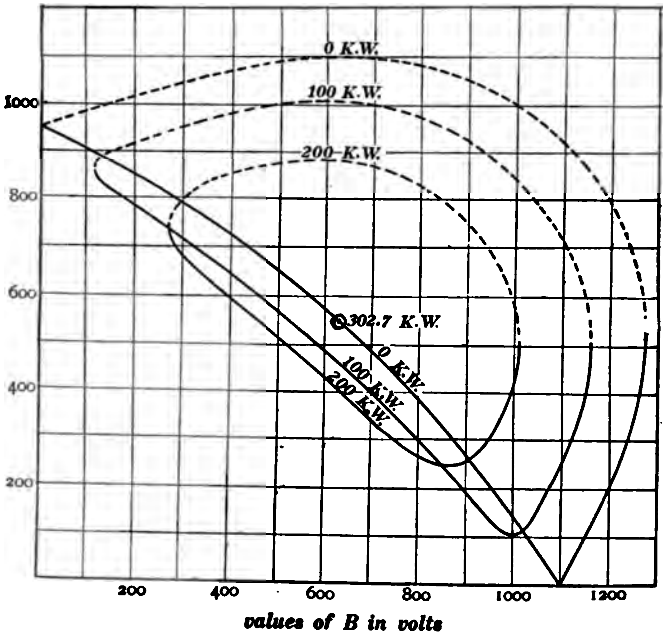


Fig. 202.

A clear idea of the change which takes place in the current intake of a synchronous motor at given load when the electromotive force B of the motor is varied may be obtained from Fig. 202. This figure shows the curves for intakes of zero kilowatts, 100 kilowatts, and 200 kilowatts, respectively, for the case in which $A = 1,100$ volts, $R = 1$ ohm, and $\omega L = 0.58$ ohm. For the greatest possible intake, 302.7 kilowatts, the curve reduces to the point enclosed in the small circle. The curves in Fig. 202 were determined by the use of the clock diagram of Fig. 192. A chosen value of B was represented to scale by the line OC in Fig. 192, the line ef was laid off at the proper angle, the distance OQ (measured from O towards e) was laid off to correspond to the assigned value of the intake of the motor in watts, and the two possible values of the resultant electromotive force OP were measured off on the diagram. The two possible values of I were then determined by calculation.

The excitation characteristic of a synchronous motor is usually plotted as follows: Values of field current of the motor are plotted as abscissas and the corresponding observed values of the current intake for given motor load are plotted as ordinates. The curve when so plotted is similar in general appearance to the curves as plotted in Fig. 202. The dotted portions of the curves in Fig. 202 correspond to unstable conditions of running and these portions of the curves cannot be determined by observation.

The efficiency of transmission of power by means of an alternating-current generator and a synchronous motor. — The ratio P''/P' of the motor intake to the generator output is called the *efficiency of transmission*. This is not the net efficiency of the system inasmuch as power consumed in field excitation of the two machines and power lost by friction and by eddy currents and hysteresis are not considered. The conditions necessary for maximum efficiency of transmission depend upon which of the quantities A, B, X, R and P'' are open to choice or capable of adjustment. The quantities X and R are ordinarily fixed in value, while A and B may be changed more or less by varying the field excitation of the respective machines, and P'' may be changed by varying the load on the motor.

1. *When the electromotive force A of the generating alternator is adjustable*, maximum efficiency is obtained when the current (and also RI^2) is a minimum; values of B, X, R and P'' being given. This minimum current is obtained when A is adjusted until B and I are opposite to each other in phase.

Proof. — The motor intake is $P'' = BI \cos (BI)$, according to equation (iv), Art. 133. Therefore, since P'' and B are given, the minimum value of I corresponds to maximum value of $\cos(BI)$. But the maximum (negative) value of $\cos(BI)$ is -1, and the corresponding value of the angle (BI) is 180°.

To calculate the value of A which will bring B and I opposite to each other in phase, for the given values of B, X, R and P'', consider Fig. 203*a*, in which I

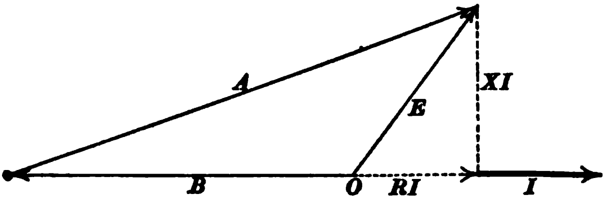


Fig. 203a.

represents the current, and E the resultant electromotive force of machines A and B. From this figure we have

$$A^2 = (B + RI)^2 + X^2I^2$$

from which A may be calculated when I is known. The value of I may be determined from the equation $P'' = BI\cos(BI)$, remembering that $\cos(BI) = -1$.

2. *When the electromotive force B of the synchronous motor is adjustable*, maximum efficiency is obtained when the current (and also RI^2) is a minimum; values of A, X, R and P'' being given. This minimum current is obtained when B is adjusted until A and I are in phase with each other. The proof of this proposition is given below.

To calculate the values of B which will bring A and I into phase with each other for the given values of A, X, R and P'', consider Fig. 203*b*, in which A

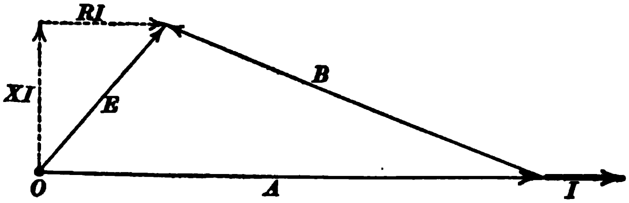


Fig. 203b.

represents the value of the electromotive force of machine A, and E represents the resultant electromotive force of machines A and B. From this figure we have

$$(A - RI)^2 + X^2I^2 = B^2$$

from which B may be calculated when I is known. The value of I may be calculated from the relation $AI\cos(AI) = P'' + RI^2$ in which $\cos(AI) = 1$.

Proof that I is a minimum when B is adjusted until A and I are in phase with each other; A, X, R and P'' being given. — Plot the curve of which the ordinates represent values of (XI) and the abscissas represent values of $[B\cos(BI)]$. This curve is a hyperbola inasmuch as $B\cos(BI) \times I = P''$, or $B\cos(BI)$

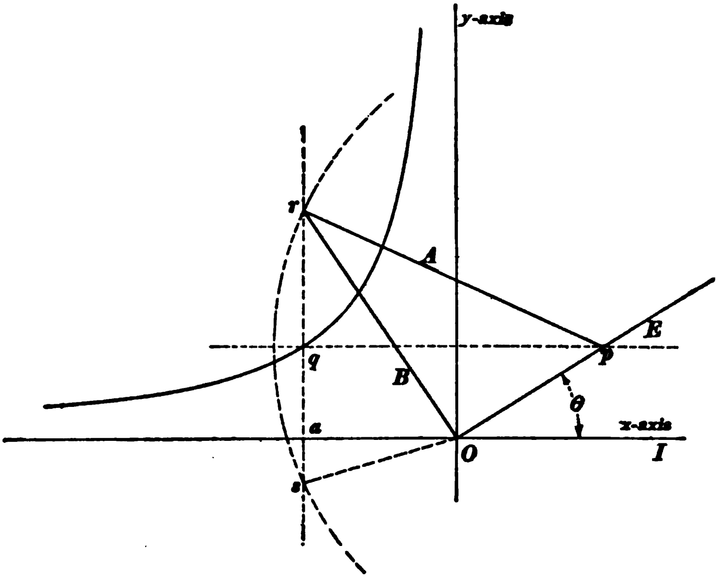


Fig. 204.

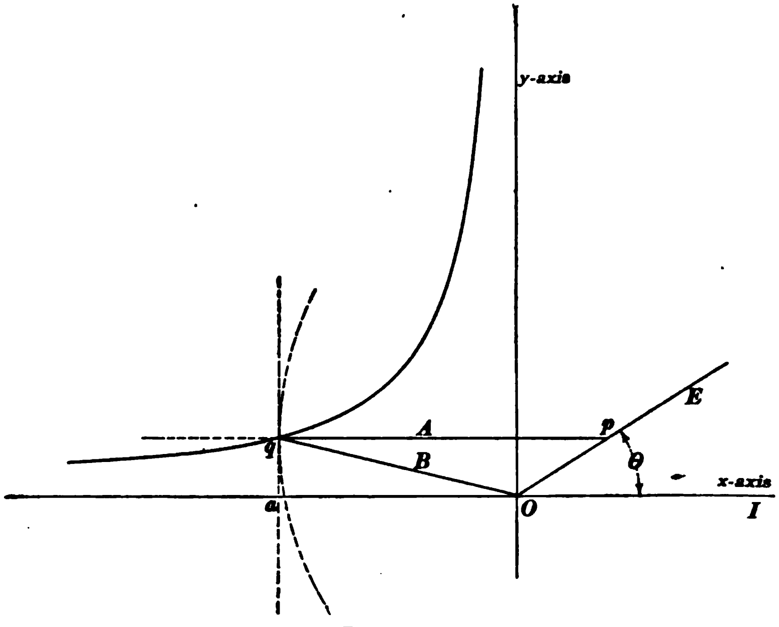


Fig. 205.

× $XI = XP'' =$ *a constant.* This curve is plotted in Fig. 204 with the values of B cos (BI) laid off to the left inasmuch as P'', being an intake, is negative. The x-axis of reference OI represents the current in the circuit; and the line OE of unknown length, making with OI the known angle θ, represents the resultant electromotive force of machines A and B.

Draw a horizontal line cutting OE in p and cutting the hyperbola in q. Then, for that particular value of the current which corresponds to the chosen ordinate aq $(= XI)$, the line Op represents the actual resultant electromotive force inasmuch as the vertical component of E is equal to XI. Describe about the point p a circle of which the radius represents A. Then the lines Or and Os represent the two possible values of B for the chosen value of XI. From this diagram it is evident that for the shortest possible value of Op (smallest possible current) the circle described about p just touches the ordinate aq, in which case the line A is horizontal and parallel to OI, as shown in Fig. 205. For any shorter value of Op (smaller value of E or I) the circle does not reach to the ordinate aq, which means that for so small a value of the current the given value of A is too small to supply the line losses RI^2 and the given motor intake P''.

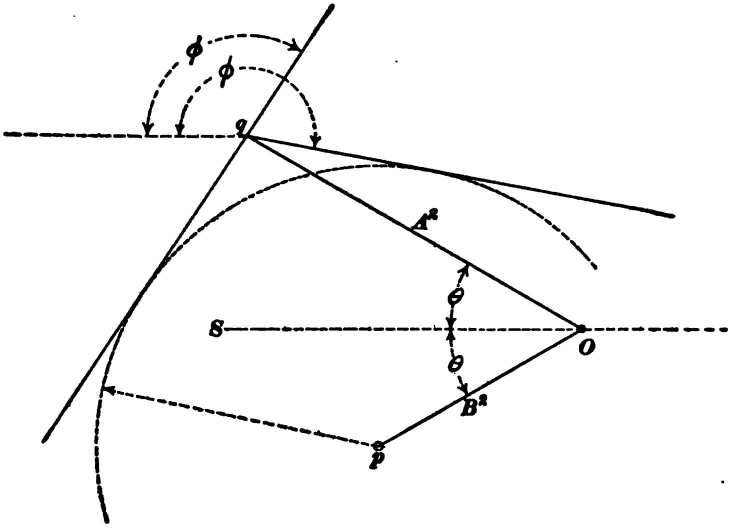


Fig. 206.

3. *When P'' is adjustable,* maximum efficiency occurs when the differential coefficient of P''/P' with respect to ϕ is zero. From equations (53) and (54) we have

$$\frac{P''}{P'} = \frac{AB \cos(\phi - \theta) + B^2 \cos\theta}{AB \cos(\phi + \theta) + A^2 \cos\theta}$$

whence, applying the condition

$$\frac{d\left(\frac{P''}{P'}\right)}{d\phi} = 0$$

we have

$$A^2 \sin(\phi - \theta) - B^2 \sin(\phi + \theta) = 2AB \sin\theta \qquad \text{(i)}$$

which determines the value of ϕ for which the efficiency of transmission is a maximum

Geometrical construction of equation (*i*). — Draw lines, Fig. 206, representing A^2 and B^2 to scale, the angle between them being 2θ. About the point p as a center describe a circle of which the radius represents $2AB \sin\theta$. From the point q draw a tangent to this circle. The angle between the line OS and this tangent is the required value of ϕ. Two tangents can be drawn from the point q. One of these tangents determines the value of ϕ (less than 180°) for which the efficiency of transmission is a maximum with machine A acting as a synchronous motor, and the other tangent determines the value of ϕ (greater than 180°) for which the efficiency of transmission is a maximum with machine B acting as a synchronous motor. The machine A is distinguished as having the greater electromotive force. The angle ϕ is the lag of B behind A.

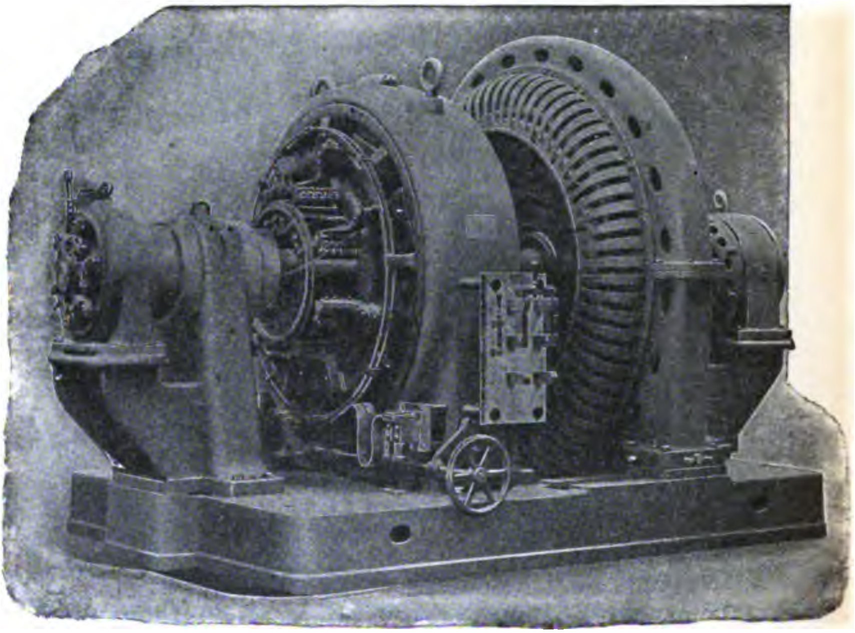
CHAPTER XIII.

THE SYNCHRONOUS CONVERTER.

144. The conversion of alternating current into direct current. — It is frequently necessary to utilize in the form of direct current the power which is delivered by an alternator, or, in other words, to convert alternating current into direct current. Thus, if the field of an alternator is to be compounded, it is necessary to rectify the alternating current in the series field winding of the machine, as explained in Art. 37. If the power delivered by an alternator is to be used for charging storage batteries it must be used in the form of direct current. If power, which is transmitted to a distance in the form of alternating current, is to be used to drive direct-current motors, such as are extensively employed in electric railway operation, it must be converted from alternating current to direct current. This conversion from alternating current to direct current is accomplished in practice in five ways; namely, (*a*) by the rectifying commutator, (*b*) by the motor-generator, (*c*) by the synchronous converter, (*d*) by the mercury-arc rectifier and (*e*) by the aluminum-valve rectifier.

(*a*) *The rectifying commutator.* — The ordinary direct-current generator is essentially an alternator with a commutator arranged to properly reverse the connections between the armature windings and the receiving circuit so that, although the current in a given armature conductor is an alternating current, the current in the receiving circuit is always in one direction The commutator of a direct-current generator, however, reverses the connections of successive *small portions* of the armature windings and its tendency to spark is thereby reduced to a minimum, whereas the rectifying commutator, strictly so-called, which is described in Art. 37, reverses the connections of the armature winding of an alternator as a whole. The rectifying commutator is not suited to heavy service, that is to say, it cannot rectify a large alternating current at a moderately high voltage because of its excessive tendency to spark.

(*b*) *The motor-generator* — Figure 207 shows a 500-kilowatt compound wound direct-current generator directly coupled to a three-phase synchronous motor with a small direct-current exciter on one end of the main shaft, and a small induction starting-motor on the other end of the main shaft. The synchronous motor takes power from a three-phase alternating-current supply, and drives the direct-current generator which delivers direct current. Generally the field excitation of the synchronous motor of a motor-generator set is supplied from the main direct-current generator without the use of a small auxiliary exciter; and when-



500 Kw., 575 Volt, Compound Wound Generator and 1,100 Volt, Three-Phase, 3,000 Alternation Synchronous Motor; 375 R.P.M. with A.C. Starting Motor and D.C Exciter. (Westinghouse Company.)

Fig. 207.

ever direct current is available for starting, the machine is brought up to speed by operating the main direct-current generator as a motor, in which case the small induction motor is not required for starting.

(*c*) *The synchronous converter.* — The device which is most frequently used for converting alternating current into direct current is the synchronous converter, which is described in Art. 145.

(*d*) *The mercury-arc rectifier.* — Figure 208*a* represents a glass bulb *DD* containing a pool of mercury *C* and a graphite electrode *A′*. The bulb is highly exhausted and contains only mercury vapor. A very high electromotive force is required to start a current in either direction through the mercury vapor, but when the current is once established *in the direction represented by the arrows* (*A′* being an anode and *C* being a cathode), it can be maintained by an electromotive force of 10 or 15 volts. Under these conditions a jet of mercury vapor *N* appears to issue from a point on the surface of the mercury. This jet is called the *cathode blast* or the *negative blast.**

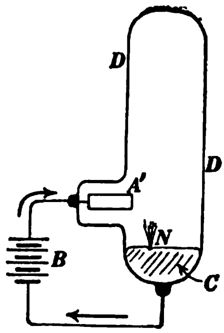


Fig. 208a.

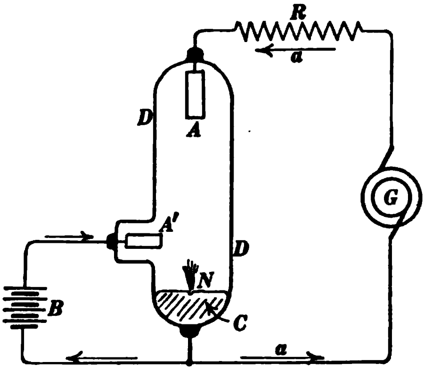


Fig. 208b.

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* See a paper on "The Electric Arc," by C. P. Steinmetz, *Transactions of the International Electrical Congress*, Vol. 2, pages 710 to 730, St. Louis, 1904; also a paper on "The Mercury Arc Rectifier," by C. P. Steinmetz, *Transactions of the American Institute of Electrical Engineers*, Vol. 24, pages 371 to 393, 1905; and a paper by P. D. Wagoner, read before the *National Electric Light Association* at Denver, Colo., in June, 1905.

Figure 208*b* represents a glass bulb, exactly like that which is shown in Fig. 208*a*, with a battery *B* (or any source of direct current) maintaining a cathode blast *N*, and an alternating-current generator *G* connected, as shown, to a graphite electrode *A* and to the mercury cathode *C*. Under these conditions, current can flow readily in the direction of the arrows *aa*, but it cannot flow in the opposite direction (unless the electromotive force of the generator *G* amounts to many thousands of volts). That is to say, the bulb *DD* with its auxiliary battery *B* for maintaining the cathode blast *N* acts like a valve, permitting current to flow in one direction but not permitting current to flow in the other direction. Such an arrangement is called a ***mercury-arc rectifier.***

The mercury-arc rectifier is arranged in practice as shown in Fig. 208*c*. In this arrangement the mercury-vapor tube *DD* has two graphite anodes *A* and *A′* from either of which current may flow to the mercury cathode *C* and thence through the storage battery *J* (which is being charged) to the point *p* which is at a potential midway between the potentials of the terminals *t* and *t′* of the supply transformer. The coils *E* and *F* are the two coils of an autotransformer. (See Art. 164.)

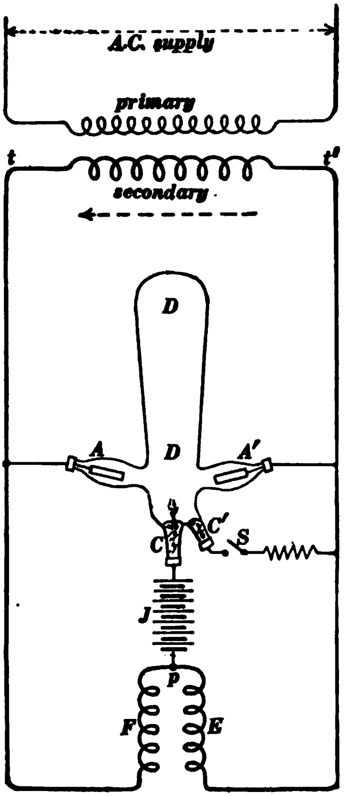

Fig. 208c.

Consider the instant when the supply voltage is in the direction of the dotted arrow, *t* being at a high potential and *t′* being at a low potential. Current enters the rectifier at A, flows to C, and thence through the storage battery J to the point p. The potential of this point p is midway between the potentials of the terminals t and t', and the current divides at p, half of it flows down-hill, as it were, through E to t' delivering energy to E as a primary coil. This energy is transferred to the coil F and pumps the other half of the current up-hill, as it were, to the terminal t.

When the supply voltage is in the opposite direction, the current enters at A' and half of it flows down-hill through the coil F delivering energy which is transferred to E, thus pumping the other half of the current up-hill to the terminal t'.

The mercury cathode C' is called the *starting cathode* and it is used as follows: The switch S is closed and the tube is gently rocked so as to bring the mercury in the pools C and C' momentarily into contact. Alternating current then starts to flow between C' and C, and the moment C becomes a cathode, current begins to enter at A and A' as above explained. The inductance of the circuit JFE maintains the current through J during the time that the alternating current is passing through zero value, thus maintaining the cathode blast. The switch S is opened after the tube is in operation.

Some idea of the voltage relations of the mercury-arc rectifier may be obtained as follows: Suppose that the supply voltage is harmonic and that its effective value is E between the terminals t and t' of Fig. 208*c*. The effective voltage A to p or A' to p is thus equal to $E/2$ and the average voltage between A or A' and p is $\sqrt{2}E/\pi$; but there is approximately 15 volts drop across from A or A' to C, so that the average voltage between C and p is $\sqrt{2}E/\pi - 15$ volts. Thus, 110 volts effective across tt' in Fig. 208*c*, gives approximately 35 volts average across Cp.

(*e*) *The aluminum valve rectifier* is a special form of electrolytic cell which permits current to flow through it in one direction only. Figure 209*a* shows a single aluminum valve CC connected in series with a storage battery B so that the battery may be charged from an alternating-current supply (secondary of a transformer). In this arrangement only one wave, say the positive wave, of the alternating current is utilized, the negative wave of current being blocked by the aluminum valve. Figure 209*b* shows four aluminum valves so arranged as to cause both waves (the positive wave and the negative wave) of the alternating current to flow in the same direction through a storage battery *B* in order to charge the battery.

In the aluminum-valve rectifier of Pollak * the electrodes are lead (Pb) and aluminum (Al) submerged in a slightly acid solution of potassium phosphate. The current flows freely through such an electrolytic cell in the direction of the arrows in Fig. 209, but a reversed current forms a thin insulating coating of aluminum oxide over the aluminum electrode which cannot be broken down unless the voltage exceeds about 150 volts. The full-load efficiency of such an aluminum-valve rectifier is about 70 per cent. when it is in good order with about 110 volts (effective) across the alternating-current supply mains.

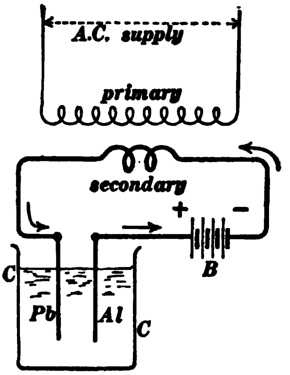

Fig. 209a.

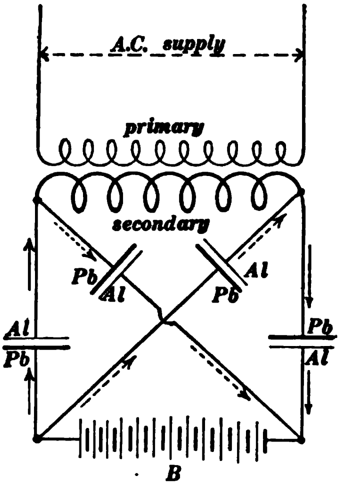

Fig. 209b.

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* See paper by J. Blondin, *Bulletin Soc. Int. Élect.*, Vol. I, pages 323–338, July, 1901. A very good abstract of this paper is given on pages 996–998 of *Science Abstracts*, Vol. IV, 1901. In some forms of the aluminum-valve rectifier, the electrolyte consists of a solution of ammonium phosphate, or ammonium borate, or ammonium bicarbonate.

Comparison of methods (*b*) *and* (*c*). — The motor-generator and the synchronous converter are the only devices which are available for heavy service in the conversion of alternating current into direct current.* The synchronous converter has a higher efficiency than the motor-generator and its cost is very much less, but there is a nearly fixed relation between the effective value of the voltage of the alternating current supplied to the synchronous converter and the steady voltage of the direct current delivered by the machine, so that the voltage of the direct-current side of the machine cannot be changed through a wide range without changing the effective voltage of the alternating-current supply. (See Art. 150.) The motor-generator, on the other hand, is very flexible in this respect, the voltage of the direct-current generator can be varied independently of the voltage of the alternating-current supply by varying the field excitation of the generator.

145. The synchronous converter. — Consider two opposite commutator bars on an ordinary two-pole direct-current dynamo. The electromotive force between these two bars alternates as the armature rotates, and if these two commutator bars are connected to two collector rings, the direct-current dynamo may be used to deliver alternating current; the machine becomes an alternator and, its commutator being undisturbed, it remains a direct-current dynamo. Such a machine may be belt driven and deliver direct current or alternating current or both; it may be driven as a direct-current motor and deliver mechanical power, or alternating current, or both; or it may be driven as a synchronous alternating-current motor and deliver mechanical power, or direct current or both. The most important use of such a machine is to convert alternating current into direct current, for which purpose it is driven as a synchronous alternating-current motor and loaded by delivering direct current; and the machine is called a *synchronous converter* * when used in this way.

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* The practical possibilities of the mercury-arc rectifier are not as yet fully established. At the present time this rectifier is built in sizes large enough to deliver thirty or forty amperes of direct current with a loss of about 14 volts in the mercury-arc. The mercury-arc rectifier is suitable for any voltage up to about 10,000 volts.

Single-phase and polyphase synchronous converters. — The machine above described, having two collector-rings, is called a two-ring or single-phase converter. The three-ring or three-phase converter has three collector rings connected to three commutator bars 120° apart (on a two-pole machine), the four-ring, or so-called two-phase converter, has four collector rings connected to four commutator bars 90° apart (on a two-pole machine) and so on. In the case of a multipolar machine the *n* collector rings of an *n*-ring converter are connected as follows: Ring No. 1 is connected to all the commutator bars † which for a given position of the armature lie midway under the north poles of the field magnet. Let *l* be the distance between adjacent commutator bars of this first set. Then ring No. 2 is connected to all commutator bars 1/*n*th of *l* ahead of the first set; ring No. 3 is con-

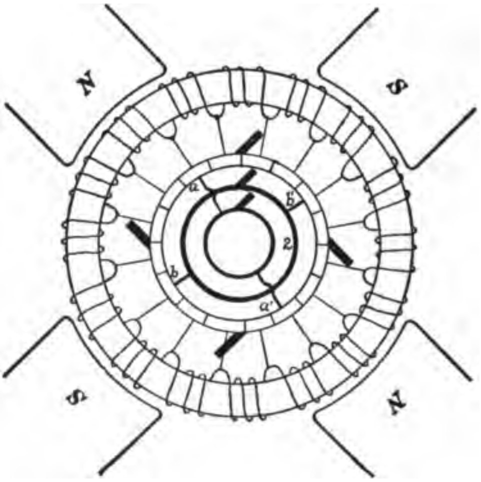

Fig. 210.

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* Also often called a *rotary converter*.

† This statement applies to that type of armature winding which is called the simplex lap winding. In the simplex wave winding each collector ring is connected to but one commutator bar whatever the number of field magnet poles may be. The multiplex winding is not adapted to the synchronous converter.

nected to all commutator bars $2/n$ths of l ahead of the first set; and so on. Thus Fig. 210 shows a four-pole two-ring converter, each ring being connected to two commutator bars.

146. The internal actions of a synchronous converter. — In its relation to the alternating-current supply a synchronous converter is like a synchronous motor, and in its relation to the direct-current receiving circuit a synchronous converter is like a direct-current generator; but in regard to (*a*) the actual motor and generator actions in the machine, (*b*) the magnetizing action of the armature current, and (*c*) the heating effect of the armature current, the machine is essentially unlike a synchronous motor and essentially unlike a direct-current generator.

The details of motor and generator action are of little practical importance and they are described in the following paragraphs of this article, but the magnetizing action and heating effects of the armature current are important, inasmuch as they determine the power rating of the machine.

*Motor and generator action in the synchronous converter.** — Let *a* and *b*, Fig. 211, be the direct-current brushes of a synchronous converter. Let the negative brush *b* be chosen as the zero or reference point of potential so that the electromotive force

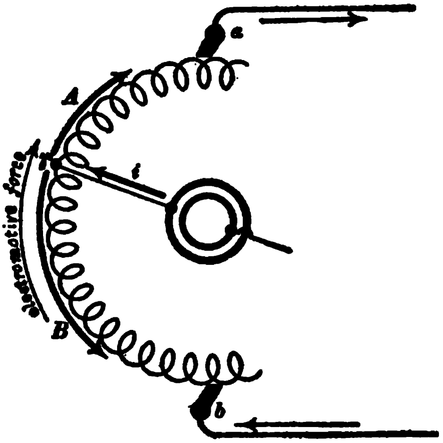

Fig. 211.

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* See a paper on "The energy transformations in the synchronous converter," by W. S. Franklin, *Trans. Am. Inst. Elec. Eng.*, Vol. 22, pages 17–33, 1903.

between *b* and any other given point may be spoken of as the potential at the given point.

Let the half-circle represent one side of the armature, and let *r* be the position at a given instant of the point of attachment of one of the collector rings. At the given instant a definite current *i* is entering the armature at *r* from the alternator, which is driving the converter; this current is entering at a potential which is less than the potential of the brush *a*, a portion *B* of the current *i* flows down hill (*against* the induced electromotive force in the windings, which electromotive force is represented by the fine line arrow) to brush *b* and delivers power to the portion *rb* of the armature in the form of motor action, and the remainder *A* of the current *i* flows up hill (*with* the induced electromotive force) to brush *a* and receives power from the portion *ra* of the armature in the form of generator action. That is to say, at each instant certain portions of the armature of a synchronous converter have motor action and develop torque in the direction of the motion of the armature, and other portions of the armature have generator action and develop torque tending to oppose the motion of the armature. In the two-ring converter the motor action at times greatly exceeds the generator action, at other times the generator action greatly exceeds the motor action, and the armature is alternately accelerated and retarded. This must be so because the single-phase alternating current delivers power to a two-ring converter in pulses. In a polyphase converter, however, the motor action is at all times very nearly equal to the generator action, being just enough in excess to supply the mechanical and magnetic friction losses in the motor.

147. The starting of the synchronous converter.* — Synchronous converters are often used in sets, one or more of the machines being always in operation. When this is the case a converter not in use may be started as a direct-current motor, taking direct current from the other machines, and its speed may be adjusted until it is in exact synchronism with the alternating-current supply by which it is to be driven, and then, when its alternating electromotive force is opposite in phase to the supply voltage, the alternating current switches may be closed. The procedure is exactly the same as the starting of a synchronous motor as described in Art. 138 and the same synchronizing devices are used.

When direct current is not available for starting a converter, an induction motor is generally used to bring the machine up to speed. Any polyphase alternator however contains the essential features of an induction motor, and therefore a polyphase alternator or synchronous converter can be started by its own induction motor action. In some cases this method of starting is employed. See Art. 178.

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* See a series of short practical papers on this subject by A. Wagner, *Electric Journal*, Vol. II, pages 436, 494 and 572, 1905.

148. The operation of the synchronous converter.—Not only in the matter of starting but also in certain matters of operation the synchronous converter is like a synchronous motor. Consider, for example, a four-ring converter. Such a converter is often called a two-phase converter because it is supplied with two alternating currents 90° apart in phase, each alternating current being delivered to a pair of opposite collector rings, that is, to collector rings which are connected to opposite points on the armature of a two-pole machine. Let B be the effective value of the total induced electromotive force between opposite collector rings, R the resistance of the armature between opposite collector rings, L the inductance of the armature between opposite collector rings plus any outside inductance that may be collected in series in each alternating-current circuit between the supply mains and the machine, and let A be the effective value of the supply voltage (each phase) and $\omega/2\pi$ the frequency.

Lay off the line OC, Fig. 212, to represent B, describe a circle about C of which the radius represents the value of A, draw the line ef making with OC the angle whose tangent is $\omega L/R$, and draw the line sOs' perpendicular to ef. Let us suppose for the sake of simplicity that the efficiency of the converter is 100 per cent. so that the intake of power may be considered equal to the output. The figure shows the phase relation between A and B (each phase) for a direct-current load equal to 2 times $\overline{OQ}$ times $B/\sqrt{R^2 + \omega^2 L^2}$ watts, where $\overline{OQ}$ is expressed in volts (of course all lengths in Fig. 212 represent volts). This relation is explained in Art. 134, and the factor 2 is introduced because the expression given in Art. 134 gives the intake of one phase only.

If the direct-current output is decreased, the point P, Fig. 212, moves towards s causing $\overline{OQ}$ to decrease; if the direct-current output is increased the point P moves towards M, causing $\overline{OQ}$ to increase; and if the direct-current output is greatly increased the point P is carried beyond the point M, and the machine falls out of synchronism and ceases to operate. The maximum intake of power is 2 times Ma times $B/\sqrt{R^2 + \omega^2L^2}$.*

The hunting action of the synchronous motor is described in Art. 137. The synchronous converter is especially prone to develop hunting oscillations as stated in Art. 137, and the methods for obviating this trouble are there described.

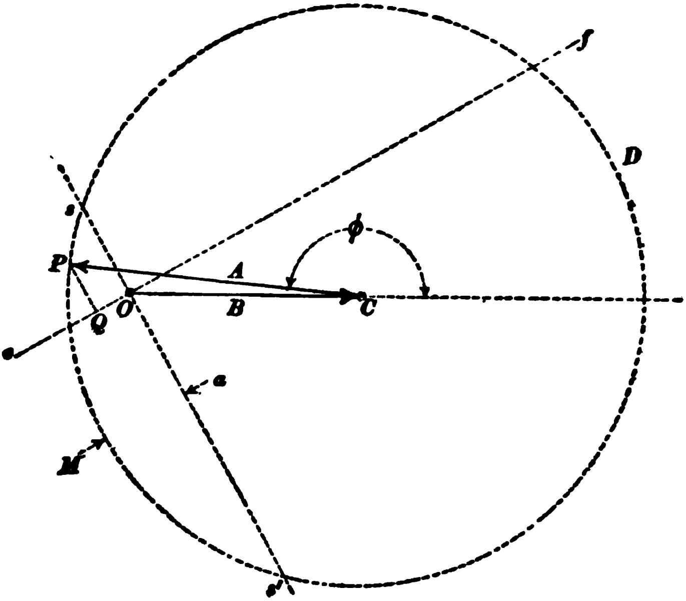

Fig. 212.

When the synchronous converter is driven as a direct-current

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* What is here said concerning the maximum intake of power of a synchronous converter and its stoppage due to overload is based on an assumption, namely, that the terminal voltage of a synchronous motor may be considered to be the sum of three distinct parts: (*a*) The "total induced electromotive force" B which is due to the armature flux produced by the field-winding alone, (*b*) the electromotive force induced in the armature winding by the flux due to the armature current alone, and (*c*) the RI drop in the armature winding. An excessive armature reaction alters the flux distribution and *thereby alters the reluctance of the magnetic circuit* so that the flux which is produced by the field-winding is to some extent altered by armature reaction.

A synchronous converter can in fact carry an excessive over-load without dropping out of synchronism.

(shunt) motor and loaded by delivering alternating current, it is called, usually, an *inverted rotary*, and when so operated the machine exhibits one peculiarity which should never be lost sight of, namely, the machine tends to run at an excessively high speed when it is overloaded. This rather paradoxical behavior is due to the weakening of the field by the demagnetizing action of the (lagging) alternating current which is taken by the machine, and, like any shunt motor, its speed rises when its field is weakened, since there is but little mechanical opposition to the motion.

149. Armature reaction of the synchronous converter.—The magnetizing action of the current in the armature of a dynamo is called *armature reaction*, and armature reaction has in general two distinct effects, namely, (*a*) it crowds the flux to one side or the other of the pole pieces of the field magnet, and (*b*) it tends to weaken (or strengthen) the field. The first is called *distortion* and the second is called *demagnetizing action.*

Consider the power-component and wattless component of the alternating current delivered to a synchronous converter, or delivered by an inverted rotary. The power-component of the alternating current has chiefly a distorting action on the field, and its action is almost completely neutralized by the direct current. The wattless component of the alternating current has chiefly a strengthening or weakening action (according as it is 90° ahead of or 90° behind B in phase), and this action is not neutralized by the direct current because the direct current has but little strengthening or weakening action when the direct-current brushes are in the neutral axis.

The effect of the unbalanced armature reaction of the wattless component of the alternating current in a synchronous converter is exemplified by the behavior of the over-loaded inverted rotary as described in the previous article, and by the behavior of the straight rotary when its field excitation is raised or lowered as described under the heading Voltage Control of the Synchronous Converter in Art. 150.

150. Electromotive force relations of the synchronous converter. — There is fairly definite relation between the value of the electromotive force between the direct-current brushes of a synchronous converter and the effective value of the alternating electromotive force between collector rings. The following discussion of these electromotive force relations is based on the assumption that the alternating electromotive force of the machine is harmonic and that the RI losses of voltage in the armature windings are negligible.

Two-ring converter. — In the two-ring converter the two rings are connected to commutator bars, one of which comes under a positive direct-current brush at the instant that the other comes under a negative direct-current brush; at this instant the electromotive force between the collector rings is at its maximum value and it is equal to the electromotive force between the direct-current brushes. Therefore the direct electromotive force E_0 of a two-ring converter is equal to the maximum value $\sqrt{2}E_2$ of the harmonic alternating electromotive force between the two collector rings. Therefore

$$E_2 = \frac{E_0}{\sqrt{2}} \qquad \text{(i)}$$

in which E_2 is the effective value of the alternating electromotive force between the collector rings.

Converter with n collector rings. — Consider all of the armature conductors which constitute one of the paths between the two rings of a two-ring converter. These conductors are evenly distributed over one half of the armature, and the harmonic alternating electromotive forces which are induced in the respective conductors are all equal in value but they are distributed in phase throughout an angle of 180° as shown by the vectors a, Fig. 213. Therefore the effective electromotive force between the collector rings of a two-ring converter is represented by the diameter E_2, Fig. 213, of the polygon whose sides are equal and parallel to the vectors a which represent the effective values of the electromotive forces in the individual armature conductors.

The electromotive force (effective) between two collector rings which are connected to the armature winding at points 120 electrical degrees apart, that is, between any two rings of a three-ring converter, is represented by that chord of the polygon, Fig. 213, which subtends an angle of 120° ; the effective electromotive force between two rings which are connected to the armature winding at points 90 electrical degrees apart, that is, between any two adjacent rings of a four-ring converter, is represented by that chord of the polygon, Fig. 213, which subtends an angle of 90° ; and, in general, the effective electromotive force between adjacent rings of an n-ring converter is represented by that chord of the polygon, Fig. 213, which subtends an angle of 360°/n. Therefore, treating the polygon of Fig. 213 as a circle, we have

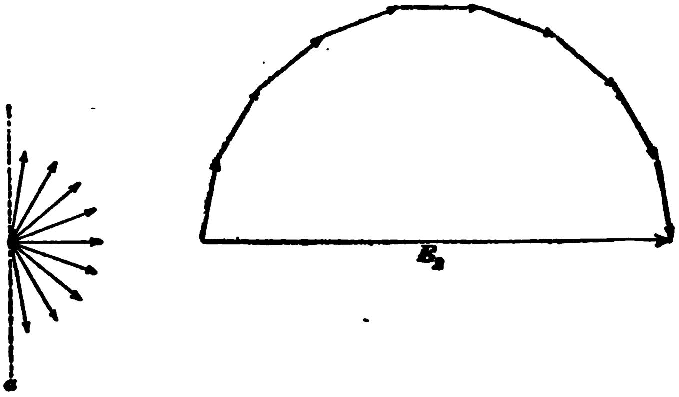

Fig. 213.

$$E_n = E_2 \sin \frac{\pi}{n}$$

or, using the value of E_2 from equation (i), we have

$$E_n = \frac{E_0}{\sqrt{2}} \cdot \sin \frac{\pi}{n} \qquad \text{(ii)}$$

which gives the following expressions for the effective values of the alternating electromotive forces between adjacent rings of a two-ring, a three-ring, a four-ring, and a six-ring converter in terms of the electromotive force E_0 between the direct-current brushes

$$E_2 = \frac{E_0}{\sqrt{2}} = 0.707E_0 \tag{55}$$

$$E_3 = \frac{\sqrt{3}E_0}{2\sqrt{2}} = 0.612E_0 \tag{56}$$

$$E_4 = \frac{E_0}{2} = 0.500E_0 \tag{57}$$

$$E_6 = \frac{E_0}{2\sqrt{2}} = 0.354E_0 \tag{58}$$

These electromotive force relations are shown geometrically in Fig. 214.

Actual electromotive force relations.—The relations given in equations (55), (56), (57) and (58) are not exactly realized in practice. The departure of an actual machine from these ideal ratios depends upon: 1st, the wave-shape of the alternating electromotive force of supply; 2d, the wave-shape of the alternating electromotive force of the converter itself, which depends chiefly upon the percentage of the armature surface that is covered by the pole faces; 3d, the resistance of the armature windings which causes losses of voltage when the machine is loaded; 4th, the position of the direct-current brushes, and 5th, the manner in

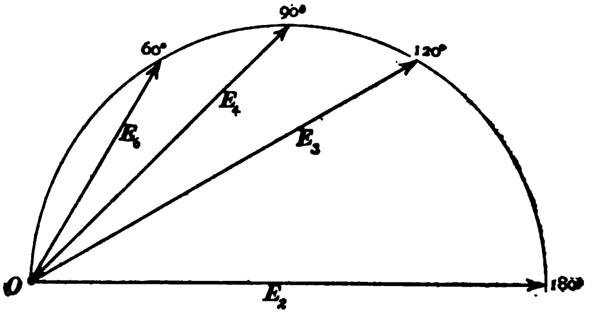

Fig. 214.

which the machine is used, that is, whether it is used to convert from alternating to direct current or *vice versa.*

When the alternating electromotive force of supply is approximately harmonic, and the pole faces of the converter cover three quarters of the armature surface, and when the direct-current brushes are in the neutral position the actual voltage ratios are approximately as follows:

TABLE.

VOLTAGE RATIO OF TWO-RING AND THREE-RING CONVERTERS (550-volt type).

	Actual Ratio E_n/E_0.			Ideal Ratio E_n/E_0.
	Zero Load.	Full Load. (Straight.)	Full Load. (Inverted.)	
E_2	0.715	0.725	0.705	0.7071
E_3	0.610	0.620	0.600	0.612

The values of E_4 and E_6 are not included in this table inasmuch as the alternating voltage of a four-ring converter is usually specified as the effective voltage between opposite rings which is the same as E_2, and the alternating voltage of a six-ring converter is usually specified either as the voltage between opposite rings $(= E_2)$ or the voltage between a given ring and the next ring but one, between rings 1 and 3 for example $(= E_3)$.

Voltage control of the synchronous converter.—It is evident from the above table that the direct-current voltage of a rotary converter is very nearly constant when the alternating-current supply voltage is constant, that is to say, the voltage regulation of a rotary converter is good. It is evident, therefore, that it is not possible to alter the direct-current voltage of a synchronous converter at will so long as the alternating-current voltage between the collector rings of the machine has a constant value. If the field excitation of a synchronous converter is reduced, the machine takes *lagging* wattless current * from the alternating-current supply, the effect of which in the armature is to *help* the field flux, and make up almost completely for the decrease of direct-current field excitation; if the field excitation of the machine is increased, the machine takes *leading* wattless current from the alternating-current supply, the effect of which in the armature is to *oppose* the field flux and counteract almost completely the increase of direct-current field excitation. If, however, an inductance is connected in the alternating-current circuit outside of the machine, the lagging wattless current in the first instance causes a considerable *drop* * of alternating-current voltage, or the leading wattless current in the second instance causes a considerable *rise* * of alternating-current voltage, and the direct-current voltage of the converter is decreased or increased correspondingly. Therefore, to be able to control the direct-current voltage of a synchronous converter by means of a field rheostat, an inductance must be connected in the alternating-current circuit.

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* The statements here made are worded to apply to the single-phase converter.

An ideal method for controlling the voltage between the direct-current terminals of the synchronous converter would be to shift the direct-current brushes; by so doing the voltage between the direct-current terminals could be varied from its full value to zero independently of any change of the voltage between the alternating-current terminals of the machine. This method of voltage control is, however, impracticable because of the sparking which is produced at the direct-current brushes when they are shifted from the neutral axis. To overcome this difficulty, Mr. J. L. Woodbridge has proposed the *split-pole converter* † in which each field pole is split into two (or three) parts. One of these parts is permanently excited, and it produces near its edge the fringe of field which is necessary for sparkless reversal as explained in Art. 96. The other portion of each field pole is arranged so that its excitation may be increased or decreased at will, thus shifting the resultant axis of the field with respect to the direct-current brushes. The result is exactly equivalent ‡ to the shifting of the direct-current brushes in so far as the voltage relations of the converter are concerned.

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* See pages 215 and 216.

† See a paper by Comfort A. Adams, *Proceedings of the American Institute of Electrical Engineers*, June, 1908, pages 899–925.

‡ The voltage ratio E_n/E_0 of a synchronous converter depends upon the wave-shape of the alternating voltage, and it is frequently stated that the variation of the

Compound field excitation of the synchronous converter.—When the field magnet of a converter is excited in part by a shunt winding and in part by a series winding (a field winding in series with the direct-current receiving circuit), then an increase of direct-current output gives an increased field excitation, but if the alternating voltage across the converter terminals is kept at a constant value this increase of field excitation is counteracted by leading wattless current in the armature and the voltage between the direct-current terminals of the machine remains sensibly unchanged in value. In order that the series field winding may produce an increase of voltage between the direct-current terminals of the machine with increase of direct-current output, an inductance must be connected in the alternating-current circuit or the split-pole converter must be used.

151. Current relations of the synchronous converter.—It is evident from the preceding article that there is a nearly fixed ratio between E_0 and E_n of a synchronous converter. The ratio of the direct-current output of a converter (I_0) to the effective value of the alternating current entering at a collector ring (I_r) is subject to greater variations than the voltage ratio. The voltage ratio must be known with considerable accuracy if one is to be able to design a transformer to deliver alternating current to a converter which in its turn is to deliver direct current at a specified voltage; an exact knowledge of the current ratio, on the other hand, is of no great importance. Indeed the current ratio is of importance chiefly in connection with the discussion of armature heating, which determines the power rating of the converter, and for this purpose an approximate value of the current ratio is sufficient.

In the discussion of the current relations of a synchronous converter we shall assume (*a*) that the efficiency of the machine

voltage ratio of the split-pole converter depends upon changes of wave-shape. This is an error.

is 100 per cent., that is, we shall assume that the intake of power is equal to the output of power, and (*b*) that the alternating current I_n flowing in the armature between two collector rings is exactly opposite in phase to the alternating electromotive force B that is induced in that portion of the armature winding.

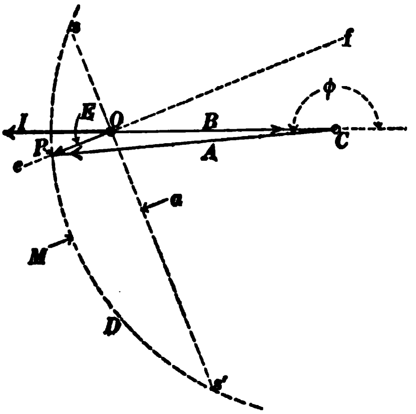

Fig. 215.

This second condition means that the synchronous converter *is assumed to operate at unity power factor*, and that the point P of the clock diagram, Fig. 212, is located as shown in Fig. 215, in which B and I are opposite in phase, so that, the angle (BI) being equal to 180°, the power intake of the machine per phase is equal to BI. This second condition can in general be realized, for any given load on the converter, by adjusting the field excitation of the machine until the alternating-current intake is a minimum as explained in section 2, of Art. 143.

Current in each section of the converter armature.—Let E_0 be the electromotive force between direct-current brushes and I_0 the direct-current output of the machine. Then the power output is E_0I_0. Let E_n be the effective value of the induced electromotive force in one section of the armature between adjacent collector rings and I_n the effective value of the alternating current in the same section of the armature winding. Then, since E_n and I_n are assumed to be exactly opposite to each other in phase, the intake of power per phase is E_nI_n and the total intake of power is nE_nI_n. Therefore, assuming 100 per cent. efficiency, we have

$$E_0I_0 = nE_nI_n$$

whence, substituting the value of E_n in terms of E_0 from equation (ii) of Art. 150 and solving for I_n, we have

$$I_n = \frac{\sqrt{2}\, I_0}{n \sin \frac{\pi}{n}} \qquad \text{(i)*}$$

whence

$$I_2 = \frac{I_0}{\sqrt{2}} = 0.707 I_0 \qquad \text{(ii)}$$

$$I_3 = \frac{2\sqrt{2}}{3\sqrt{3}} I_0 = 0.544 I_0 \qquad \text{(iii)}$$

$$I_4 = 0.500 I_0 \qquad \text{(iv)}$$

$$I_6 = 0.471 I_0 \qquad \text{(v)}$$

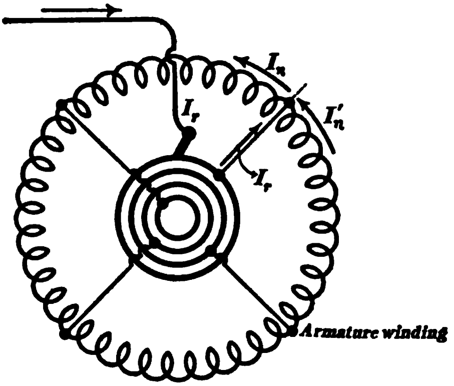

Fig. 216.

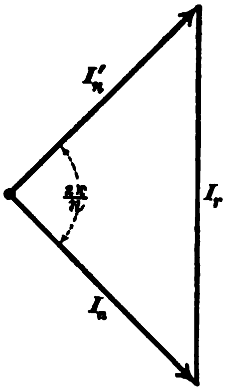

Fig. 217.

Current entering at each collector ring. — Figure 216 shows the points of attachment of collector rings to the armature winding of a synchronous converter, *the two arrows I_n and I_n'* show the *directions in which the currents in the two armature sections are considered as positive, and the arrow I_r shows the positive direction of the*

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* This entire article applies to the two-pole machine (armature winding having two paths). In case of a multipolar machine having p' current paths through the armature this equation becomes

$$I_n = \frac{\sqrt{2} I_0}{p' n \sin \frac{\pi}{n}}$$

current I_r. From this figure it is evident that $I_r = I_n - I_n'$ (the vector difference of course). Therefore, since I_n and I_n' differ in phase by the angle $2\pi/n$, as shown in Fig. 217, it is evident that the value of I_r is

$$I_r = 2I_n \sin\frac{\pi}{n} \tag{vi}$$

whence using the value of I_n from equation (i) above, we have

For a two-ring converter $I_r = 1.414 I_0$
For a three-ring converter $I_r = 0.942 I_0$
For a four-ring converter $I_r = 0.707 I_0$
For a six-ring converter $I_r = 0.471 I_0$

152. The armature heating of the synchronous converter. Synchronous converter ratings. — Inasmuch as the armature reaction of a synchronous converter is negligible,* the power rating of such a machine is determined by armature heating. This power rating is always expressed in terms of the rating the same machine would have if used as an engine-driven direct-current generator, and the difference in rating of a machine when used as an engine-driven direct-current generator and when used as a synchronous converter is determined wholly by the RI^2 losses in the armature windings, inasmuch as the eddy current and hysteresis loss in the armature core is the same (nearly) in the two cases.

The following table gives the ratings of synchronous converters expressed in terms of the rating of the same machine when used as an engine-driven direct-current generator. It will be noted, for example, that the rating of a six-ring converter is nearly twice as great as the rating of the same machine when used as an engine-driven direct-current generator, and it must be remembered that this increased rating refers to the increase of permissible direct-current output, the voltage of the machine being fixed. Thus a six-ring converter rated as a 1,000-ampere direct-current generator would be rated as a 1,920-ampere synchronous converter and *this increased current rating necessitates a much larger commutator so as to permit of a greatly increased contact surface be-*

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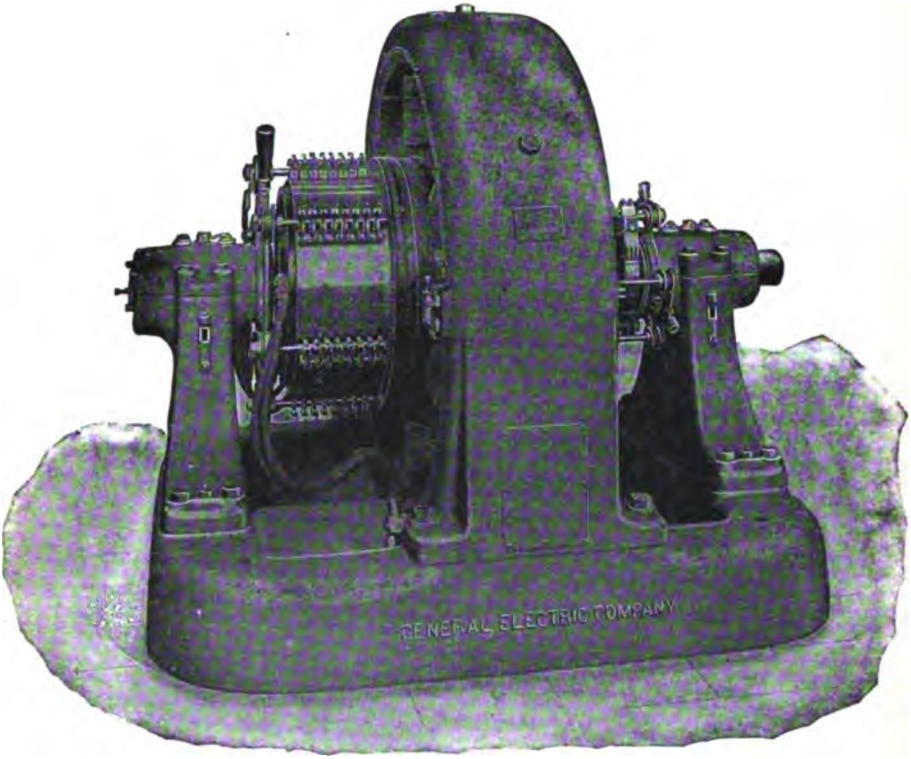
* Field distortion, which has chiefly to do with sparking as explained in Art. 96, is here referred to. See Art. 149.

tween the direct-current brushes and the commutator. In fact a striking feature of the synchronous converter is the large size of its commutator as compared with the commutator of a direct-current generator having the same armature and field magnet



300 kw. Three-phase Rotary Converter.
Fig. 218.

TABLE.*

POWER RATINGS OF SYNCHRONOUS CONVERTERS.

a As engine driven generator.	*b* As a two-ring converter.	*c* As a three ring converter.	*d* As a four-ring converter.	*e* As a six-ring converter.
1.00	0.85	1.32	1.62	1.92

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* The calculation of these power ratings is explained in Franklin and Esty's *Elements of Electrical Engineering*, Vol. II, pages 189–191.

structure. Thus, Fig. 218 shows a 300 kilowatt, three-ring, synchronous converter of the General Electric Company.

153. The use of the double-current machine as a direct-current three-wire generator. — When the synchronous converter is driven by an engine and used to deliver both alternating current and direct current, it is called a *double-current generator.* Such a machine can be used to deliver direct current to an Edison three-wire distributing system. This use of the double-current generator is due to Dobrowolsky. The essential features of this arrangement are shown in Fig. 219 for a two-ring machine. The voltage between the direct-current brushes *a* and *b* is, say, 220 volts, and it is desired to supply direct current to an Edison three-wire system and maintain 110 volts (direct) between the middle main and each outside main. To do this the current in the middle main must be delivered to the armature at a potential midway between the potentials of the direct-current brushes.

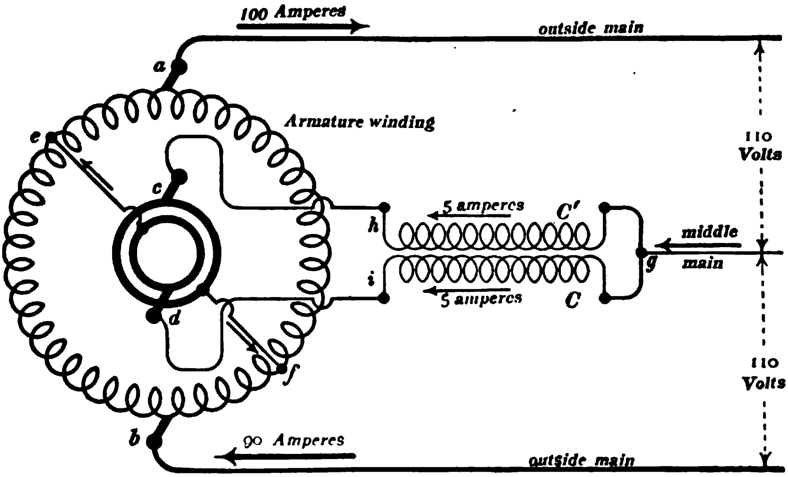

Fig. 219.

The two coils *C* and *C'* represent the two similar coils of a transformer so wound that the point *g* is, as it were, the middle of one continuous winding, the terminals of which, namely, *h* and *i*, are connected to the two alternating-current brushes *c* and *d* of the machine. The terminals *h* and *i* are at the same potentials as the points *f* and *e* of the armature winding respectively. Half of the inflowing current in the middle main flows down hill, as it were, through coil *C′* to the point *f* and delivers energy to coil *C′*. This energy is transformed into coil *C* and it causes the other half of the current in the middle main to flow up hill, as it were, through coil *C* to the point *e*. The current which enters the armature at the point *e* enters at a "level" such that the electromotive force induced in the rotating armature between *e* and *a* is sufficient to carry the current up from *e* to *a*, and the current which enters the armature at the point *f* enters at a "level" such that the electromotive force induced in the rotating armature between *f* and *a* is sufficient to carry this current up from *f* to *a*.

It is interesting to compare the action of the transformer *CC′* in Fig. 219 with the action of the transformer as ordinarily used (see Art. 154). Direct current flows steadily through both coils *C* and *C′* and the pulses of power are transferred alternately from *C′* to *C* and from *C* to *C′*. When a transformer is used as described in Art. 154, however, alternating current flows through both of its coils and the pulses of power are always transferred in the same direction, not back and forth as in Fig. 219.

CHAPTER XIV.

THE SIMPLE THEORY OF THE TRANSFORMER.

154. The ideal transformer.—The action of a transformer would be very simple (*a*) if the coils of wire had zero resistance, (*b*) if there were no magnetic leakage, that is, if all of the magnetic flux which links with one coil also linked with the other coil, and (*c*) if the magnetic reluctance of the iron core were zero. A transformer satisfying these conditions would be called an *ideal transformer.* In an actual transformer, however, the coils always have more or less resistance; some of the magnetic flux, the so-called leakage flux, links with one coil and does not link with the other; and a certain amount of magnetomotive force (ampere-turns) is necessary to force the magnetic flux through the transformer core. The effects of these things on the action of a well-designed commercial transformer are small and therefore the very simple theory of the ideal transformer is sufficiently exact for most practical purposes. All operating engineers in dealing with transformers, make use of the very simple theory of the ideal transformer, whereas designing engineers find it necessary to take account of the effects of coil resistance, magnetic leakage, and core reluctance.

Transformer action.—In the following discussion N' represents the number of turns of wire in the primary coil and N'' represents the number of turns of wire in the secondary coil; the coils are assumed to have negligible resistance, and all of the flux which passes through one coil is assumed to pass through the other coil also. The transformer core, however, is not assumed to have zero reluctance.

Ratio of primary current to secondary current. — Aside from resistance, the only thing which opposes the flow of current through the primary coil is the *reacting* electromotive force induced in the primary coil by the reversals of magnetization of the core. The greater the range of this magnetization the greater the value of the reacting electromotive force. *The combined magnetizing action of the primary and secondary coils is always such as to magnetize the core to that degree which will make the reacting electromotive force in the primary coil equal to the electromotive force of the alternator which is forcing current through the primary coil.* Resistance of primary coil neglected.

When the secondary coil is on open circuit, just enough current flows through the primary coil to produce the degree of magnetization above specified. Let this value of the primary current, which is called the *magnetizing current,* be represented by M. When current I'' is taken from the secondary coil, additional current I', called the *load current,* flows through the primary coil. The current M still suffices to magnetize the core, and *the magnetizing action of* I'' *is exactly neutralized by the equal and opposite magnetizing action of* I'. The magnetizing action of I'' is measured by the product $N''I''$, and the magnetizing action of I' is measured by the product $N'I'$, so that, ignoring algebraic signs, we have

$$N'I' = N''I''$$

or

$$\frac{I'}{I''} = \frac{N''}{N'}\,. \tag{59}$$

Ratio of primary electromotive force to secondary electromotive force. — The rapid reversals of magnetization of the iron core induce a certain electromotive force a in each turn of wire surrounding the core. Therefore the total electromotive force induced in the primary coil is $N'a$. This is the reacting electromotive force in the primary coil and it is equal and opposite, as pointed out above, to the electromotive force E' which is pushing current through the primary coil; so that, ignoring signs, we have

$$E' = N'a$$

Similarly, the total electromotive force E'', induced in the secondary coil, is

$$E'' = N''a$$

Therefore

$$\frac{E'}{E''} = \frac{N'}{N''} \qquad (60)$$

The above discussion should in strictness refer, primarily, to *instantaneous* values of I' and I'' and to *instantaneous* values of E' and E''. Thus i' and i'' are at each instant opposite to each other and in the ratio N''/N'; and e' and e'' are at each instant opposite to each other and in the ratio N'/N''. Therefore I' and I'' are opposite or 180° apart in phase, and E' and E'' are opposite or 180° apart in phase.

Approximate equality of input aud output of power. — The output of power by an ideal transformer would be exactly equal to the input of power. In an actual transformer the output of power is usually 97 per cent. or more of the power input when the transformer is fully loaded; that is, the RI^2 losses in the coils and the eddy current and hysteresis losses in the core are together usually less than three per cent. of the full-load output of a good transformer. This high efficiency of the actual transformer shows how closely the actual transformer approximates to the ideal transformer in its action.

155. Particular cases of transformer action. (Harmonic electromotive forces and currents.) — The above discussion is not limited to any particular wave-form of electromotive force and current, but in order to show the voltage and current relations of a transformer in a clock diagram it is necessary to consider the case of harmonic electromotive forces and currents, inasmuch as non-harmonic electromotive forces and currents cannot be represented in a clock diagram.

The magnetizing current M of a transformer is not harmonic and it cannot, therefore, in strictness, be represented by a line in a clock diagram. In Figs. 220, 221 and 222, however, M is represented by a line. The current so represented is called the "equivalent harmonic magnetizing current."

Case 1. Non-inductive receiving circuit. — In this case the current I'' produced in the receiving circuit is in phase with E'', as shown in Fig. 220, and, according to Art. 154, I' must, therefore, be in phase with E'. The line OM represents the magnetizing current in the primary coil, and the vector sum of OM and I' represents the total primary current. The vector $O\Phi$ represents the harmonically varying core flux. Figures 220 to 223 all represent a 2 : 1 step-down transformation.

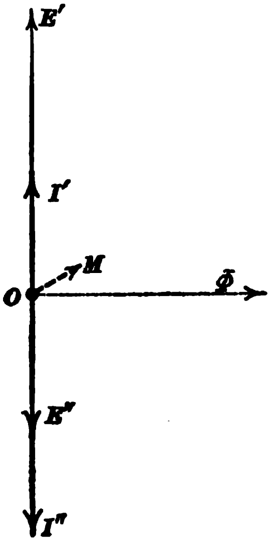

FIG. 220.

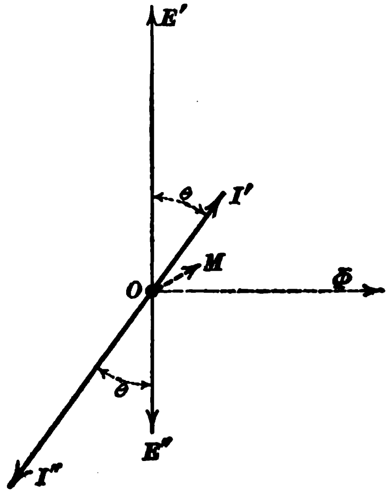

FIG. 221.

Case 2. Inductive receiving circuit. — In this case the current I'' produced in the receiving circuit lags behind E'' by an angle θ of which the tangent is equal to X/R, where X is the reactance and R is the resistance of the receiving circuit. The load current I' in the primary coil being opposite to I'' in phase is θ behind E', as shown in Fig. 221.

Case 3. Receiving circuit containing a condenser. — In this case, which is shown in Fig. 222, I'' is ahead of E'' in phase and I' is ahead of E' in phase, as indicated in the figure.

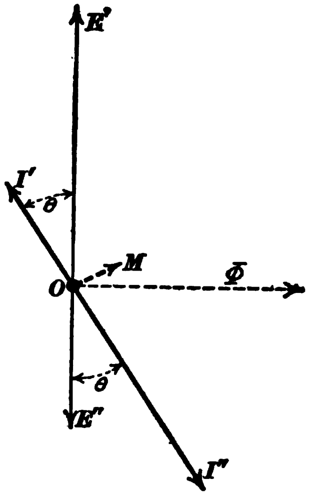

Fig. 222.

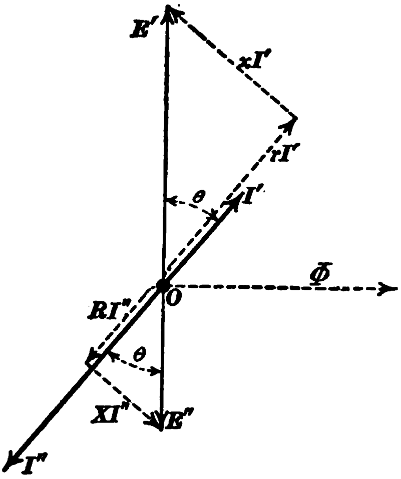

Fig. 223.

156. Equivalent resistance and reactance of an ideal transformer feeding a given receiving circuit. — The primary of a transformer takes from the supply mains a definite current at a definite phase lag when the secondary of the transformer is delivering current to a given circuit. *Consider a simple circuit, of resistance* r *and reactance* x, *which, if connected directly to the supply mains, would take the same current that is delivered to the primary of the transformer as above and at the same phase lag.* This simple circuit is equivalent to the transformer and the secondary receiving circuit, and r and x are called *the equivalent primary resistance and the equivalent primary reactance, respectively, of the secondary receiving circuit.*

In using the ideas of equivalent primary resistance and reactance, magnetizing current is always ignored.

Let R be the actual resistance of the secondary receiving circuit, X its actual reactance, N' the number of primary turns, and N'' the number of secondary turns; then

$$r = \left(\frac{N'}{N''}\right)^2 R \qquad (61)$$

and

$$x = \left(\frac{N'}{N''}\right)^2 X \tag{62}$$

To derive these two equations, resolve the primary electromotive force E', Fig. 223, into components parallel to and perpendicular to I' as shown. The component parallel to I' is rI' and the component perpendicular to I' is xI'. The triangle whose sides are E', rI' and xI' is similar to the triangle whose sides are E'', RI'' and XI'', therefore

$$\frac{xI'}{XI''} = \frac{E'}{E''}$$

and

$$\frac{rI'}{RI''} = \frac{E'}{E''}$$

But

$$\frac{E'}{E''} = \frac{N'}{N''}$$

and

$$\frac{I'}{I''} = \frac{N''}{N'}$$

so that

$$r = \left(\frac{N'}{N''}\right)^2 R$$

$$x = \left(\frac{N'}{N''}\right)^2 X$$

Example of the use of the ideas of equivalent primary resistance and reactance. — A resistance of 150 ohms and a reactance of 100 ohms are connected in series with the primary coil of a 10 : 1 step-down transformer which takes current from 2,000-volt supply mains, and the secondary coil of the transformer delivers current to a receiving circuit of which the resistance is 2.5 ohms and the reactance is 2.0 ohms. Required the values of I', I'', E', E'', and all the phase angles. Multiplying 2.5 ohms $(= R)$ and 2.0 ohms $(= X)$ by $(\frac{10}{1})^2$ we have the primary equivalent values, namely, 250 ohms $(= r)$ and 200 ohms $(= x)$, so that the primary current flows, as it were, through a circuit of which the resistance is 150 ohms + 250 ohms and of which the reactance is 100 ohms + 200 ohms. That is, the impedance of this equivalent simple circuit is $\sqrt{(400)^2 + (300)^2} = 500$ ohms, so that the current is 2,000 volts ÷ 500 ohms or 4 amperes. Taking this current as the reference axis, the components of the supply voltage are 4 amperes × 400 ohms = 1,600 volts parallel to the current and 4 amperes × 300 ohms = 1,200 volts perpendicular to the current; the components of E' (voltage across the primary coil) are 4 amperes × 250 ohms = 1,000 volts parallel to the current and 4 amperes × 200 ohms = 800 volts perpendicular to the current; the components of E'' are $\frac{1}{10} \times 1{,}000$ volts and $\frac{1}{10} \times 800$ volts parallel to and perpendicular to the current respectively; and the value of I'' is of course 40 amperes. The student should represent these details in a clock diagram.

157. Relation between core flux and primary voltage. — The electromotive force in abvolts induced in each turn of wire of the primary coil of a transformer is at each instant equal to the rate of change of the core flux ϕ, so that the total induced electromotive force in abvolts is equal to $N' \cdot d\phi/dt$, and this induced electromotive force is equal and opposite* to the primary voltage e' at each instant (resistance of primary coil negligibly small), therefore

$$e' = N' \frac{d\phi}{dt} \tag{i}$$

where e' is expressed in abvolts.

Example of core flux for a particular non-harmonic electromotive force. — The relation between core flux and primary voltage is simplest for the case in which the electromotive force curve is rectangular like the full-line curve in Fig. 224. Suppose, for example, that a 50-volt storage battery is connected through a reversing switch to the primary coil of a transformer, imagine that the switch is reversed 25 times per second, and let it be required to determine the curve of which the ordinates represent the varying value of the core flux, the primary coil having 100 turns of wire.

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* In fact the induced electromotive force should properly be considered equal to $-N'd\phi/dt$ so that, placing e' equal and opposite to the induced voltage, we have equation (i).

During each twenty-fifth of a second that the battery voltage continues to act steadily on the primary coil the core flux must change at the constant rate of fifty million lines per second according to equation (i) above, the resistance of the primary coil being ignored. Therefore (*a*) the curve of core flux curve must be an inclined straight line during each twenty-fifth of a second, and (*b*) the total change of flux during each twenty-fifth of a second must be $\frac{1}{25} \times 50{,}000{,}000$, or two million lines. In every case, however, the core flux of a transformer pulsates between equal positive and negative maximum values, so that the total change of flux in a half-cycle is 2Φ, where Φ is the maximum value of the core flux, positive or negative. Therefore, in the case here considered, the maximum value of the core flux is one million lines as shown by the dotted curve in Fig. 224.

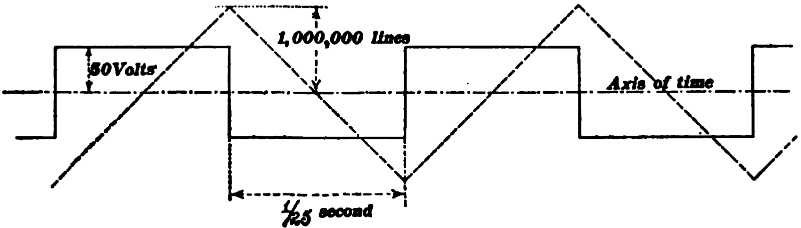

Fig. 224.

Example of core flux for harmonic electromotive force. — The relation between core flux and primary voltage in this case is most easily established by starting with a given harmonically varying core flux ϕ and finding the corresponding primary voltage. Therefore let

$$\phi = \Phi \sin \omega t \qquad \text{(ii)}$$

Differentiating this expression with respect to time we have

$$\frac{d\phi}{dt} = \omega\Phi \cos \omega t \qquad \text{(iii)}$$

and substituting this value of $d\phi/dt$ in equation (i) we have

$$e' = \omega N'\Phi \cos \omega t \qquad \text{(iv)}$$

from which it is evident: (*a*) That e' is harmonic; that is to say, a harmonically varying core flux induces a harmonic electromotive force in the primary coil (or a harmonic primary electromotive force always leads to the production of a harmonically varying core flux); (*b*) that the primary voltage is 90° ahead of the harmonically varying core flux in phase, as shown in Figs. 220, 221 and 222; and (*c*) that the maximum value of e', namely $\mathbf{E}'$, is equal to $\omega N'\Phi$, that is,

$$\mathbf{E}' = \omega N'\Phi$$

or, substituting the effective value $E'(=\mathbf{E}'/\sqrt{2})$, and solving for Φ we have

$$\Phi = \frac{\sqrt{2}E'}{\omega N'} \qquad (63)$$

in which E' is the effective value in abvolts of a harmonic voltage acting on the primary coil of a transformer, N' is the number of turns of wire in the coil, ω is the frequency in radians per second ($= 2\pi$ times frequency in cycles per second), and Φ is the maximum positive and negative value between which the core flux pulsates.

The maximum flux density $\mathcal{B}$ reached in the transformer core is of course equal to Φ divided by the area of the cross-section of the core in square centimeters.

158. Loss of power in transformers. — The power output of a transformer is always less than its power intake because of the loss of power in the transformer. This loss of power in a transformer consists of two parts, namely, (*a*) the iron or core loss due to eddy currents and hysteresis; and (*b*) the copper loss due to the resistances of the primary and secondary coils.

The iron losses are practically the same in amount at all loads, and they depend upon the frequency, upon the range of the flux density $\mathcal{B}$, upon the quality and volume of the iron, and upon the thickness of the laminations.

The hysteresis loss in watts is

$$P_h = aVf\mathcal{B}^{1.6} \tag{64}$$

where f is the frequency in cycles per second, $\mathcal{B}$ is the maximum flux density in lines per square centimeter, V is the volume of the iron in cubic centimeters, and a is a constant depending upon the magnetic quality of the iron. For annealed refined wrought iron the value of a is about 3×10^{-10}.

The eddy current loss in watts is

$$P_e = bVf^2l^2\mathcal{B}^2 \tag{65}$$

where l is the thickness of the laminations in centimeters, and b is a constant depending upon the specific electrical resistance of the iron. For ordinary iron the value of b is about 1.6×10^{-11}. Insufficient insulation of laminations causes an excessive eddy current loss.

Equations (64) and (65) may be used for calculating the hysteresis and eddy current losses in any mass of laminated iron subjected to periodic reversals of magnetization, such as alternator armature cores and the rotor and stator iron in an induction motor.

The copper loss is

$$P_c = R'I'^2 + R''I''^2 \tag{66}$$

This loss is nearly zero when the transformer is not loaded; it increases with the square of the current, and becomes excessive when the transformer is greatly overloaded.

159. Efficiency of transformers.—The ratio *power output* ÷ *power intake* is called the efficiency of a transformer.

The efficiency of a given transformer is very low when the output is small; it increases as the output increases, reaches a maximum, and falls off again when the output is very great. This falling off of efficiency when the output is great is due to the great increase of copper losses. Figure 225 shows the efficiency of a transformer at various loads.

The efficiency of a well-designed transformer at full load ranges from about 95 per cent. for a one-kilowatt * transformer, to about 97 per cent. for a 15-kilowatt transformer, to 98.5 or 99 per cent. for very large transformers.

All-day efficiency.—A transformer is usually connected to the mains continuously, although current may be delivered by the secondary for a few hours only each day. In such a case the iron loss is continuous and may represent a very considerable loss of energy each day. The copper loss on the other hand is not appreciable except when the transformer is loaded. Therefore a transformer which is always connected to the mains, but which is loaded only a short time each day, should be designed to have as small a power loss as possible in the iron, even though the design may involve a considerable increase of the power loss in the copper.

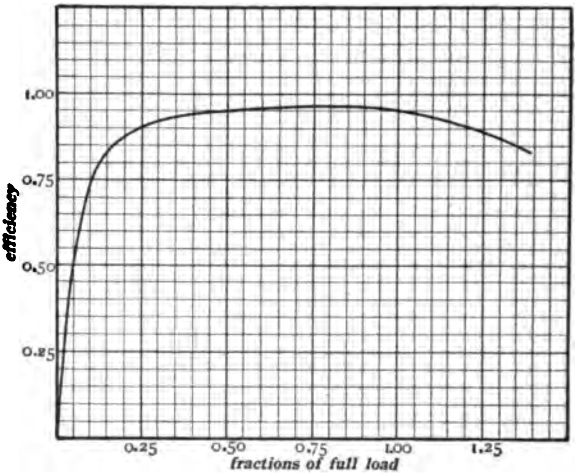

Fig. 225.

The all-day efficiency of a transformer (or of any machine) is defined as the quotient obtained by dividing the total energy delivered by the machine in a day by the total energy delivered to the machine in a day. For example, consider a transformer of which the iron loss is 300 watts and the copper loss at full load is also 300 watts, the full-load output of the transformer being 10 kilowatts. The transformer is connected to the mains all day and the total energy lost in the core in a day is 24 × 300 watt-hours. The transformer is used at full load for 1½ hours each day, during which time the copper loss is 1½ × 300 watt-hours, and the copper loss during the remaining 22½ hours is negligible. The total energy delivered by the transformer in a day is 10,000 × 1½ watt-hours, and the total energy delivered to the transformer in a day is of course [10,000 × 1½ + 24 × 300 + 1½ × 300] watt-hours, so that the all-day efficiency of the transformer under the given conditions is 15,000 watt-hours divided by 22,650 watt-hours, or 66.2 per cent.

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* See Art. 161.

Calculation of efficiency. — Given the details of design of a transformer to calculate its efficiency for given value and frequency of E' (or E'') and given current output I''.

The power output is equal to $E''I''$ on non-inductive load. If the receiving circuit is inductive the power output is $E''I''$ multiplied by the power factor of the receiving circuit.

The power input is equal to the power output plus the losses, so that, if the losses can be calculated, the power input is known, and then the efficiency may be calculated by dividing output by input.

The copper loss may be easily found from the equation

$$P_c = R'I'^2 + R''I''^2$$

In order to determine the iron loss, calculate Φ from equation (63), Art. 157, and divide by the sectional area of core to find the maximum flux density $\mathcal{B}$. Then, knowing volume of iron V, thickness of laminations l, frequency f, and the two empirical constants a and b in equations (64) and (65) of Art. 158, the values of P_h and P_e may be calculated.

160. Transformer regulation. — When a transformer is at zero load the ratio of primary to secondary voltage is almost exactly equal to the ratio N'/N'', but with increasing load the secondary terminal voltage decreases, the supply voltage being constant, or, in other words, the secondary terminal voltage increases in value as the load on the transformer is decreased. The degree of regulation is usually specified by expressing the increase of secondary terminal voltage from full load to zero load in per cent. of full-load voltage. Thus a given transformer supplied from constant voltage mains gives a secondary terminal voltage of 108.5 volts at full load and 112 volts at zero load and its percentage regulation is

$$\frac{112 - 108.5}{108.5} \times 100 = 3.22 \text{ per cent.}$$

The decrease of secondary terminal voltage of a transformer with increase of load is due to coil resistances and to magnetic leakage. When the receiving circuit is non-inductive the decrease of secondary terminal voltage is due almost entirely to coil resistances, when the receiving circuit is very highly inductive the decrease of secondary terminal voltage is due almost entirely to magnetic leakage, and when the receiving circuit is a condenser the effect of magnetic leakage is to cause the secondary terminal voltage to increase with increase of load.*

161. Transformer rating.—An increase of the current output of a transformer, with given value and frequency of E', causes a decrease of secondary terminal voltage and an increased generation of heat in the transformer coils. Increase of E' or decrease of frequency, on the other hand, causes an increase of Φ according to equation (63), Art. 157, and this increase of Φ causes an increase of iron loss according to equations (64) and (65), Art. 158.

The voltage rating, frequency rating, and current rating of a transformer are therefore determined by the allowable heating and by the required degree of constancy of secondary voltage, and usually the allowable heating is the determining factor.

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* The theory of transformer regulation is given in Chapter XI, Volume II, of Franklin and Esty's *Elements of Electrical Engineering*.

A transformer is usually rated by specifying the value and frequency of the primary voltage with which it is intended to be used, and by specifying its ratio of transformation and the power it can deliver steadily (at rated voltage) to a non-inductive receiving circuit.

The rating of a transformer is by no means rigid, and it is allowable to use a transformer under conditions differing very greatly from the conditions specified in its rating, but when so used a transformer should be watched so that it may be disconnected if its temperature becomes excessive.

162. The constant-current transformer.—When a number of arc lamps (or other receiving units) are operated in series, individual lamps must be short-circuited to cut them out of service, and in order that the lamps may be independent of each other, the circuit must be supplied with a constant current. When alternating current is used to operate arc lamps in series, the *constant-current transformer* is used. This transformer takes current from a constant voltage source and delivers constant current to the arc-lamp circuit. Figure 226 shows a constant-current transformer of the General Electric Company. The essential features of this transformer are shown in Fig. 227, in which *CCC* is the laminated iron core, *PP* is the stationary primary coil, and *SS* is the secondary coil which is suspended from a counterpoised lever system so as to move up and down with great ease. When lamps are short-circuited, the slight increase * of current causes an increased repulsion between *P* and *S*, the coil *S* moves upwards, and the increase of leakage flux across the regions *LL* lessens the useful flux through the secondary coil *S*, which reduces the induced voltage in *S*, thus counteracting the tendency for the current to increase.

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* An initial increase of current must be produced in order to start the moving coil upwards, but the lever system may be so designed as to keep the equilibrium value of the current almost exactly constant with decrease of resistance of receiving circuit, or even to cause the equilibrium value of the current to decrease with decrease of resistance of receiving circuit.

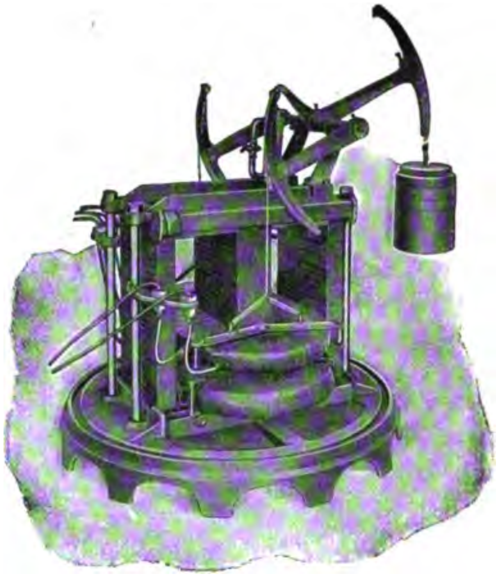

Fig. 226.

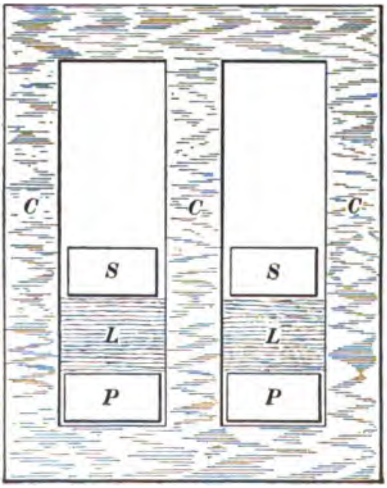

Fig. 227.

163. Simple transformer connections. Connections in parallel. Connections in series. — When transformers are used to supply current to groups of lamps or motors at constant voltage, stepping down from constant-voltage transmission lines, the primary of each transformer is connected directly across the high-voltage mains, and each transformer delivers current to a separate group of lamps or motors as shown in Fig. 228. This arrangement is called the parallel connection of transformers.

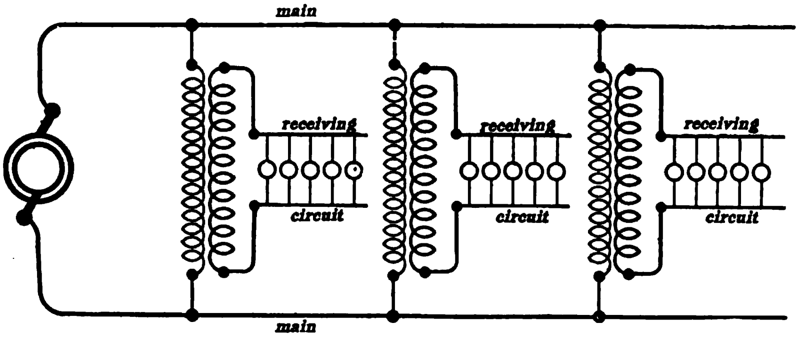

Fig. 228.

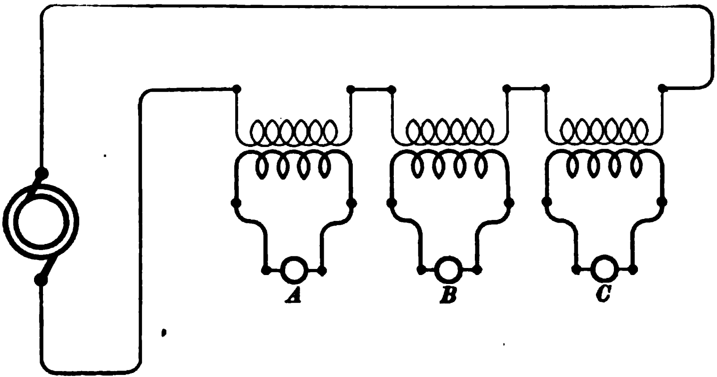

Fig. 229.

The primary coils of several transformers may be connected in series with each other as shown in Fig 229. This arrangement, which is called the series connection of transformers, is seldom used.

Transformers with divided coils. — Alternators in moderately small lighting plants give usually 1,100 or 2,200 volts electromotive force, and the standard voltages for incandescent lamps are 110 volts or 220 volts. Small transformers for lighting plants are therefore usually made with two-part primary coils and two-part secondary coils, so that the two parts of the primary may be connected in series with each other for 2,200 volts or in parallel with each other for 1,100 volts, and so that the two parts of the secondary may be connected in series with each other for 220 volts or in parallel with each other for 110 volts.

In connecting the two parts of a secondary coil of a transformer in parallel with each other the utmost care must be taken to connect the parts so that one part does not act as a short-circuit for the other, and in connecting the two parts of the primary coil of a transformer in series with each other the utmost care must be taken to be sure that the two parts work together in opposing the primary impressed voltage. Failure in either case to make the proper connections may lead to serious damage to the transformer when it is connected to the mains.

164. The autotransformer.* — Consider the supply mains which bring current from an alternator, and the service mains which deliver current to a customer, as shown in Fig. 230. There are

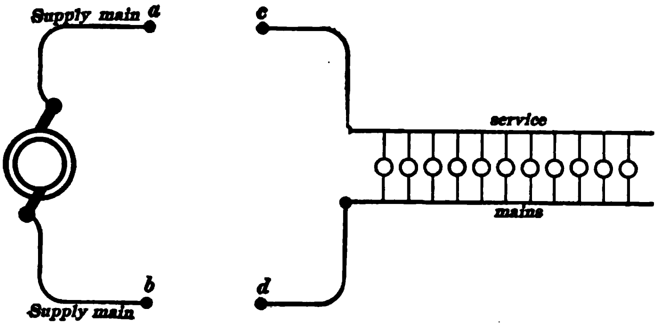

Fig. 230.

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* It is misleading to speak of *the* autotransformer inasmuch as a so-called autotransformer is exactly the same thing as an ordinary transformer, and it acts in the same way. The name autotransformer refers to a scheme of connections.

three methods for establishing communication from *ab* to *cd*, as follows: (*a*) By conductive connections, that is, by connecting wires from *a* to *c* and from *b* to *d* in Fig. 230, (*b*) by transformer connections, that is, by connecting the primary of a transformer to *ab* and the secondary to *cd*, and (*c*) by a combination of conductive and transformer connections. Any transformer which is connected up for this combination method is called an *autotransformer.*

The action of the autotransformer may be brought out most clearly by considering a numerical example. Let the coils *A* and *B*, Fig. 231, represent the fine-wire coil and the coarse-wire coil respectively of an ordinary 10 to 1 transformer. The fine-wire coil is connected across 100-volt supply mains, and of course an electromotive force of 10 volts is induced in the coarse-wire coil. One terminal *c* of the receiving circuit is connected directly to one of the supply mains as shown, and *the other terminal b is connected through the coil BB to the supply main a.* This connection may be made so that the 10 volts in *BB* is *added* to the supply voltage thus giving 110 volts across *bc*, or it may be made so that the 10 volts in *B* is *subtracted* from the supply voltage thus giving 90 volts across *bc*. In the first case we have auto-step-up transformation, and in the other case we have auto-step-down transformation. In the first case the coil *A* acts as a primary, receiving power from the supply mains; and the coil

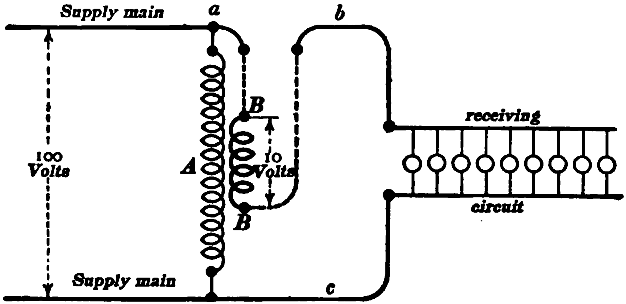

Fig. 231.

BB acts as a secondary, delivering power to the receiving circuit. In the second case the coil *BB* acts as a primary, receiving power from the current which is forced through it in opposition to its voltage; and the coil *A* acts as a secondary, delivering power back to the supply mains. In both cases the power which is actually transformed from *A* to *B* or from *B* to *A* is equal to 10 volts multiplied by the current *I* which is delivered to the receiving circuit, whereas the total amount of power delivered to the receiving circuit is $110 \times I$ in the first case, and $90 \times I$, in the second case. Therefore a 10 to 1 transformer rated as a 1-kilowatt transformer would suffice for the delivery of 11 kilowatts to the receiving circuit in the first case, or 9 kilowatts in the second case. This illustrates the most important feature of the autotransformer when the supply voltage is to be only slightly raised or lowered, namely, that a small transformer suffices for the delivery of a large amount of power, and of course the losses of power are less than would be involved in a large transformer used in the ordinary way.

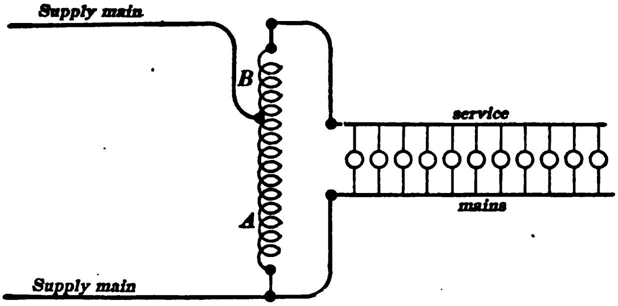

Fig. 232a.

The action of the autotransformer in altering a supply voltage is most clearly represented as shown in Figs. 232*a* and 232*b*;* Fig. 232*a* represents auto-step-up transformation, and Fig. 232*b* represents auto-step-down transformation. In each of these figures the two coils A and B are related to each other as parts of one continuous winding, and the voltage of the whole winding (A and B together) is to the voltage of any given part of it as the whole number of turns is to the number of turns in the given part.

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* Figures 232*a* and 232*b* show two of the four possible arrangements of a given transformer as an autotransformer.

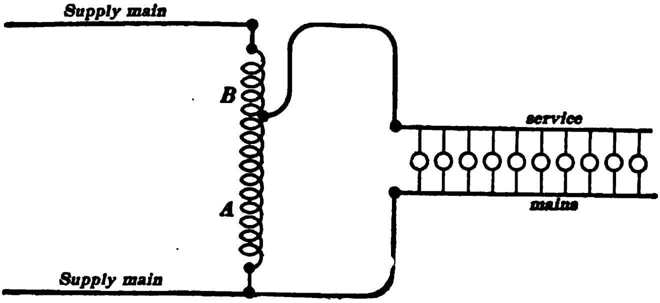

Fig. 232b.

165. The arrangement of transformers in polyphase systems. (*Without changing the number of phases.*) — Step-up and step-down transformation in polyphase systems is accomplished, in general, by using an independent transformer for each phase, that is, by transforming each phase by its individual transformer.

When three transformers are used in a three-wire three-phase system, the primary coils of the three transformers may be Δ-connected or Y-connected to the supply mains, and in either case the secondary coils may be Δ-connected or Y-connected to the service mains, so that four arrangements are possible, namely, the ΔΔ-connection, the ΔY-connection, the YΔ-connection, and the YY-connection. The ΔΔ-connection is, however, the usual arrangement, for, in this case, the system remains operative when one of the transformers is disconnected; thus with the ΔΔ-connection the burning-out of one of the transformers need not render the system inoperative, although the two remaining transformers would operate at a disadvantage, not only because the two transformers would have to carry the load of three, but because the currents in the two transformers are increased in a greater ratio than 2 : 3.* i.e. each transformer can carry Gvr 58% rated Load

The use of two transformers for three-phase transformation. — Figure 233 shows the connections of two transformers *Aa* and *Bb* for three-phase transformation. It is evident at once that this arrangement maintains the proper voltage (proper, that is, in value and in phase) between service mains 1 and 2, and between service mains 2 and 3; but a proper voltage between 1 and 2 together with a proper voltage between 2 and 3 gives necessarily a proper voltage between 3 and 1. The use of two transformers as shown in Fig. 233 for three-phase transformation is not usually desirable because the currents in the transformer windings are out of phase with the voltages across the transformer terminals even if the receiving circuits are non-inductive so that the full power-rating of the two transformers is not available even when they are delivering current to receiving circuits of which the power factor is unity However, two transformers are thus used for three-phase transformation on switchboards for supplying voltmeters, synchronizing indicators, and wattmeters, because in this case the power capacity of the transformers is not important.

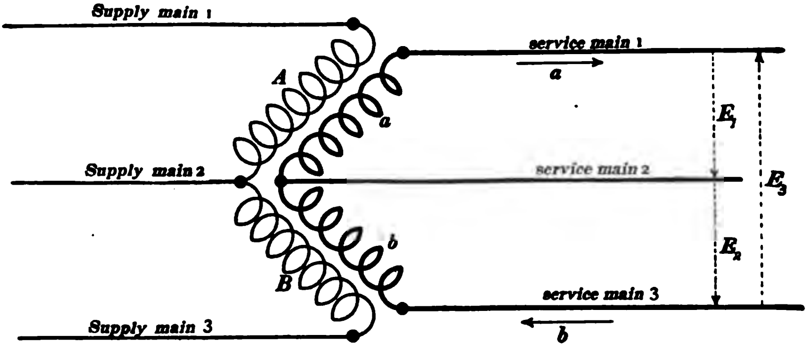

Fig. 233.

Consider the transformation of three phases by the use of three ΔΔ-connected transformers. If one of these transformers becomes damaged and is therefore disconnected, the other two transformers are then connected as shown in Fig. 233, and the system remains operative except that the two transformers may be greatly overloaded.

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* See pages 222–223 of Franklin and Esty's *Elements of Electrical Engineering*, Vol. II.

166. Polyphase transformers. — It is possible to combine the magnetic circuits of transformers which are used for polyphase transformation. Thus Fig. 234 shows what may be called a two-phase transformer. It consists of two similar transformers *A* and *B*, the magnetic circuits of both being completed through the portion *c* of the core. When used for two-phase step-up or step-down transformation, transformer *A* is used for one phase and transformer *B* for the other phase. In this case the magnetic flux in core *a* is in phase-quadrature with the magnetic flux in core *b*, and the return flux in core *c* is related to the fluxes in *a* and *b* as shown in the clock diagram in Fig. 235.

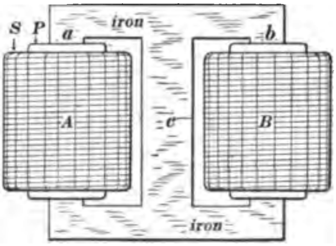

Fig. 234.

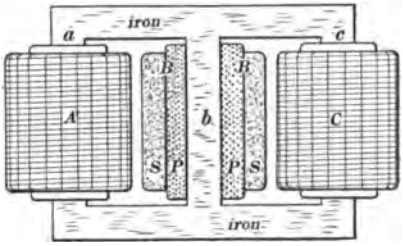

Fig. 236.

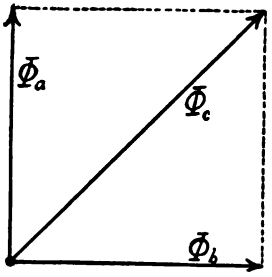

Fig. 235.

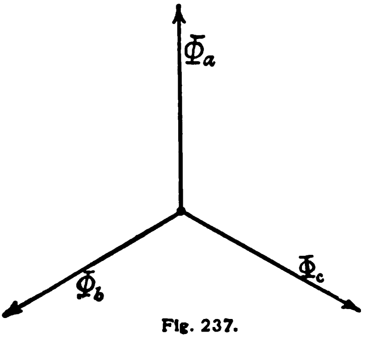

Fig. 237.

Figure 236 shows what may be called a three-phase transformer. It consists of three similar transformers *A*, *B* and *C*, with a single core-structure as shown. When used for three-phase step-up or step-down transformation, transformer *A* is used for one phase, transformer *B* for another phase, and transformer *C* for the third phase. In this case the magnetic fluxes in cores *a*, *b* and *c* are 120° apart in phase, as shown in the clock diagram of fluxes, Fig. 237, and each core may be looked upon as the return path for the flux in the other two.

167. Transformation of phases.—Transformers in polyphase systems are often arranged so as to change the number of phases and at the same time to produce step-up or step-down transformation. Thus at one of the large power stations at Niagara Falls, two-phase alternators are used, and the step-up transformers which supply the Buffalo transmission line are arranged to change from two-phase to three-phase so as to take advantage of the fact that three-phase transmission requires less copper than two-phase transmission, for the same voltage between wires. Also when it is desired to operate, from an *n*-phase supply, a synchronous converter having *m* collector rings, the step-down transformers must be arranged to change the number of independent phases.

It is impossible by means of transformers to convert a single-phase supply into a polyphase supply, because a transformer cannot convert the pulsating power delivered by a single-phase alternator into the steady flow of power required in the polyphase system. A transformer (that is, an ideal transformer) delivers energy at each instant at the same rate that it receives energy.

The principle involved in the transformation of phases may be best brought out by considering the transformation from two phases to any number of phases, say, three; although any arrangement of transformers which will transform from two-phase to *n*-phase will serve to transform from *n*-phase to two-phase.

Let A and B, Fig. 238, represent the two electromotive forces of a two-phase alternator, and let E_1 be any given electromotive force which it is desired to produce. The component of E_1 parallel to A is $E_1 \sin \beta$, and the component of E_1 parallel to B is $E_1 \cos \beta$; and *to produce the desired electromotive force* E_1 *it is only necessary to produce the components of* E_1 *in the respective secondaries of two transformers, of which the primaries are supplied from* A *and* B *respectively, and to connect these secondary coils in series.* Thus Fig. 239 shows two separate transformers with their primary coils connected to phase A and phase B, respectively, of a two-phase alternator, the secondary coil a has a number of turns sufficient to produce the electromotive force $E_1 \sin \beta$, the secondary coil b has a number of turns sufficient to produce the electromotive force $E_1 \cos \beta$, and the two coils a and b connected in series give the required electromotive force E_1. Similarly, the electromotive forces E_2 and E_3, Fig. 238, may be produced each by a pair of properly

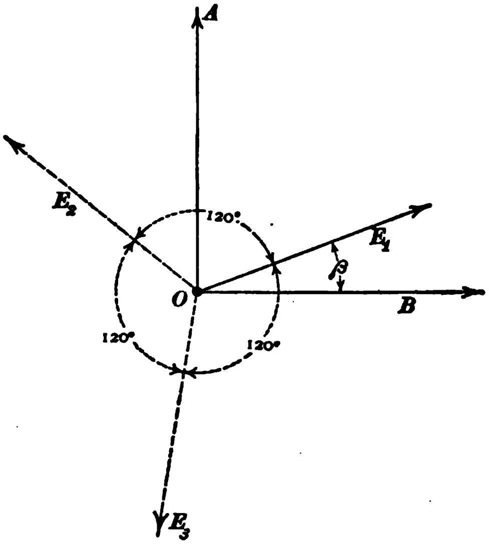

Fig. 238.

proportioned secondary coils. Therefore, *the two-phase, three-phase transformer consists of two transformers with their primary coils supplied from the two-phase system, and having, in general, six secondary coils which are connected in pairs in series, each pair supplying one of the three-phase voltages.* Such an arrangement can be used to change from two-phase to three-phase or to change from three-phase to two-phase.

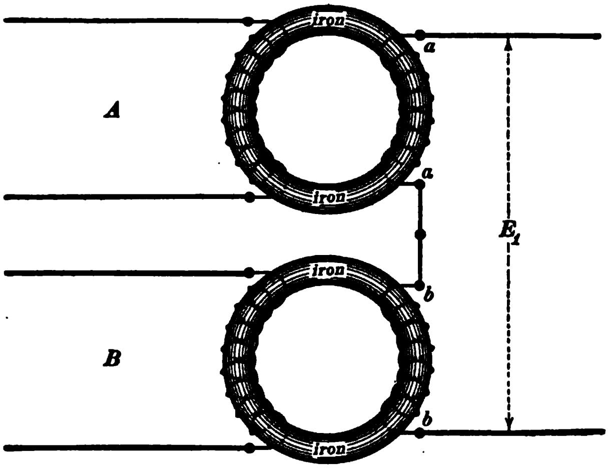

Fig. 239.

The Scott transformer. — The general type of two-phase three-phase transformer above described may be greatly simplified, and the simplest arrangement is that due to Chas. F. Scott. To understand the Scott transformer it is necessary to consider an even more general type of two-phase three-phase transformer than that described above, and to make use of the following proposition: Consider three units of any kind, alternators, or combinations of transformer secondaries, *a*, *b* and *c*, which are Y-connected to three mains 1, 2 and 3, as shown in Fig. 240; *if the lines a, b and c, Fig. 241, which represent the electromotive forces generated in a, b and c, Fig. 240, terminate at the apices of an equilateral triangle, then the Y-connected units give symmetrical three-phase voltages between the mains.* This is evident when we consider that the voltage from main 1 to main 2, Fig. 240, is the vector difference $a - b$, that the voltage from main 2 to main 3

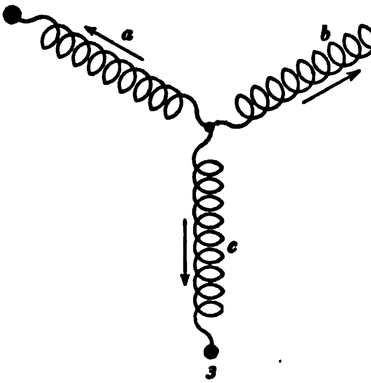

Fig. 240.

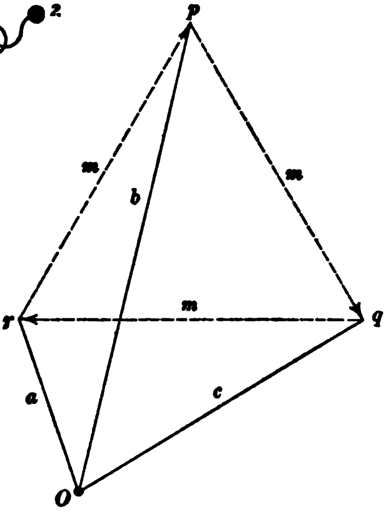

Fig. 241.

is the vector difference $b - c$, and that the voltage from main 3 to main 1 is the vector difference $c - a$.

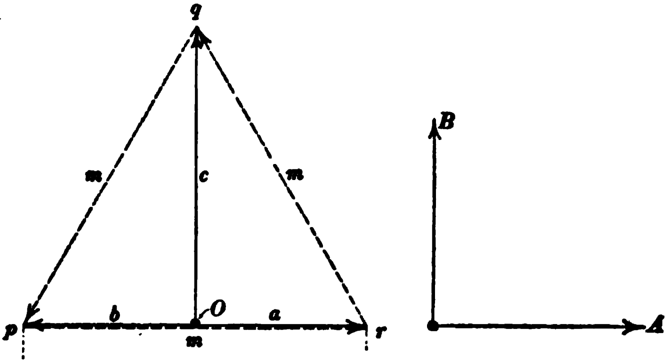

Fig. 242.

In the Scott transformer the point O, Fig. 241, lies midway between the points p and r as shown in Fig. 242, so that the lines a and b are parallel to each other and at right angles to the line *c*. Then the voltage *c* may be produced by the single secondary of a transformer supplied from phase *B* of the two-phase system, and the voltages *a* and *b* may be produced by two secondary coils (one for *a* and one for *b*) of a transformer supplied from phase *A* of the two-phase system, the three secondaries *a*, *b* and *c* being Y-connected or, as it is sometimes expressed, T-connected, to the three-phase mains as shown in Fig. 243. The one condition that is necessary is that the coils *a* and *b* be related to each other as halves of one continuous winding.

Example of a Scott transformer.—The similar primary coils of two transformers have 1,000 turns of wire each and they are connected to the respective phases of a 1,000-volt two-phase supply; and it is desired to deliver three-phase currents at 240 volts between mains. Each side of the equilateral triangle *pqr*, Fig. 242, therefore, must represent 240 volts, so that each of the secondary coils *a* and *b*, Fig. 243, must generate 120 volts, and have 120 turns of wire; and the secondary coil *c*, Fig. 243, must generate $\sqrt{3}/2$ times 240 volts or 208 volts and have 208

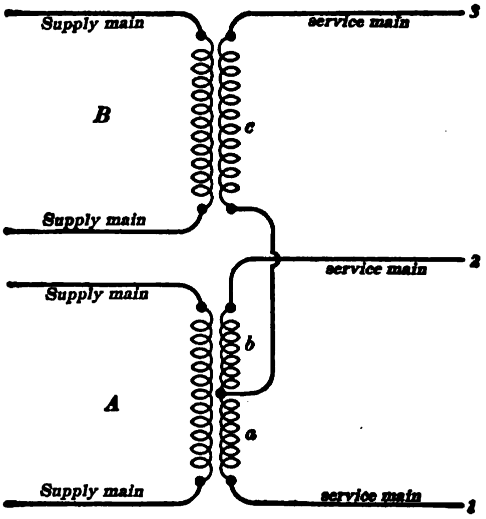

Fig. 243a.

turns of wire. The coils *a* and *b* constitute one continuous winding of 240 turns of wire, with a lead coming out from its middle point.

The diagram of connections of the Scott transformer is sometimes arranged as shown in Fig. 243*b*, the service mains being connected to the points 1, 2, and 3. This diagram is more easily understood than Fig. 243*a* because the positions of the various transformer coils in the diagram suggest the phase relations. The connections of the secondary coils in the Scott transformer constitute what is sometimes called the T-connection.

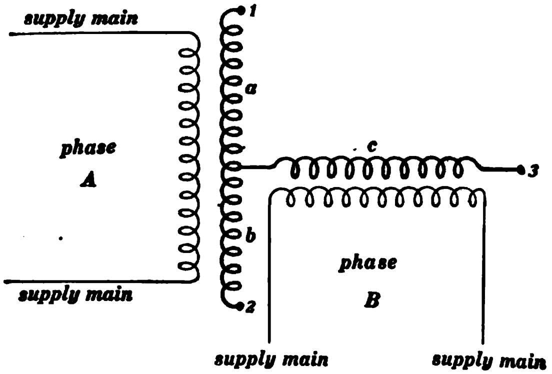

Fig. 243b.

168. Transformer connections for synchronous converters.—An *n*-ring converter must be supplied with alternating current from an *n*-wire system, the supply wires being connected to the rings in proper order. Thus a four-ring converter (sometimes called a two-phase converter) cannot be supplied from a three-wire two-phase system, but the two phases must be entirely separate, each transmitted over a separate pair of wires.

Three-ring converter.—A three-ring converter is supplied with alternating currents from a three-wire three-phase system, the three phases being stepped down to the proper voltage, and the three service wires being connected to the three rings. The order of connections of service wires to rings depends on the direction in which the converter is to be driven. If the alternating-current supply is two-phase it is necessary to transform to three-phase as explained in Art. 167.

The four-ring converter.—A four-ring converter must be supplied with alternating currents from a four-wire two-phase system. If the two-phase currents are transmitted over a three-wire line, the step-down transformers must deliver currents to four service wires, two separate wires for each phase. The wires of phase *A* are connected to two opposite * collector rings, and the wires of phase *B* are connected to the other two rings. The reversal of the connections of either phase will reverse the direction of driving.

If the alternating current supply is three-phase it is necessary to transform to two-phase as explained in Art. 167.

Six-ring converter.—Any converter having $2n$ rings can be supplied with alternating currents from an n-phase system, provided the n armature windings of the n-phase supply generator are *not* electrically connected with each other, ***each generator winding must have its own separate pair of collector rings*** ($2n$ ***rings in all***) ***and must deliver current over a separate pair of transmission wires.*** If the n-phase generator has n collector rings, the n-phases being transmitted over n wires, then *a $2n$-ring con-*

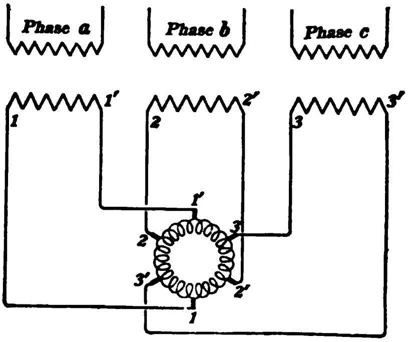

Fig. 244.

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* Opposite collector rings are those which are tapped into opposite points of a two-pole armature winding.

verter may be supplied, if the n-phases be kept entirely separate on the secondary side of the step-down transformers, that is, if each of the *n* secondary coils delivers current to the converter over a separate pair of wires. This is exemplified by what is said above concerning the four-ring converter. It is also exemplified by the supply of currents to a six-ring converter from a three-phase supply, the three phases being separate on the secondary side of the transformers and each delivered to the converter over two wires. To supply a six-ring converter from a two-phase system it is necessary to transform to three-phase.

The simplest method of connecting a six-ring converter to a three-phase (six-wire) system is to connect each of the three phases to opposite collector rings of the six-ring converter. This is called the diametrical connection and it is shown in Fig. 244.

CHAPTER XV.

THE INDUCTION MOTOR.

169. The induction motor.—The induction motor consists of a primary member, to the windings of which polyphase* alternating currents are supplied, and a secondary member with short-circuited windings. The primary member is generally stationary and it is called the *stator*, and the secondary member generally rotates and it is called the *rotor*. The alternating currents in the stator windings produce a *rotating state of magnetism* in the laminated iron of the stator, this rotating magnetism induces currents in the short-circuited rotor windings, and the stator magnetism exerts force on these induced currents and causes the rotor to revolve.

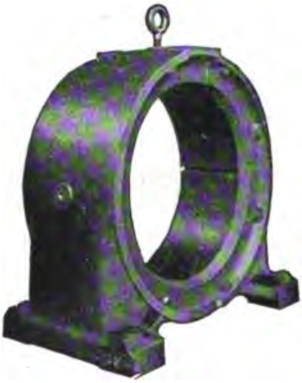

Fig. 245.

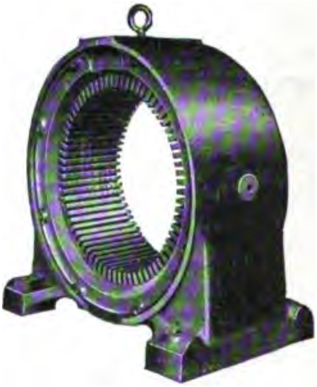

Fig. 246.

The structure of the induction motor is shown in Figs. 245 to 249, which show the details of a small motor manufactured by the Westinghouse Electric Company. Figure 245 shows the cast-iron frame for holding the stator laminations, Fig. 246

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* The single-phase induction motor is described later.

shows the stator laminations assembled, Fig. 247 shows the stator complete with its windings, Fig. 248 shows the laminated core of the rotor mounted on its shaft, and Fig. 249 shows the rotor complete. The rotor shown in Fig. 249 is of the so-called *squirrel-cage* type, and its windings consist of heavy copper rods placed in nearly closed slots and connected at both ends to massive short-circuiting rings of copper. Figure 250 shows a complete three-phase induction motor, the three wires for supplying the three-phase currents to the stator are shown at the lower left-hand side in the figure.

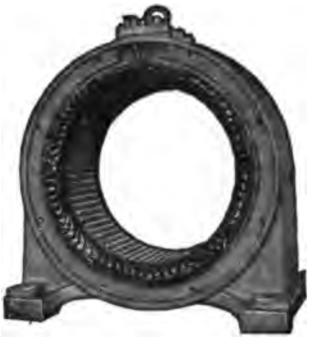


Fig. 247.

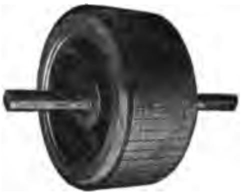


Fig. 248.

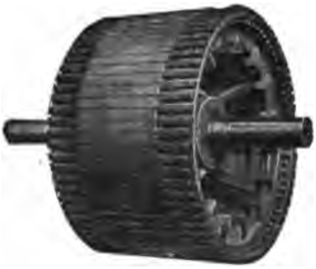
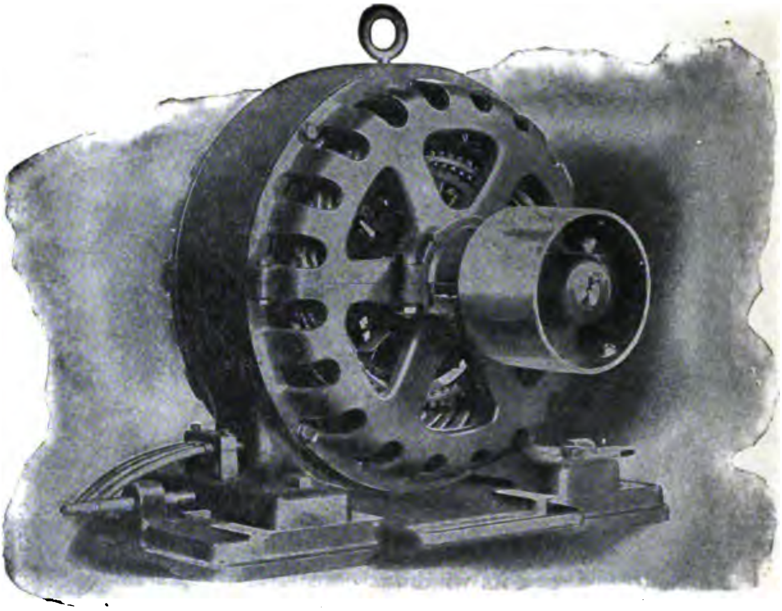


Fig. 249.

In some cases, especially for induction motors of large size, the rotor is wound like a polyphase alternator armature, and the winding is short-circuited through an adjustable resistance. Thus Fig. 251 shows a wire-wound rotor with a device in its interior for varying the short-circuit resistance of the windings; the resistance is usually controlled by pushing on a knob at the end of the rotor shaft. In Fig. 251 the resistance is controlled by a lever (not shown in the figure) which engages the ring-like projection on the short sleeve. Figure 252 shows a wire-wound rotor with



Westinghouse Induction Motor complete.

Fig. 250.

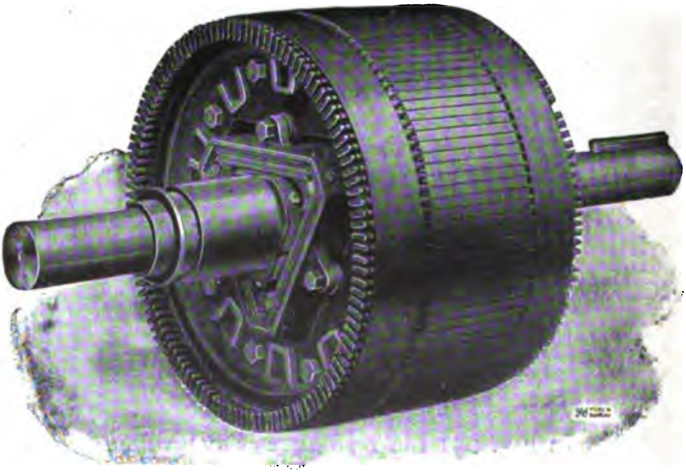


Fig. 251.

the terminals of the winding brought out to three collector rings through which the windings may be connected to an external adjustable resistance. Figures 251 and 252 are taken from the publications of the General Electric Company.

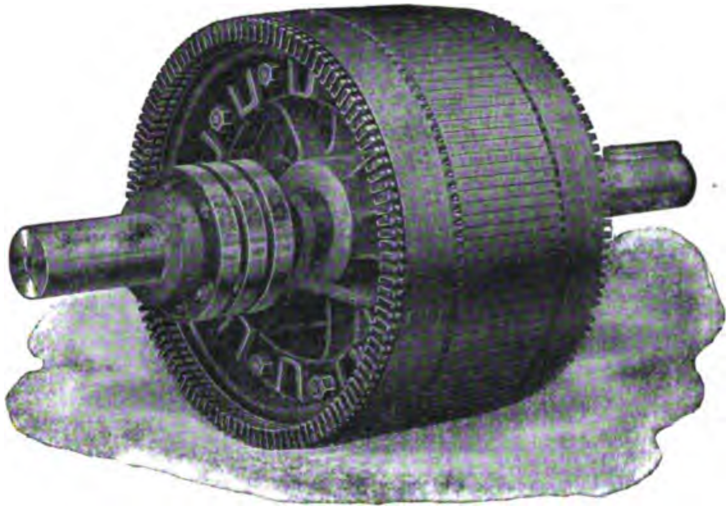


Fig. 252.

Stator windings. — The stator windings of an induction motor are exactly similar to the armature windings of a polyphase alternator. Figure 253 is a diagram of a two-phase stator winding for the production of four rotating magnet poles, a four-pole winding, as it is called; and Fig. 254 is a diagram of a three-phase stator winding for the production of four rotating magnet poles. In each of these diagrams the complete connections are shown for one phase only, namely, the *A* phase. The other phase or phases are exactly like the *A* phase, and each phase takes current from a separate source, as explained in Chapter X. The short radial lines in Figs. 253 and 254 represent the portions of the windings which lie in the slots of the stator iron, the curved lines outside of the radial lines represent the connections at one end of the stator, and the curved lines inside of the radial lines represent the connections at the other end of the stator.

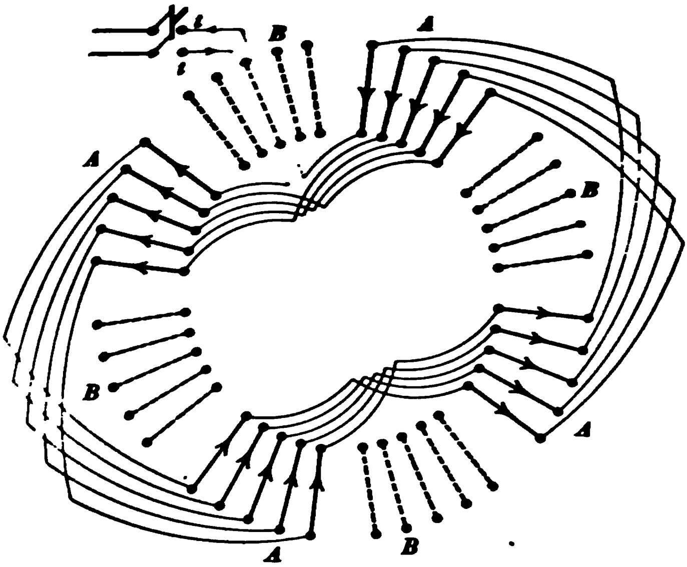


Fig. 253.

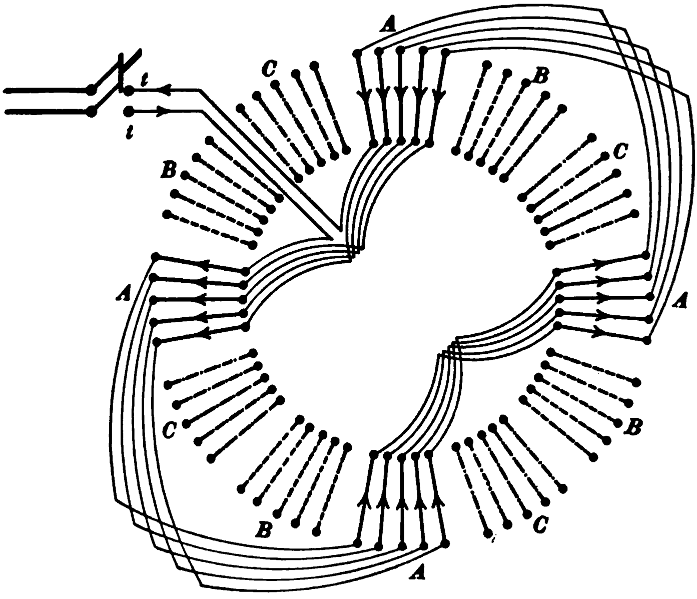


Fig. 254.

170. The action of the induction motor. — The action of the induction motor may be described in two steps, namely, (*a*) the production of rotating magnetism by the stator windings, and (*b*) the action of this rotating magnetism on the short-circuited rotor. Action (*b*) is very simple and may be stated as follows: Imagine a squirrel-cage rotor to be placed between the poles of a mechanically rotated direct-current-excited field magnet as indicated in Fig. 255. The action of this revolving magnet upon the short-circuited rotor windings is exactly equivalent to the action of the stator of an induction motor upon the rotor. The revolving magnet induces currents in the short-circuited rotor rods, and because of these currents the rotor rods are pushed sidewise by the magnetic flux which emanates from the revolving magnet poles.

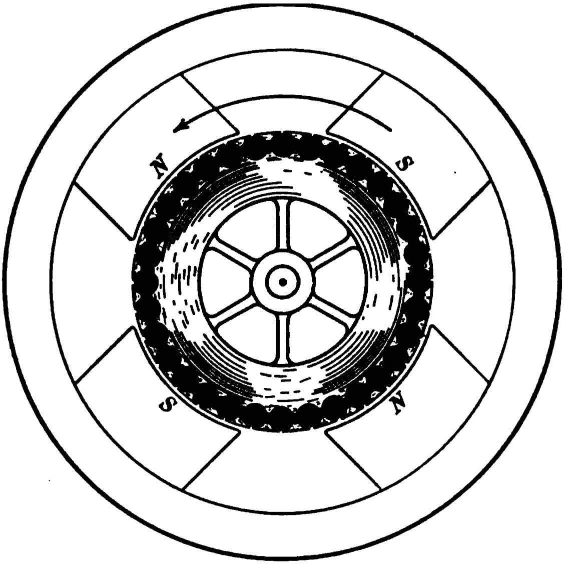


Fig. 255.

The production of rotating magnetism by the stator windings. — Figures 256 represent an end view of a two-phase induction motor with a stator winding like that which is represented in Fig. 253 (four-pole winding). The rotor slots and conductors are omitted for the sake of clearness, and the bands of stator conductors are shown slightly separated from each other so that they

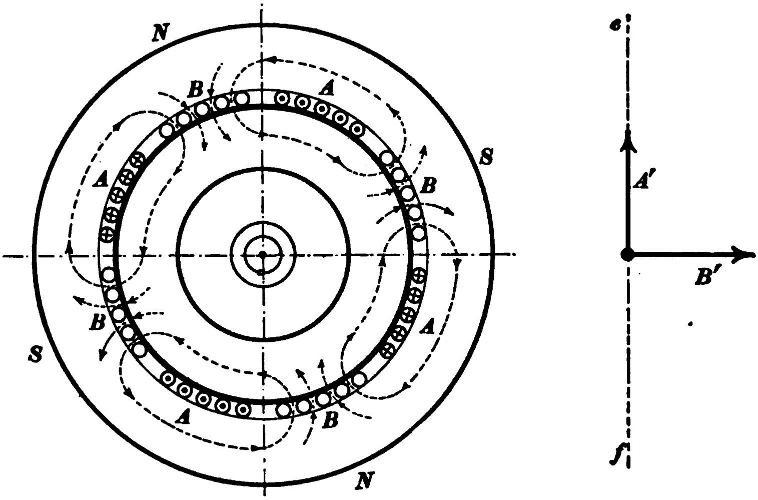


Fig. 256a.

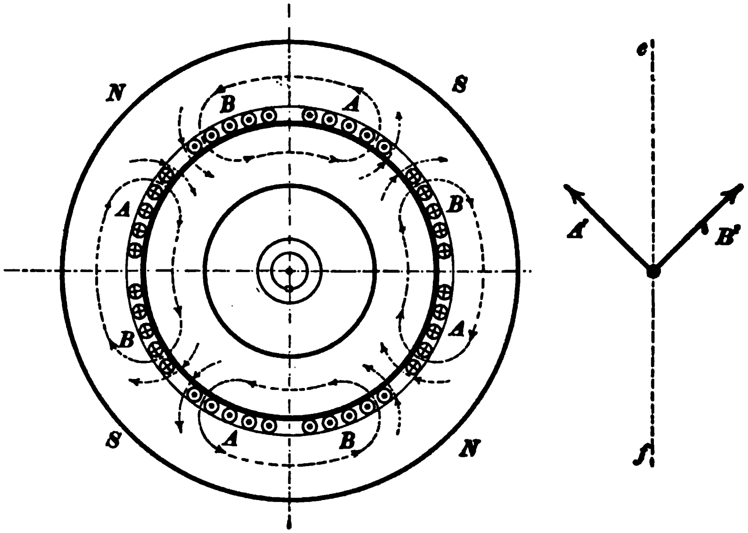


Fig. 256b.

may be easily distinguished. Before proceeding to the discussion of the action of the stator windings it is necessary to consider the

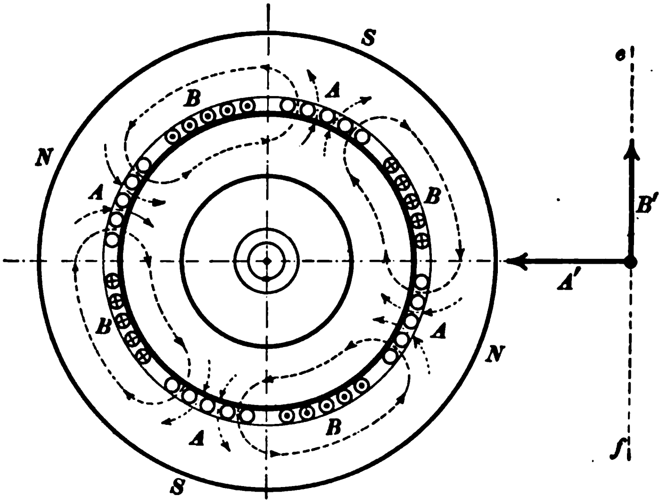
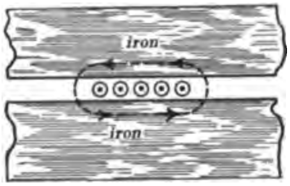


Fig. 256c.

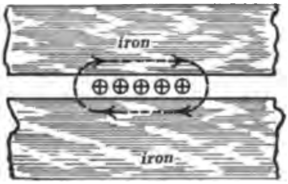
effect of current in a band of conductors between two masses of iron. This effect is shown in Figs. 257*a* and 257*b*. The small circles with dots represent conductors carrying up-flowing currents and the small circles with crosses represent conductors with down-flowing currents. *The action of the currents in these bands of conductors is to produce magnetic flux along the dotted lines in the directions indicated by the arrows.*

The lines *A'* and *B'* in Figs. 256 are clock-diagram lines to be thought of as rotating in a counter-clockwise direction ; and



up flowing currents

Fig. 257a.



down flowing currents

Fig. 257b.

their projections on the fixed line *ef* represent at each instant the values of the currents which flow in the *A* bands and *B* bands of conductors respectively.

At the instant represented in Fig. 256*a* the current in the *A* bands is at its maximum value, the current in the *B* bands is zero, and the magnetic flux due to the stator windings is at this instant represented by the dotted lines in Fig. 256*a*. Where the flux passes from stator iron into rotor iron, the stator iron has north magnetic polarity; and where the flux passes from rotor iron into stator iron, the stator iron has south magnetic polarity. The location of these polar areas at the given instant is indicated by the letters *NSNS*.

When the lines A' and B' have rotated one eighth of a revolution (one eighth of a cycle) as shown in Fig. 256*b*, the current in the *A* bands has decreased and the current in the *B* bands has increased, the two currents have become equal in value; and the magnetic flux due to the stator windings has shifted as indicated by the dotted lines in Fig. 256*b*.

When the lines A' and B' have rotated one quarter of a revolution (one quarter of a cycle) as shown in Fig. 256*c*, the current in the *A* bands has decreased to zero, the current in the *B* bands has increased to its maximum value, and the magnetic flux due to the stator windings has shifted still farther as indicated by the dotted lines in Fig. 256*c*.

As a matter of fact, the magnetic flux due to the stator windings shifts *continuously* forwards, and by comparing Fig. 256*a* with Fig. 256*c*, it may be seen that the forward shift of the polar regions during one quarter of a cycle is equal to one half the distance from *N* to *S* in Fig. 256*a*. In a whole cycle, therefore, the forward shift of the polar regions is two times the distance from *N* to *S*, which is equal to the distance from *N* to *N*. That is to say, the stator magnetism makes a complete revolution in $p/2$ cycles where p is the number of polar regions on the stator iron. Therefore if the alternating currents supplied to the stator windings have a frequency of f cycles per second, we have

$$n = \frac{2f}{p} \tag{67}$$

in which n is the speed of the stator magnetism in revolutions per second.

A clear idea of the action of the induction motor may be obtained with the least difficulty by considering an induction motor

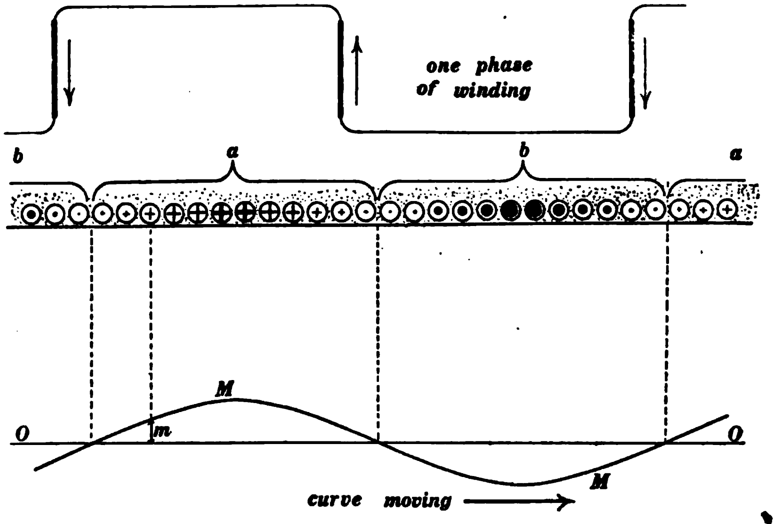


FIG. 258.

of which the stator is supplied with an indefinitely large number of phases of alternating current, each phase being of course supplied by a separate source. Fig. 258 shows a scheme for representing the flow of currents in the stator conductors of such a multiphase induction motor. The small circles with dots and crosses represent the stator conductors (end view), and one phase of the winding is indicated in the upper part of the figure.* Consider the sine curve MM, Figure 258, and imagine it to travel in the direction of the arrow along the axis OO. *The ordinate* m *of this curve at any point represents the value of the current at the given instant in the stator conductor which is directly above* m. Thus, the different sized crosses in the group a of stator conductors represent currents of different values flowing away from the reader; and the different sized dots in the group *b* of stator conductors represent currents of different values flowing towards the reader. The *group a* of stator conductors is a group in which currents flow away from the reader, the *group b* of stator conductors is a group in which currents flow towards the reader, and these two *groups* travel along with the curve *MM* as the curve moves to the right, although of course, the stator conductors themselves remain stationary.

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* Compare the upper part of Fig. 258 with Fig. 253 or 254 and it will be seen that a band of conductors belonging to one phase in Fig. 253 or 254 corresponds to a single conductor in Fig. 258.

Figure 259 represents portions of the stator and rotor of a multiphase induction motor, and the ordinates of the moving curve *MM* represent the values of the current in the stator conductors as in Fig. 258. By comparing Fig. 259 with Figs. 257 it will be seen that the groups *a* and *b* of stator conductors in Fig. 259 produce magnetic flux which crosses the gap space as indicated in Fig. 259, so that the regions *SNS*, etc., in Fig. 259 are regions of south polarity, north polarity, south polarity, respectively, of the stator iron, and these polar regions travel along with the moving curve *MM*. The flux density at any point in the gap space in Fig. 259 is represented by the ordinate *b* of the curve *BB*. The currents in the stator conductors which are represented by the dots and crosses in Fig. 259 constitute what are called the

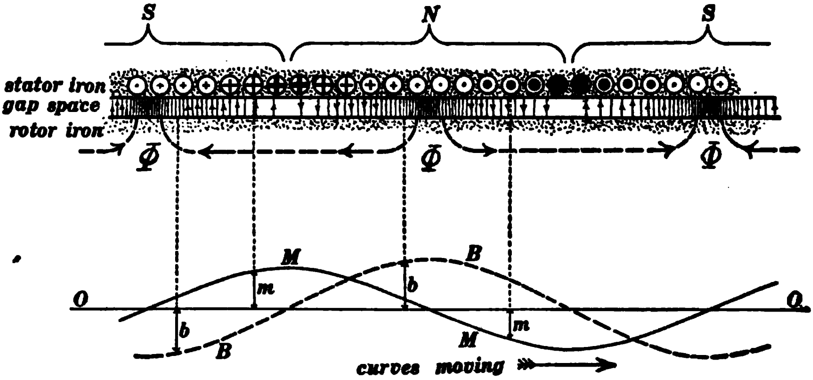

Fig. 259.

magnetizing currents of the motor inasmuch as their sole effect is to produce the moving state of magnetism which is represented by the moving curve *BB*.

Figure 260 shows both stator conductors and rotor conductors. The rotor is understood to travel at a slower speed than the magnetism *BB* of Fig. 259, and therefore the rotor conductors are being continually cut by the travelling flux so that electromotive forces are induced in the rotor conductors. These electromotive forces are largest in the middle of the polar regions *SNS*, etc., and they produce currents in the rotor conductors as represented by the dots and crosses in Fig. 260. The magnetizing action of these rotor currents is balanced by them agnetizing action of *additional currents* in the stator conductors. These additional currents in the stator conductors are called the *load currents* of the motor to distinguish them from the currents which are represented in Figs. 258 and 259 (magnetizing currents). The rotor currents in Fig. 260 are pushed sidewise by the magnetic flux in the gap space, thus developing mechanical power in the rotor.

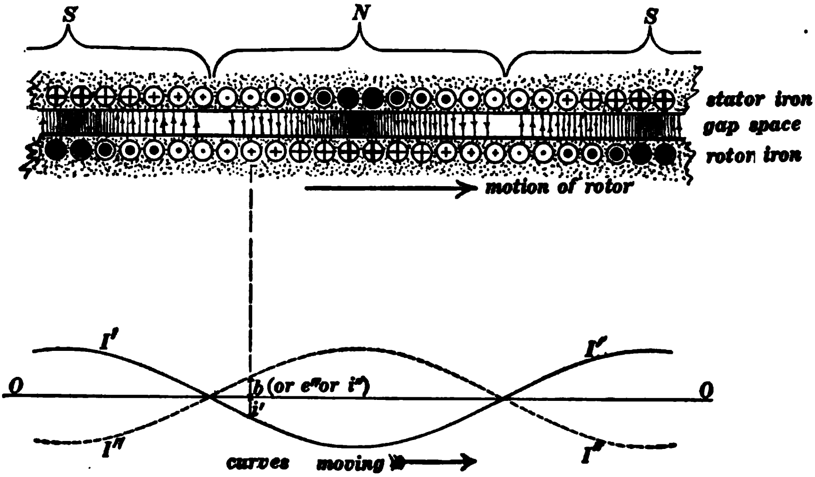

Fig. 260.

A careful study of Figs. 259 and 260 will enable one to understand the time-phase relations of all the electromotive forces and currents involved in the action of the induction motor, and it will also show the interesting space relations between the magnetizing currents and the load currents in the stator winding. Thus the magnetizing currents in the stator windings are at their maximum values in the regions between adjacent polar areas as shown in Fig. 259, whereas the load currents in the stator windings are at their maximum values in the regions in the middle of the polar areas as shown in Fig. 260. It may be seen furthermore, that the currents in the rotor conductors are at their maximum values in the regions in the middle of the polar areas where the flux density in the gap space has its maximum value.

171.[*] Relation between speed and torque of the polyphase induction motor.—When an induction motor is running at zero load, but little torque is required to drive it, and the rotor revolves at a speed but very little less than the speed of the stator magnetism (the speed of the stator magnetism is called the synchronous speed). The rotor cannot,[†] however, reach full synchronous speed, because at synchronous speed the rotor conductors would move along with the stator magnetism; there would be no electromotive force, and therefore no current induced in the rotor conductors; and consequently the stator magnetism could exert no driving force on the rotor conductors.

When an induction motor is running with a load, the driving torque must be large, and therefore the current in the rotor conductors must be large in order that the stator magnetism may exert the necessary driving force upon the rotor conductors; furthermore the induced electromotive forces in the rotor conductors must be sufficient to produce the necessary rotor currents, and consequently the rotor must run appreciably below synchronism in order that sufficient electromotive forces may be induced in the rotor conductors by the more rapidly moving stator magnetism.

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[*] The method of calculating the details of performance of an induction motor are given in Chapter XIII, Vol. II, of Franklin and Esty's *Elements of Electrical Engineering*.

[†] The effect of magnetic hysteresis in the rotor iron is to produce a certain slight torque the value of which is independent of the relative speed of stator magnetism and rotor, and this hysteresis torque may, and in some cases does, bring the rotor of an induction motor up to full synchronous speed when the motor is without load. See second footnote to Art. 178.

It is at once evident from the foregoing statement that the speed of the rotor at full load will be but little less than synchronous speed if the short-circuited rotor windings have a very low resistance, for, in this case, but little electromotive force need be induced in the rotor conductors to produce sufficient current to enable the stator magnetism to exert the necessary driving torque. If, however, the resistance of the short-circuited rotor windings is large, then the rotor speed will fall off greatly with increase of load. That is, low rotor resistance is necessary for good speed regulation of an induction motor.

If the magnetic flux Φ which enters the rotor from one of the polar regions of the stator were constant in value, that is, if it did not decrease in value with increase of load, then the difference between synchronous speed n and rotor speed n' would be exactly proportional to the torque to be exerted on the rotor, that is to say, the speed-torque curve of the induction motor would be a straight line *AB*, as shown in Fig. 261. This is

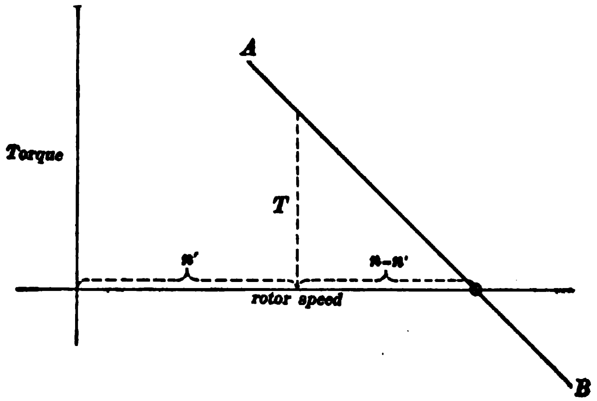

Fig. 261.

evident if we consider that to double the value of $n - n'$ would, if Φ were constant, double the induced electromotive forces in the rotor windings, this would double the rotor currents, and the doubled rotor currents would be acted upon by a doubled driving force.

As a matter of fact, however, the flux Φ decreases with increase of load because of the demagnetizing action of the rotor currents, and this decrease of Φ causes a two-fold decrease of torque inasmuch as (*a*) it causes a decrease of induced electromotive forces and therefore a decrease of rotor currents at a given rotor speed, and (*b*) these decreased rotor currents are acted upon by the reduced flux to produce a greatly reduced torque.* A true speed-torque curve of an induction motor is shown in Fig. 262. The torque is zero at synchronous speed ($n' = n$), and the torque increases, reaches a maximum, and then falls off as n' is decreased.

Break-down of the induction motor.—If the load on an induction motor is increased until the motor develops its maximum torque, then any further increase of load will cause the motor to

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* A good understanding of the actual speed-torque curve of the induction motor may be obtained on the basis of the arrangement shown in Fig. 255, an induction motor rotor acted upon by a revolving field magnet. The torque acting on the rotor is proportional to the product of the rotor current I and the flux Φ' which passes into the rotor from each field pole. Therefore we may write

$$T = k\Phi' I \qquad \text{(i)}$$

The total flux Φ emanating from a field pole is fixed in value, and the leakage flux $\Phi - \Phi'$ is proportional to I. Therefore we may write

$$\Phi - \Phi' = k' I$$

or

$$\Phi' = \Phi - k' I \qquad \text{(ii)}$$

The rotor current I is proportional to $(n - n')$ and to Φ'. Therefore we may write

$$I = k'' (n - n')\Phi' \qquad \text{(iii)}$$

Eliminating Φ' and I from equation (i) with the help of equations (ii) and (iii) and reducing, we have

$$T = \frac{a(n - n')}{1 + b(n - n')^2}$$

in which a and b are constants depending upon Φ, k, k' and k''. The curve corresponding to this equation is shown in Fig. 262.

come to a standstill. This phenomenon is called the *breaking down* of the motor.

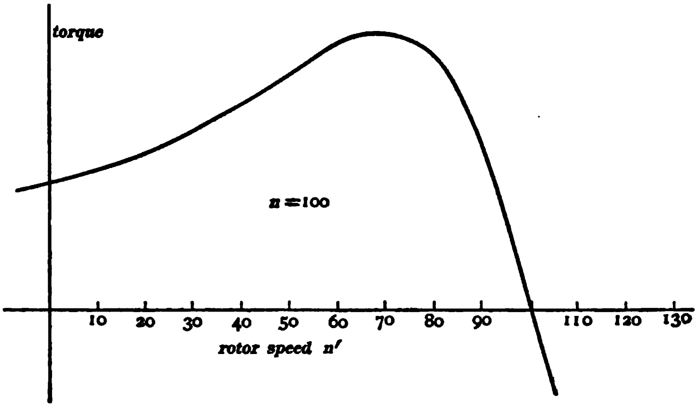

Fig. 262.

Use of a starting resistance in the rotor windings.—Curves 1, 2, 3, 4 and 5 in Fig. 263 show the relation between speed and torque of an induction motor whose rotor windings are short-circuited through an adjustable resistance; curve 1 shows the relation between speed and torque when the rotor resistance is very small, and curves 2, 3, 4 and 5 show the relations between speed and torque for greater and greater rotor resistance.* A certain value of rotor resistance gives the maximum torque at zero speed, *and therefore when an induction motor is being started it is desirable to increase the rotor resistance;* this is especially true if the motor starts under load (see discussion of Figs. 251 and 252).

* The exact change of the speed-torque curve of an induction motor due to an increase of rotor resistance is to increase in the same ratio the values of $(n - n')$ for the same values of torque. This is evident from the following considerations: Suppose the rotor resistance to be doubled, then, if the flux Φ' entering the rotor be unchanged, the electromotive force induced in the rotor windings will be doubled if $(n - n')$ is doubled and therefore the rotor current I would be unchanged. But if the rotor current is unchanged the value of Φ' is in fact unchanged and therefore the torque which is proportional to $\Phi' I$ is unchanged.

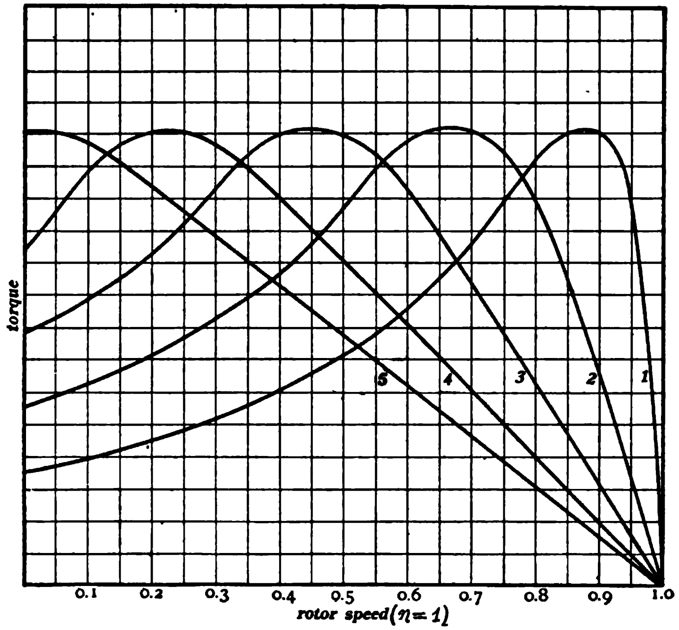

Fig. 263.

It may be seen from Fig. 263 that the speed at which an induction motor runs under given load (given torque) may be reduced at will by inserting resistance into the rotor windings (see Art. 175).

172. Relation between speed and efficiency of an induction motor. — Of the total power delivered to an induction motor, part is lost in heating the stator windings and in heating the stator iron on account of eddy currents and hysteresis, and the remainder is delivered to the rotor; this power which is delivered to the rotor is partly lost in heating the rotor windings, and the remainder * appears as mechanical power in driving the rotor; and this mechanical power which drives the rotor is partly lost in mechanical friction in the motor bearings and in air friction, and the remainder is delivered by the motor as its mechanical output. *Let us for the present ignore all of the losses of power except the power lost in heating the rotor windings.*

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* Eddy current and hysteresis losses in the rotor iron are generally quite small and these losses are here ignored.

Under this assumption all of the power delivered to the stator windings is to be thought of as being used to produce the rotating stator magnetism, exactly as if this delivered power were mechanical power used to revolve an ordinary field magnet as indicated in Fig. 255. Let T be the torque which this revolving field magnet exerts on the rotor, then of course the rotor reacts and opposes the rotation of the field magnet with the same torque T. The power required to drive the field magnet at speed n against this opposing torque T is equal to $2\pi nT$,* and the power developed by the torque T in driving the rotor at speed n' is equal to $2\pi n'T$. That is, the power input is $2\pi nT$ and the power output is $2\pi n'T$, so that (ignoring the various losses above specified, losses which are, however, by no means really negligible) the efficiency of the motor, output divided by input, is n'/n.

Efficiency and rotor resistance. — According to Art. 171 an induction motor with a given load runs the more nearly at synchronous speed the less the resistance of the short-circuited rotor windings. Therefore the efficiency of the motor, which is approximately equal to n'/n, is the greater the less the resistance of the short-circuited rotor windings; and therefore, when provision is made for having a large rotor resistance at starting, as explained in Art. 171, it is very desirable to arrange for the cutting out of this resistance when the rotor reaches nearly full speed, in order to give high efficiency, and also to lessen the tendency for the motor to change its speed with change of load.

Relation between the mechanical power developed in turning the rotor and the electrical power developed in the rotor windings. — The power $2\pi nT$ required to drive the "revolving field magnet" against the opposing torque T, as above explained, is all delivered to the rotor, but the mechanical power developed in the rotor is $2\pi n'T$ (less than $2\pi nT$), and the remainder, $2\pi nT - 2\pi n'T$, is the electrical power developed in the rotor windings. Therefore, ignoring the losses above specified, the power P delivered to the stator of an induction motor, the mechanical power P_m developed in the rotor, and the electrical power P_e developed in the rotor windings are proportional to n, n' and $n - n'$ respectively, that is,

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* If T is expressed in pound-feet and n in revolutions per second, then $2\pi nT$ expresses the power in foot-pounds per second.

$$P : P_m : P_e \text{ as } n : n' : n - n'$$

Analogy between the induction motor and the friction clutch. — A friction clutch, the two parts of which continue to slip on each other so that the driving part A runs continuously at speed n and the driven part B runs continuously at a slower speed n', is analogous in some respects to the induction motor. If we ignore the friction losses in the two bearings at either side of the clutch, the total power required to drive the part A is $2\pi nT$, and the mechanical power developed in part B is $2\pi n'T$, where T is the torque with which the driving part A drags the driven part B; and the difference, $2\pi nT - 2\pi n'T$ is the energy developed as heat where the slipping occurs. The efficiency of transmission of such a clutch would evidently be equal to n'/n.

173. Rating of induction motors. — An induction motor is always rated to carry a specified load in horse-power when supplied with a specified number of phases of alternating current at a specified frequency and a specified voltage per phase, and in addition, either the speed or the number of poles is given. Thus, one would speak of a ten horse-power, two-phase, four-pole, 60-cycle, 220-volt induction motor.

To overload an induction motor, frequency and voltage of supply being normal, causes the motor to take excessive current at a reduced power factor, and the motor runs at a slightly reduced speed. Excessive overload causes the motor to break down as described in Art. 171.

An induction motor may be operated quite satisfactorily at a frequency or voltage considerably different from normal. A change of frequency produces a proportional change of speed; and a change either of voltage or frequency causes a change in the value of the magnetic flux Φ which emanates from each polar region of the stator iron, a change in the magnetizing (zero-load) current, and a change in the eddy current and hysteresis losses, which losses occur chiefly in the stator iron.*

The behavior of an induction motor when it is operated by an abnormally high or low voltage is especially important. In fact the torque that is developed by an induction motor at a given relative speed $n - n'$ of stator magnetism and rotor is approximately proportional to the square of the voltage. This is evident when we consider that to increase the voltage in the ratio of 1 to k causes the same ratio of increase of flux Φ, of rotor electromotive forces, and of rotor currents (for given value of $n - n'$); but, since the torque is proportional to the product of flux and rotor currents, it is evident that the torque is increased in the ratio of 1 to k^2.

174. The starting of the induction motor. — (*a*) *Small induction motors* with squirrel-cage rotors are nearly always started by connecting their stator windings directly to the polyphase supply mains.

(*b*) *Large induction motors* with wire wound rotors are usually started by inserting resistance in the rotor windings, as explained in Art. 169 (Fig. 251). The advantages of using a high resistance in the rotor windings at starting are (*a*) that the starting torque is thereby increased and (*b*) the stator currents at starting are reduced in value.

(*c*) *The starting compensator* consists of two or three autotransformers (two for two-phase supply and three for three-phase supply) with switches arranged for conveniently supplying the motor with low voltage polyphase currents at starting, which voltage is increased in steps as the motor speeds up, until finally the full supply-voltage is connected to the motor. The objection to the use of the starting compensator is that the starting torque is very small when the voltage is small; indeed, the starting compensator can scarcely be used in the case of a motor which is to be started under load. The advantage of the starting compensator is that the motor takes very much smaller currents from the supply mains at starting than it would if connected directly to the supply mains.

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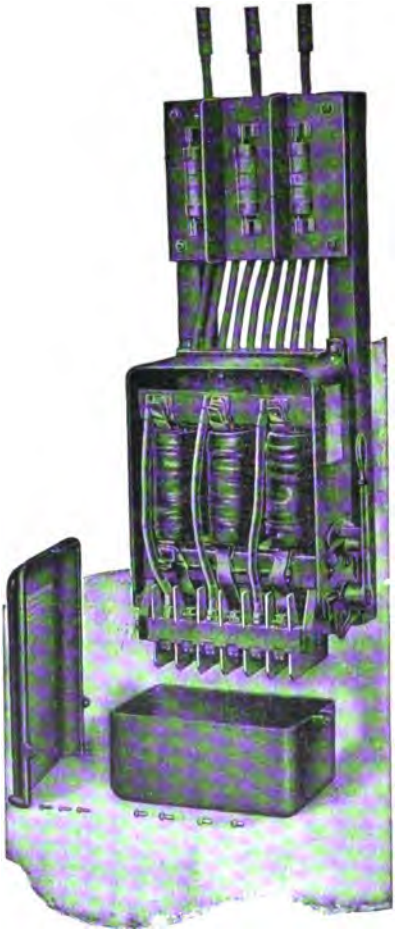
* See G. B. Werner, On the effect of voltage and frequency variations on induction motor performance, *Electric Journal*, Vol. III, page 400, July, 1906.



Three-phase Starting Compensator.
Fig. 264.

The starting compensator is nearly always used when it is desired to start a polyphase alternator or a polyphase synchronous converter by inherent induction motor action, as explained in Art. 178.

Figure 264 shows the interior of a three-phase starting compensator in the lower part of which is a cylinder switch for making the successive connections above mentioned. Figure 265 is a diagram of connections of a three-phase compensator in which one intermediate voltage is provided for. To start the motor, switch *S'* is closed, and then switch *S* is closed on the starting side, thus connecting the low voltage taps of the compensator to the motor. When the motor reaches a moderate speed, switch *S* is suddenly thrown to the running side, thus connecting the full supply voltage to the motor, and then switch *S'* is opened, leaving the compensator entirely disconnected. The trip coils actuate a trigger which opens a circuit breaker if the current exceeds a prescribed value.

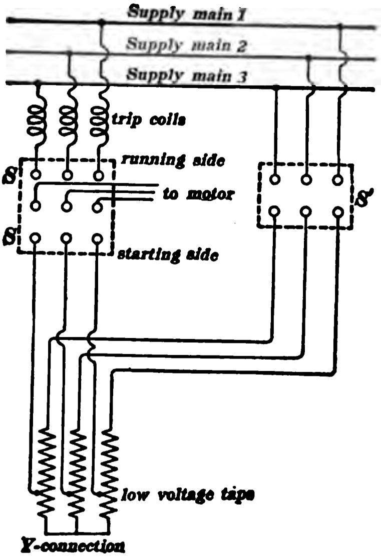


Fig. 265.

175. Speed control of the induction motor. — The speed *n* of the stator magnetism of an induction motor is fixed by the frequency *f* of the alternating-current supply and by the arrangement of the stator windings (number of polar regions on stator). Therefore the zero-load speed of an induction motor, which is of course equal to the speed of the stator magnetism, is invariable, unless the frequency of supply or the arrangement of the stator windings is changed.

On the other hand, the rotor speed *n'* of an induction motor with given load may have any value between zero and normal full speed if the rotor resistance is increased to the proper value.

There are therefore two feasible methods for controlling the speed of an induction motor which is supplied with alternating currents at a given frequency, namely, (*a*) to arrange the stator windings so that they may be changed quickly from, say, a four-pole winding to an eight-pole winding by means of switches, and (*b*) to bring out the rotor windings to collector rings, as shown in Fig. 252, and short-circuit the rotor through external adjustable resistances.

The difficulty with the first method of speed control is that the arrangement of the stator windings becomes very complicated if more than two speeds are provided for; and the difficulties with the second method of speed control are (*a*) that the motor is very inefficient at low speeds, and (*b*) that the speed varies greatly with changes of load when the rotor resistance is great enough to give a low speed with a given load.

176. Use of the induction motor as a frequency changer. — If we consider that the effect of the stator windings of an induction motor is to produce a rotating state of magnetism which acts upon the rotor exactly as would a mechanically revolving field magnet, it is evident that the electromotive forces in the rotor windings are zero in value and zero in frequency when the rotor speed is equal to the speed of the stator magnetism, that the rotor electromotive forces increase in value and in frequency as the rotor speed decreases, that when the rotor stands still the frequency of the rotor electromotive forces is the same as the frequency of the alternating currents supplied to the stator windings, and that the rotor electromotive forces are still further increased both in value and in frequency if the rotor is driven backwards by an external source of power.

Under the ordinary operating conditions of an induction motor the rotor speed n' is nearly equal to the speed n of the stator magnetism. In this case the rotor electromotive forces are small, their frequency is very low, and the amount of electrical power developed in the rotor windings is but a small fraction $[(n - n')/n']$ of the mechanical power developed in the rotor. When, however, n and n' are very different, that is, when the relative speed $n - n'$ of stator magnetism and rotor is large, then the rotor electromotive forces are large, and if the rotor windings are brought out to collecting rings as shown in Fig. 252 the rotor may deliver a very considerable amount of electrical power to external receiving circuits at any desired frequency. When an induction motor is used in this way it is called a *frequency changer.* As usually employed the frequency changer is used to change a low frequency alternating-current supply, say 25 cycles per second, to a higher frequency, say 60 cycles per second for operating electric lamps. When so used the rotor of the frequency changer is driven backwards by a synchronous motor or by an induction motor taking power from the low frequency supply mains.

When the rotor runs backwards the relative speed of stator magnetism and rotor is $n + n'$ and the frequency of the rotor electromotive forces is $(n + n')/n$ times the primary frequency.

In order to determine the relation between the electrical power delivered to the stator of the frequency changer and the mechanical power required to drive its rotor backwards at speed n', let us consider the torque action T between stator and rotor, and let us for a moment think of the stator as a mechanically revolved field magnet. Then, ignoring losses, $2\pi nT$ is the power supplied electrically to the stator, $2\pi n'T$ is the power supplied mechanically to the rotor in driving it backwards at speed n', and $2\pi(n + n')T$ is the electrical power developed in the rotor windings.

Example. — A four-pole, three-phase induction motor has a Δ-connected stator winding supplied from 25-cycle, 220-volt, three-phase mains. The rotor winding is also three-phase, it is Δ-connected to the collector rings, and it has the same number of conductors as the stator winding, so that when the rotor is standing still the voltage between the collector rings is 220 volts. The speed of the stator magnetism is 12½ revolutions per second, the rotor is driven backwards at a speed of 17½ revolutions per second; and, therefore, the rotor frequency is 60 cycles per second, and the voltage between collector rings is 528 volts $[= (n + n')/n \times 220$ volts$]$. Suppose that the rotor delivers a total electrical output of 100 kilowatts, then, since electrical output of rotor : electrical intake of stator : mechanical intake of rotor as $n + n' : n : n'$, it follows that 41⅔ kilowatts of power is delivered directly to the stator windings from the 25-cycle mains, and that 58⅓ kilowatts of mechanical power is delivered to the rotor by the motor which drives it.

177. The induction generator. — When the rotor of an induction motor is driven at a speed n' which is greater than the speed n

of the stator magnetism, a torque T is developed which opposes the motion of the rotor, so that mechanical power amounting to $2\pi n' T$ must be used to drive the rotor. A portion of this mechanical power appears as electrical power developed in the rotor windings, and the remainder $2\pi n T$ is delivered to the stator, where, ignoring losses, it is delivered *to* the alternating-current supply mains in the form of alternating-current output from the stator windings. When used in this way the induction motor is called an *induction generator*. To operate as a generator the induction motor must be connected in parallel with an ordinary alternator.

178. The starting of the polyphase synchronous motor (or converter) by inherent induction motor action.—The windings of a polyphase alternator armature are like the stator windings of an induction motor, and the polyphase current in such an armature produces a rotating state of magnetism in the armature core exactly as the stator windings of an induction motor produce a rotating state of magnetism in the stator iron.

When an alternator is in operation either as a generator or as a synchronous motor or converter, the magnetism due to the armature currents rotates with reference to the armature structure in a direction opposite to that in which the armature rotates,* and at the same speed as the armature. Therefore the armature magnetism due to armature currents is stationary in space (see Arts. 128 and 129).

If polyphase currents are delivered to the armature of a polyphase alternator or synchronous converter at stand-still, the rotating magnetism produced in the armature core acts upon the field structure in the same way that the rotating stator magnetism of an induction motor acts upon the rotor. The torque action between the armature and field of the alternator under the above conditions sets the armature into motion, the armature increases in speed until it is nearly in synchronism, and the slight torque due to hysteresis * causes the speed to come up to full synchronism.

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*This statement is made to refer to the machine with stationary field and rotating armature for the sake of simplicity.

This induction-motor action is frequently used in the starting of the polyphase synchronous converter. The polyphase supply voltages are reduced to a fraction of their normal value by means of a starting compensator (see Art. 174), and connected to the armature terminals of the polyphase machine. The armature, being entirely free to rotate, speeds up and reaches full synchronous speed as above explained, and the field switch of the machine is then closed. When the field magnetism is built up to the limit that is possible with the reduced alternating-voltage supply,† the supply voltage (alternating) is increased to the full value in one or more steps. The machine is then in full operation.

The rotating armature magnetism produces a pulsating flux through the field coils of a polyphase machine, which is being started in the manner here described, and this pulsating flux induces a very large electromotive force in the many turns of wire which constitute the field winding. To avoid the rupture of the insulation of the field winding by this large electromotive force, a multi-pole switch is arranged to break the field circuit at a number of points so as to avoid the accumulation of the total induced electromotive force at the terminals of the complete winding where it would endanger the insulation.

179. The single-phase induction motor.—The polyphase induction motor, once in operation, continues to run and carry a moderate load when all of the stator windings (phases) but one are disconnected from the supply mains. When so operated, the induction motor is called a *single-phase induction motor.* The curve *A*, Fig. 266, shows the relation between torque and speed of a two-phase induction motor with both phases connected to the supply mains, and curve *B* shows the relation between torque and speed of the same motor with but one phase connected to the supply mains. This figure shows that when driven by single-phase current (at normal voltage), the motor runs at a slightly lower speed to develop a given torque, it shows that the maximum or break-down torque occurs at a higher speed and is less in value than the break-down torque of the same motor when driven by two-phase currents, and it shows that the single-phase induction motor develops zero torque at stand-still. In fact, the single-phase motor must be started either as a polyphase induction motor or by other means as described in the next article.*

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* In the present case the field magnet of alternator has polar projections and a momentary torque of considerable value exists when the polar areas on the armature are passing out from under the polar projections of the field. This momentary torque jerks the armature into exact synchronous speed when ordinary induction motor action has brought it nearly to synchronous speed.

† Any tendency of the converter to build up to its full degree of field magnetization is counteracted by the very heavy leading currents which its armature takes from the low voltage alternating-current supply and as the value of the alternating supply voltages is increased, these leading currents decrease and allow the field magnetism to build up to its full value.

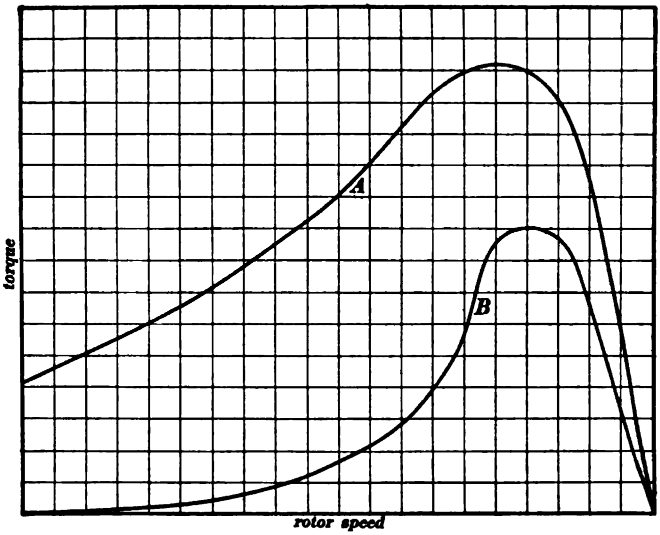

Fig. 266.

180. Starting of the single-phase induction motor. (*a*) *Hand starting.*—Small single-phase motors are sometimes intended to be started by closing the supply switch and giving a pull on the belt which connects the motor to the machine which it is intended to drive.

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* The action of the single-phase induction motor is described in Chapter XIII, Vol. II, Franklin and Esty's *Elements of Electrical Engineering.*

(*b*) *Split-phase starting.* — Small single-phase induction motors are sometimes started as polyphase induction motors as follows: The stator of the motor is provided with a two-phase winding, and these two phases are connected in parallel to the single-phase supply mains with a resistance or a condenser in series with one of the phases. Under these conditions the currents in the two stator windings are out of phase with each other sufficiently to produce the desired starting torque, and when the machine reaches moderate speed, one of the stator windings is disconnected, and the machine continues to operate as a single-phase motor. The split-phase method of starting is incorporated in many single-phase alternating-current fan-motors by means of a device called a *shading coil.* Figure 267 is an end view of such a single-phase fan-motor. The polar projections of the stator iron are wound with coils of wire which are connected in series and receive current from the single-phase supply mains, and one tip of each pole piece is surrounded by a short-circuited coil of wire or frame of copper, as indicated in the figure. This coil, or

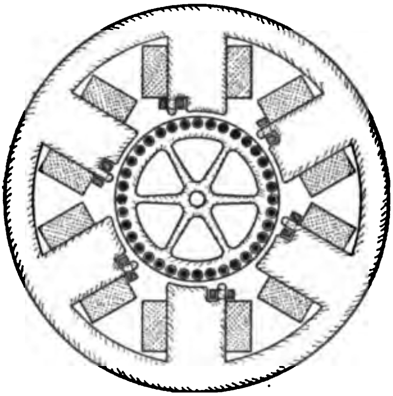

Fig. 267.

copper frame, is called a *shading coil* and it causes a phase difference between the pulsating flux that emanates from the main portion of each polar projection and the pulsating flux which emanates from the pole tip, thus introducing a two-phase action on the rotor which is sufficiently pronounced to start the motor.

(*c*) *Repulsion motor starting.* — The best method of starting a single-phase induction motor is to design the motor so that it can be operated at starting as a repulsion motor and be quickly converted into an induction motor after it reaches approximately full speed. The single-phase induction motor of the Wagner Electric Company operates in this way. It is described in Chapter XVI.

CHAPTER XVI.

THE SINGLE-PHASE SERIES MOTOR.

181. Behavior of a direct-current motor when the supply voltage is suddenly reversed. — Any direct-current motor will continue to run in an unchanged direction when the supply voltage is reversed, because the reversal of the supply voltage reverses both the armature current and the field current. Therefore any direct-current motor might be operated by a repeatedly reversed supply voltage, that is, by an alternating supply voltage, if it were not for certain effects which are produced at the moment of voltage reversal.

In the case of a shunt motor a reversal of the supply voltage would be followed by a slow dying away of the previously existing field current and a slow growth of a reversed field current, because of the very large inductance of the field winding; whereas the current in the armature would quickly drop to zero and an excessive momentary flow of reversed current would take place. This difference in the behavior of the field current and armature current of a shunt motor when the supply voltage is suddenly reversed would show itself as a very considerable phase difference between field current (field magnetism) and armature current if the attempt were made to operate a shunt motor from alternating-current supply mains, and the motor would develop but little torque. The shunt motor cannot be operated satisfactorily from alternating-current supply mains.

In the case of a series motor a reversal of the supply voltage would not be followed by an excessive flow of current, because the armature and field windings are in series; and there can be no phase difference between armature current and field magnetism of a series motor which is operated from alternating-current supply mains.

Sparking. — The most serious difficulty that is encountered when the attempt is made to operate an ordinary series motor by alternating current is the tendency to excessive sparking at the brushes. Whenever a brush touches two commutator bars, an armature coil or section is short-circuited; at the moment of this short-circuit the armature coil surrounds a field pole * as shown by the heavy line in Fig. 268, so that the armature coil is related to the field winding as the short-circuited secondary coil of a transformer of which the primary coil is the field winding. Therefore an excessive short-circuit current is produced through the armature coil and through the tip of the brush; and the result is an increased heating of the armature coils, commutator, and brushes, and an excessive sparking at the brush tips as the commutator bars pass from under the brushes.

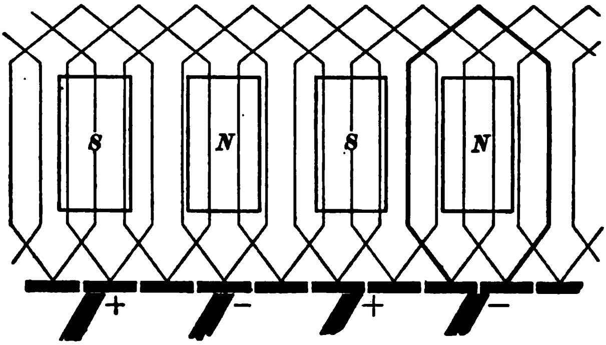

Fig. 268.

Low power factor. — A less serious difficulty in the operation of a series motor by alternating current is that the inductance of the field and armature windings cause the current to lag behind the supply voltage in phase, so that the power factor of the series motor when driven by alternating current is always considerably less than unity.

The single-phase series motor. — The ordinary series motor when designed so as to operate satisfactorily with alternating current is called the *single-phase commutator motor*, or the *single-phase series motor*. The peculiar features of design and the special devices for reducing sparking and for increasing the power factor of such a motor are described in a subsequent article. Every part of the magnetic circuit of a single-phase series motor must be made of laminated iron.

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*This particular statement applies to the lap drum winding, but the effect referred to exists in every variety of armature winding.

182. The clock diagram of the single-phase series motor. — Let the line OI, Fig. 269, represent the current flowing through the field winding and armature of a series motor, let X_f be the reactance of the field winding, X_a the reactance of the armature winding, R_f the resistance of the field winding, and R_a the resistance of the armature winding. The components of the electromotive force F which is required to force the current I through the field winding are X_fI and R_fI, as shown in Fig. 269, and the components of the electromotive force A which is required to overcome the reactance and resistance of the armature winding are X_aI and R_aI as shown.

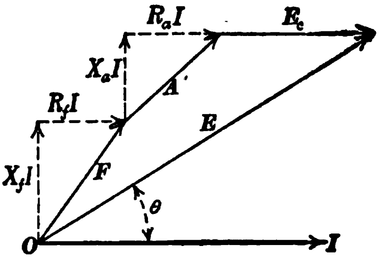

Fig. 269.

The effect of the current I in flowing through the field winding is to produce an alternating flux Φ through the armature. This flux is in phase with the current and it induces in the armature windings an electromotive force E_c which is equal to $\Phi Z'n$, where Φ is the flux entering the armature from one field pole, Z' is equal to $pZ/p' \div 10^8$, and n is the speed of the armature in revolutions per second; p is the number of field poles, Z is the total number of armature conductors, and p' is the number of current paths between positive and negative brushes. Since Φ is an alternating flux which is in phase with the field current I, it is evident that E_c (equals $\Phi Z'n$) is an alternating electromotive force of the same frequency and in phase with I. Therefore since E_c is a counter electromotive force, the portion of the supply voltage which overcomes it is equal to it and is represented by the line E_c in Fig. 269, and the total supply voltage is represented by the line OE which is equal to the vector sum of F, A and E_c.

The power lost in the field winding is R_fI^2, the power lost in the armature is R_aI^2, and the power which is used to force the current against the counter electromotive force E_c, namely, E_cI, appears as mechanical power. Eddy current and hysteresis losses are ignored in Fig. 269.

Design of single-phase series motor to give large power factor. — It is evident from Fig. 269 that a large power factor (small value of the angle θ) depends upon a large value of E_c in comparison with X_fI and X_aI; of course R_fI and R_aI should both be as small as possible because they represent energy losses in the windings.

(*a*) In the first place the reactance X_a of the armature winding may be reduced nearly to zero by the use of the neutralizing winding as explained in the following article. A neutralizing winding is therefore an important feature of the single-phase series motor.

(*b*) In the second place, it is to be noted that the reactance voltage X_fI which is lost in the field winding is proportional to the product of the number of field turns and the maximum value of the alternating field flux Φ, exactly as in the case of the primary coil of a transformer, and we may assume that we have a motor in which a specified maximum flux is to be produced, then to make the reactance voltage X_fI a minimum, the machine must be designed so that the desired flux may be produced by the least possible number of field turns. That is to say, the reluctance of the magnetic circuit must be small, or in other words the gap space between the pole faces and the armature core must be short and the iron of the field magnet and armature core must be operated at a low degree of magnetic saturation. Of course there is no clearly defined value of magnetic saturation which gives the best results, and all that can be said in regard to this second point is that low degree of magnetic saturation and short air gaps are important features of the single-phase series motor.

(*c*) In the third place the value of E_e at a given speed may be made fairly large, even though the value of Φ be rather small, by increasing the number of armature conductors. Therefore the use of a large amount of armature copper is an important feature of the single-phase series motor.

(*d*) In the fourth place, suppose that we have a single-phase series motor running at given speed with *given value* (*effective*) *of current* and given value (effective or maximum) of field flux. Under these conditions, the counter electromotive force is definite in value (effective), the mechanical power E_eI is definite, and a reduction of the frequency of the supply voltage would involve a proportional reduction of the reactance voltages X_fI and X_aI without affecting the value of E_e; that is, a reduction of frequency would increase the power factor of the motor. Therefore it is important to use a low frequency alternating current for a single-phase series motor. A low frequency will be found to be desirable in order to reduce the tendency to spark at the brushes.

183. The neutralizing of armature inductance. — A winding of wire on an iron core has considerable inductance, that is, a current in the winding magnetizes the iron core and causes a considerable amount of magnetic flux through the core. The inductance of such a winding may be annulled by anything which counteracts the magnetizing action of the winding on the iron core. Thus, Fig. 270 shows a layer of wire *WW* wound on an iron rod, the magnetic flux due to the current in the winding is indicated by fine lines; and Fig. 271 shows a second layer of wire *aa* wound over *WW* and so connected in series with *WW* that the current flows in opposite directions around the rod in *aa* and in *WW*, as indicated by the dots and crosses. The magnetizing action of the winding *WW* is neutralized by the opposite magnetizing action of the auxiliary winding *aa*, the only magnetic flux produced is the very small amount of flux between *W* and *a*, and therefore the inductance of *WW* and *aa* combined is nearly zero.

If an alternating current is made to flow through the winding *WW*, Fig. 271, and if the auxiliary winding *aa* is short-circuited, then the magnetizing action of the current in *WIW* will be very nearly neutralized by the current which is induced in the short-circuited winding *aa*; in this case the number of turns in the auxiliary coil *aa* need not be the same as the number of turns of wire in the winding *WW*.

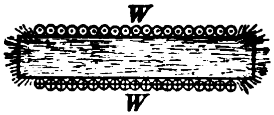

Fig. 270.

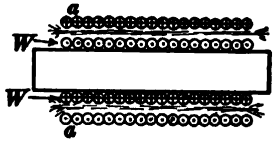

Fig. 271.

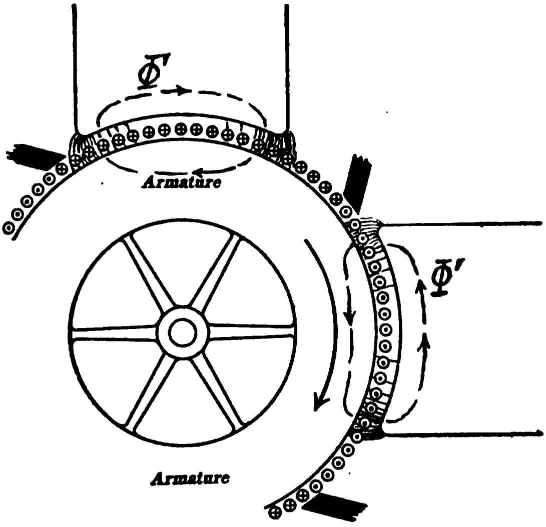

Fig. 272.

The auxiliary winding *aa*, Fig. 271, which is used to annul the inductance of the given winding *W W* is called a ***neutralizing*** or *compensating winding*. When the auxiliary winding is connected in series with the given winding, we have what is called *forced neutralization* or *forced compensation*, and when the auxiliary winding is short-circuited we have what is called *inductive* or *transformer neutralization*. Forced compensation can be used both for direct current and for alternating current, whereas transformer compensation can be used only for alternating current.

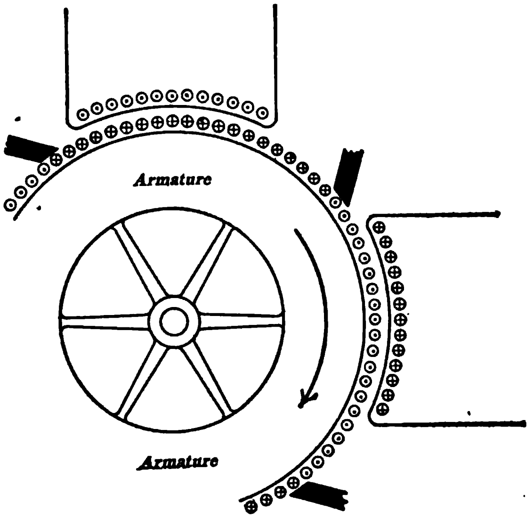

Fig. 273.

The magnetizing action of the armature current of a direct-current motor or of a single-phase series motor produces magnetic flux, as shown in Fig. 272, the inductance of the armature winding is due to this flux, and this inductance may be annulled by a neutralizing winding placed in slots in the pole faces, as shown in Fig. 273.* This neutralizing winding is connected, as shown in Fig. 274, so that the current flows in one direction in all of the conductors which lie under north poles and in the opposite direction in all of the conductors which lie under south poles as indicated by the dots and crosses in Fig. 273 and by the arrows in Fig. 274. In the single-phase series motor this neutralizing winding may be connected in series with the armature

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* See Art. 95.

(forced compensation), or it may be short-circuited by connecting the terminals *t* and *t'*, Fig. 274, together (transformer compensation).

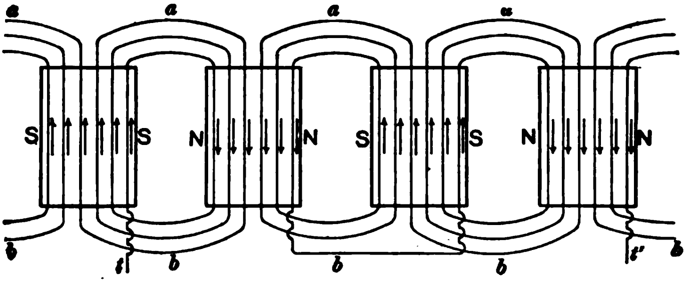

Fig. 274

Figures 275 and 276 show the field member of a four-pole single-phase series motor of the Westinghouse Electric and Manufacturing Company. The field windings and the neutralizing windings are both shown in position in Fig. 275, and in Fig. 276 the field coils are removed so as to expose the neutralizing windings to view. The neutralizing windings are placed in slots in the laminated pole pieces, and the portions *aaaa* and *bbbb*,

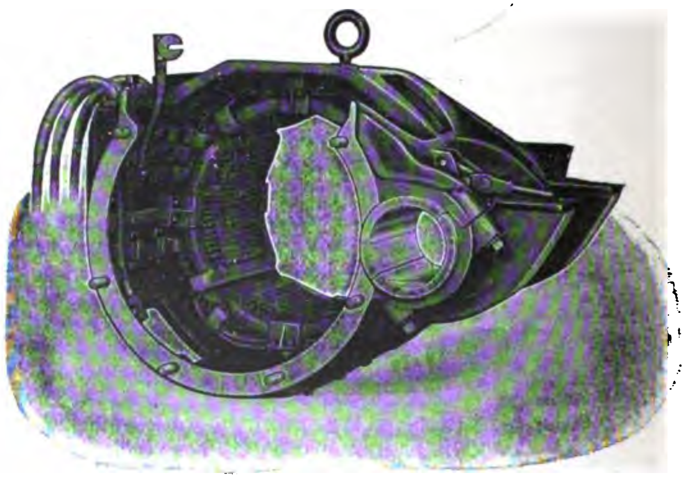

Fig. 275.

Fig. 274, of the neutralized windings are bent back in Figs. 275 and 276 to make room for the ends of the field coils.

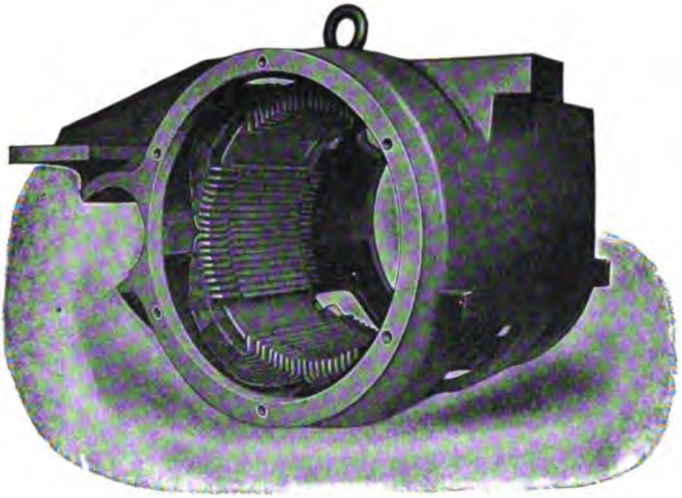

Fig. 276.

184. Prevention of sparking in the single-phase series motor. — A number of undesirable effects are included under the general term of sparking, as follows: (*a*) The effects that are presented in the direct-current dynamo* and which are due to the necessity of a sudden reversal of the current in a given armature section as the terminal bars (commutator bars) of the section pass under a brush; and (*b*) the effects of the excessive current produced in each short-circuited section by the electromotive force which is caused by the pulsations of the field flux. The effects (*a*) need not be considered because they are less serious than effects (*b*), and because anything which obviates the great difficulties due to effects (*b*) will likewise obviate the lesser difficulties due to effects (*a*).

The effects of excessive short-circuit currents due to pulsations of the field flux are, to heat the armature winding, to heat the commutator, to heat the brushes, and to roughen the commutator by sparking, and these short-circuit currents represent a considerable loss of power. The cure for these undesirable effects is, of course, to prevent excessive short-circuit currents and to shorten their duration as much as possible.

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* See Arts. 95, 96 and 97.

The electromotive force induced in a short-circuited armature section by the pulsating field flux is proportional to the number of turns of wire in the section, proportional to the maximum value reached by the pulsating field flux, and proportional to the frequency of the pulsating flux. Therefore the short-circuited current may be reduced by reducing the number of turns of wire per armature section (the resistance of the short circuit not being reduced in proportion to the turns because of the relatively large resistance at the brush contacts), by reducing the maximum value of the pulsating flux, and by reducing the frequency. It is essential, therefore, in the design of the single-phase series motor to provide for few turns of wire per armature section (many commutator bars), to provide for small field flux per pole (many field poles), and to use alternating current of low frequency.

The duration of a short circuit of an armature section by a brush is proportional to the thickness of the brush, and therefore it is desirable to use thin brushes. If the armature winding is of the simplex type, the necessary thickness of the brushes is determined solely by the necessary area of brush contact to collect the given current. If the armature winding is of the multiplex type, the brush must never touch fewer than n commutator bars, where n is the number of constituent windings in the multiplex winding. In the multiplex winding, however, adjacent commutator bars do not form terminals of an armature section, and therefore the touching of two bars by a brush does not short-circuit the armature section of a multiplex winding; in fact the brush must touch $n + 1$ commutator bars of an n-plex winding to short-circuit an armature section.* Therefore thick brushes may be used with multiplex armature windings.

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* Short circuits are produced in a multiplex winding by the simultaneous connecting of two of the constituent windings by two brushes. Some of these short circuits pass through one constituent winding from a positive to a negative brush and back to

Special devices for preventing sparking. — The foregoing discussion refers to those general features of design which tend to reduce short-circuit currents in the armature sections of single-phase series motors. It does not seem to be possible, however, to produce a single-phase series motor which will operate satisfactorily without special devices for preventing excessive short-circuit currents. The simplest device for this purpose is the insertion of a moderate amount of resistance in each commutator lead, and a number of special inductive devices have been proposed, of which, perhaps, the simplest is the balanced choke-coil arrangement of Mr. S. S. Seyfert.

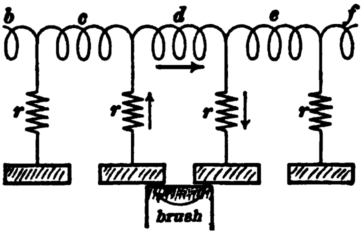

Fig. 277.

The resistance leads. — The coils *bcdef*, Fig. 277, represent the sections of an armature winding, and *rrrr* represent resistances inserted in the commutator leads. The figure shows the armature section *d* short-circuited by a brush, and the heavy arrow represents the electromotive force induced in the section *d* by the pulsating field flux. The short-circuit current must flow through two of the resistances *r* in series, as shown by the short arrows, whereas the useful current which enters or leaves the armature through the brush must flow through two of the resistances *r* in parallel. When the brush touches but one commutator bar there is no short circuit, and the whole of the useful current must flow through one of the resistances *r*. The resistances *r* are entirely idle except at the moments when the bars to which they are connected pass under a brush.

Resistance leads are objectionable because they involve a loss

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the positive brush through another constituent winding; some of them pass through one constituent winding from one positive (or negative) brush set to another positive (or negative) brush set and back through another constituent winding, and some of them pass through the winding from one positive (or negative) brush set to another positive (or negative) brush set and back through the brush leads.

of power, and because of the difficulty of finding room for them in a compact armature structure. While a motor is in actual operation the resistance leads come into service in rapid succession, and the heat is distributed among all of the leads; at starting, however, there is a possibility of the motor being stuck fast, so that a given resistance lead may be in circuit for an indefinite time. In view of this possibility it is necessary to make the resistance leads of large current-carrying capacity, and to place the leads so that they can be easily repaired in case of their being burned out. The resistance leads are usually made of heavy strips of German silver thoroughly insulated and placed in the bottoms of the armature slots.

Balanced choke coils. — The essential features of the balanced choke coil arrangement for preventing sparking may be seen in Fig. 278, in which *c* is a section of the armature winding, *ss* are two commutator bars and *tt′* are two choke coils through which the commutator leads pass. The two coils *tt′* are wound on a small iron core, and they are so connected that their magnetizing actions on the iron core are equal and opposite when equal currents flows into (or out of) the armature winding through both coils; whereas their magnetizing actions work together when current tends to flow out of the armature winding through one coil and into the armature winding through the other coil. The consequence of this arrangement is that current can flow into or out of the armature through both of the coils *tt′* without being choked, whereas opposite currents in the coils *t* and *t′* (the short-circuit current in the section) are greatly choked.

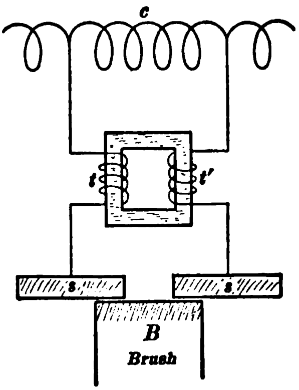

Fig. 278.

The complete arrangement of the balanced choke coils is shown in Fig. 279; each commutator lead is split into two branches, and each branch contains a choke coil which is balanced against a choke coil in an adjacent commutator lead. With this arrangement a brush must never touch less than two commutator bars, because of the excessive choking of useful current when it has to enter the armature through the two choke coils in one lead; but if the brush never touches less than two bars, it must at certain instants touch three bars, and when this occurs, a short circuit is established which is unchoked except by the combined * ohmic resistance of four of the choke coils in series.

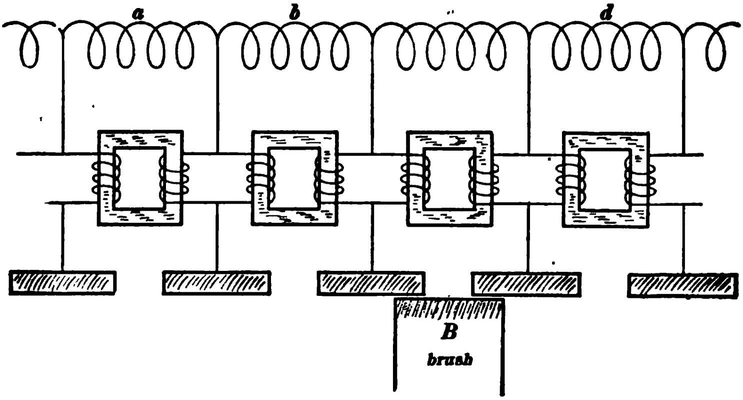

Fig. 279.

185. Characteristics of the single-phase series motor. — The single-phase series motor is very much like the direct-current series motor in its speed and torque characteristics. Thus, the curves shown in Figs. 104, 110 and 111 represent approximately the behavior of a single-phase series motor with a given value of supply voltage. The chief difference between the direct-current motor and the single-phase motor grows out of the fact that an alternating voltage of a given value gives less current than a direct voltage of the same value (power factor of single phase motor less than unity).

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* This is due to transformer action as may be readily seen by tracing the short-circuit current in two armature sections.

Advantages of single-phase series motor for railway service.— The operation of a railway by electric motors involves two distinct problems, namely, (*a*) the problem of transmitting power from a central station to the motors and (*b*) the problem of converting the electrical power into mechanical power. The first problem is a problem in long distance transmission when the railway exceeds a few miles in length and high voltage must be used. The second problem seems to admit of no other practical solution than by the use of the series motor which develops a large starting torque and operates at a fairly high efficiency throughout a wide range of speed. The series motor, however, is essentially a low-voltage machine, and indeed the single-phase series motor cannot operate at as high a voltage as the direct-current series motor on account of the greater tendency of the single-phase machine to spark. Therefore the operation of a railway by electric power requires high-voltage transmission and step-down transformation, and the choice may be said to lie between (*a*) the series motor supplied with direct current at a voltage not exceeding about 1,000 volts, and (*b*) a series motor supplied with single-phase alternating current at a voltage not exceeding about 500 volts. If the direct-current motor is used, synchronous-converter sub-stations have to be placed a few miles apart all along the line of railway, and not only is the total cost of the electrical equipment high, but the synchronous converters must operate a large portion of the time at light load so that the power losses must be great and the efficiency low. If the single-phase series motor is used, the trolley wire can itself be the high voltage transmission line, and a step-down transformer on the car or locomotive can deliver current at any desired voltage to the motors.

The most serious difficulty involved in the use of the single-phase series motor is its large size as compared with a direct-current motor of the same power rating. Thus, the electric locomotives of the New York Central Railway Company have direct-current motors of sufficient capacity to slip the wheels, whereas the electric locomotives of the New York, New Haven and Hartford Railway Company, which are about the same weight as the New York Central locomotives and which are equipped with single-phase series motors, can exert a tractive effort not to exceed 50 per cent. of that which would correspond to slipping of wheels.

The series motor requires a very low voltage at starting, and the voltage across the motor terminals must be increased as the speed increases. When direct current is used, this variation of motor voltage can be accomplished only by the use of rheostat control, as explained in Art. 80, but when alternating current is used, a number of taps can be brought out from the secondary coil of the step-down transformer so that the low voltage required at starting can be taken directly from a few secondary turns, and the voltage can be increased at will by bringing a larger and larger number of secondary turns into action as the speed increases. Rheostat control is therefore not necessary when alternating current is used and the loss of power in rheostats is obviated. Furthermore, the series parallel system of direct-current motor control, the object of which is to reduce the range of rheostat control of voltage, is unnecessary when alternating current is used.

186. The repulsion motor.—The repulsion motor consists of a primary or stator member which is identical to the stator of the single-phase induction motor, and a secondary or rotor member which is like an ordinary direct-current armature with a commutator and brushes. The primary member is supplied with single-phase alternating current, and the secondary member is short-circuited by connecting its brushes directly together. The essential features of the repulsion motor are shown in Fig. 280, which represents a two-pole model. The stator winding produces through the armature a pulsating flux in the direction of the arrow Φ, and, in order to describe the action of the motor, this flux may be resolved into two component parts, one of which T is parallel to the line of the brushes bb, and the other of which *F* is perpendicular to *bb*. The stator winding may also be considered in two parts, namely, the part *t* which produces

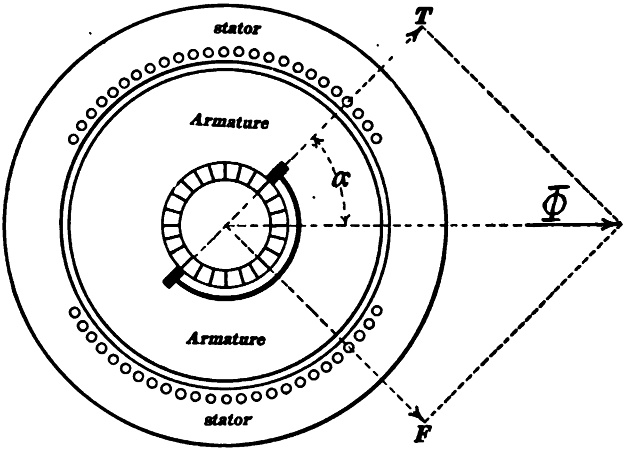

FIg. 280.

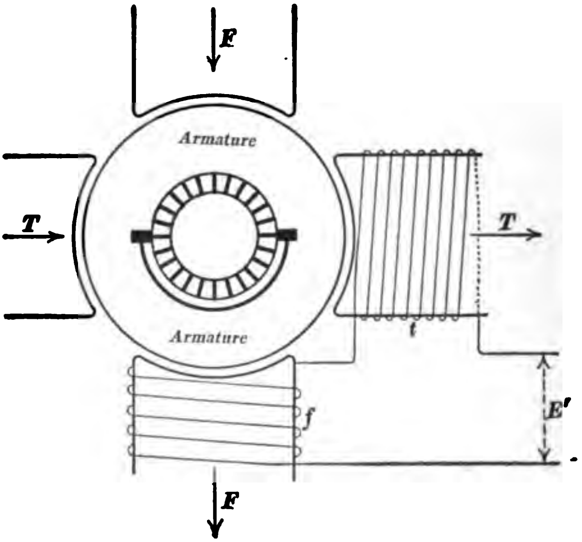

FIg. 281.

the flux *T* and the part *f* which produces the flux *F*. It is impossible to actually separate the stator winding shown in Fig.

280 into the two parts *t* and *f*, because certain portions of the stator winding help to produce both *T* and *F*. Figure 281, however, shows a more or less fanciful arrangement in which there are two separate stator windings *t* and *f* with their axes at right angles to each other.

The repulsion motor may be shown to be fundamentally equivalent to the simple series motor in its action and in its torque-speed characteristics as follows: The stator winding *t*, Fig. 281, produces the pulsating flux *T* and acts like the primary of a transformer of which the secondary is the short-circuited armature winding. Let us assume that this transformer action is ideal (zero reluctance for flux *T*) and let us assume that the ratio of transformation is 1 : 1. Then the armature current is identical in value and in phase * to the current in the coil *t* *just as if the armature were included in the primary circuit.*

The stator winding *f* produces the pulsating flux *F*. The armature is an open circuit with respect to the electromotive forces induced by the pulsations of this flux, but the electromotive forces produced by the cutting of *F* by the rotating armature act through or around the short circuit formed by the brushes, and these electromotive forces are reproduced in the stator winding *t* by transformer action *just as if the armature itself were included in the primary circuit.*

The simple series motor is different from the repulsion motor in that the transformer flux *T* does not exist in the simple series motor, and this transformer flux has an interesting effect on the rotating armature conductors as follows: The armature sections which are short-circuited under the brushes have electromotive forces induced in them by the pulsation of the field flux *F* exactly as in the simple series motor but these short-circuited sections are *cutting* the transformer flux *T* at the time of their short circuit and the electromotive force due to this cutting is opposed to the electromotive force produced by the pulsation of the field flux F. In fact, the electromotive forces due to cutting of the transformer flux T are equal and opposite to the electromotive forces due to pulsation of the field flux F at synchronous speed, and therefore the repulsion motor has no great tendency to spark at synchronous speed.

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* The secondary current of a transformer is usually thought of as exactly opposite in phase to the primary current, but whether it be considered as *opposite to* or *in phase with* the primary current depends on a mere convention as to the choice of the direction in the secondary coil which is to be considered as the positive direction.

Repulsion-motor-starting of the single-phase induction motor.— The repulsion motor has not come into practical use to any great extent because the simple series motor seems to be on the whole more satisfactory. Indeed the most valuable feature of the repulsion motor is that it can be easily and quickly converted into a single-phase induction motor by bringing a metal ring into contact with all of its commutator bars, thus converting the armature into what is substantially a squirrel-cage rotor. The single-phase induction motor of the Wagner Electric Company, for example, is started and brought up to speed as a repulsion motor, and then converted into a single-phase induction motor.

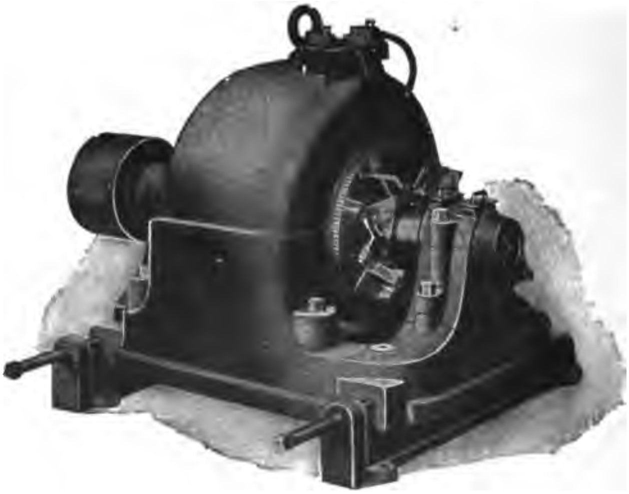

Fig. 282.

A general view of a four-pole Wagner motor is shown in Fig. 282. The commutator is radial so that the brushes can be lifted off the commutator by a movement parallel to the armature shaft. At starting the brushes (which are connected by short heavy conductors) rest against the commutator and short-circuit the armature as above explained, and the machine acts as a repulsion motor. When a certain speed is reached the weights *V*, Fig. 283, are thrown outwards by centrifugal action, and the connecting rods *U* push the sleeve *I* along the shaft, thus bringing the copper ring *K* into contact with the inner ends of the commutator segments and lifting the brushes off the commutator. The machine then operates as a single-phase induction motor. The lifting of the brushes off the commutator is not necessary but desirable in order to reduce friction and wear.

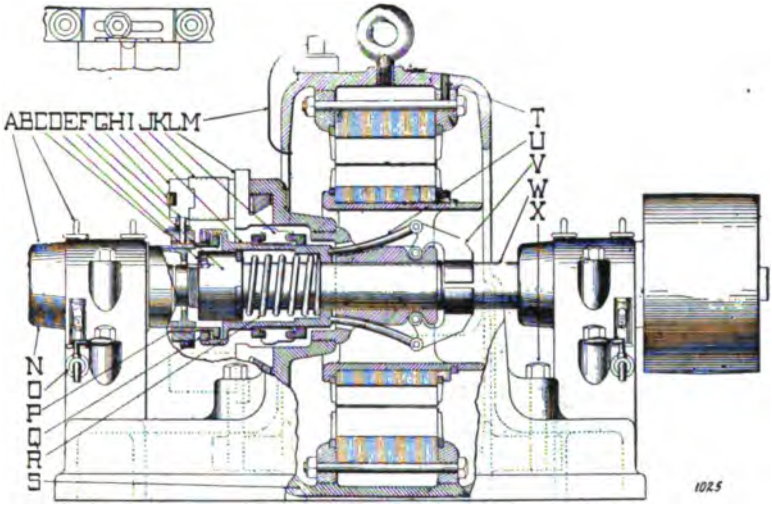

Fig. 283.

PART V.

STATION ARRANGEMENTS AND OPERATION.

CHAPTER XVII.

SWITCHBOARDS AND SWITCHBOARD APPLIANCES.

187. Recapitulation. — The preceding chapters are devoted chiefly to the discussion of individual machines, and but little has been said of the combination of machines into complete systems for the generation, transmission and utilization of power. This chapter refers to the general arrangement of apparatus in generating and receiving stations. Most of the details of arrangement in generating and receiving stations refer to the accessory apparatus which must be used for controlling and operating the various machines. Thus, a group of alternating-current generators* operating in parallel require the following accessory apparatus. One or more exciters with their field rheostats and switches, ammeters, voltmeters, "potential" and "current" transformers (see Art. 42), switches for opening and closing the main circuits of the generators, and synchronizing devices (see Art. 138). Quick-acting circuit-breakers are preferably inserted in the various feeder circuits because a quick-acting circuit-breaker in the main generator circuit throws the entire system out of operation in case of a short-circuit.

A pair of direct-current or single-phase-alternating-current feeders, or a set of polyphase-alternating-current feeders leading out from a generating station to a given locality is generally provided with the following accessory apparatus in the generating station: Switches, circuit breakers or fuses, ammeters, ground detectors and lightning arresters. A voltage regulator and voltmeter are sometimes installed in connection with a set of feeders as explained in Art. 190.

A direct-current motor is usually provided with a main switch, a starting rheostat, and an automatic overload and dead-line release, as explained in Art. 78. In case of a small motor these accessory devices are usually mounted in what is called a *starting box* which is placed conveniently near to the motor; in case of a large motor these accessory devices are sometimes mounted upon a slate or marble panel, as shown in Fig. 101*a*.

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* The arrangements for operating a group of direct-current generators in parallel are described in Art. 70.

The synchronous motor is generally provided with switches and fuses, an ammeter, an exciter with its switches and rheostats, a synchronizer, and an arrangement for starting. In some cases an auxiliary motor is used as a starter. A polyphase synchronous motor may be started by inherent induction-motor action as explained in Art. 178. A synchronous converter requires the same accessory apparatus as the synchronous motor with the exception of the separate exciter, and with the addition of a complete direct-current switch-board equipment.

A small induction motor is usually provided with a switch and fuses, no starting rheostat or compensator of any kind being required. A large induction motor is usually provided with switches and fuses, an ammeter, and a starting compensator as explained in Art. 174.

Wattmeters are frequently installed in connection with individual alternating-current generators in order that the station attendant may have a knowledge of the relative power outputs of the several generators inasmuch as the power output of an alternating-current generator cannot be inferred from ammeter and voltmeter readings.

When an over-excited synchronous motor is used to supply the wattless current at a receiving station, a power factor meter* is usually installed so that the field excitation of the synchronous motor can be adjusted until the power factor of the entire receiving station as indicated by the power-factor meter is approximately equal to unity.

Watt-hour meters are sometimes installed in a generating station to record the total output of energy of the station, and watt-hour meters are frequently installed in connection with lamps and motors in order to record the amount of energy delivered to the customer.

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* Power factor meters of various types are described by Wm. H. Browne, in the Trans. Am. Inst. Elec. Eng., Vol. XVIII, pages 287–338, 1901.

Each generator, each direct-current exciter (in case the generator is an alternator) and each pair of feeders must be provided with accessory apparatus as above specified, and it is the usual practice to group all of the accessory devices belonging to one machine or to one pair of feeders on one switchboard panel; thus we have what are called *generator panels*, *exciter panels* and *feeder panels*.

When a station is equipped with several generators which are to be operated in parallel, the main conductors to which the generators are to be connected back of all the switchboard panels and these main conductors are called the *main bus-bars*.

Whenever it is desirable to have a ground connection at each switchboard, a single wire is installed back of all the switch boards and connected to ground. This wire is called the *ground bus*.

An auxiliary set of bus-bars is always installed back of the generator panels in an alternating-current generating station, and so arranged that a single synchronizing device may be used when any one of the alternating-current generators is to be put into operation and connected in parallel with the others. These auxiliary bus-bars are called the *synchronizing buses*.

188. Direct-current generator panels. — The conditions which must be met in the operation of direct-current generators in parallel are discussed in Art. 70 and the essential connections of three compound generators operating in parallel are shown in Fig. 91. The equalizing bus bar is a connection which places the three series field windings in parallel with each other independently of the parallel connections of the armatures of the three machines as shown in Fig. 91. The two dotted rectangles in Fig. 284 show the arrangement of two switchboard panels and two compound generators G' and G''.

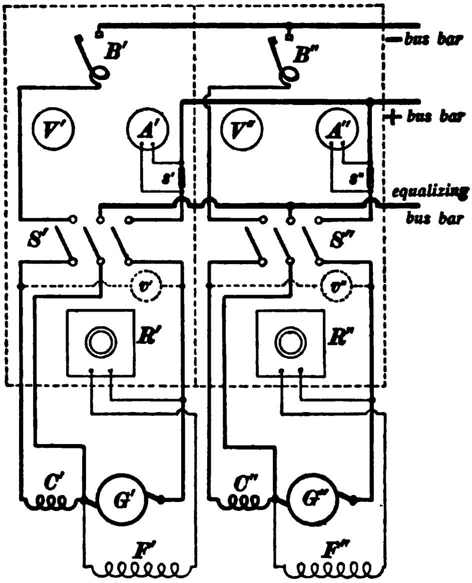

Fig. 284.

G' and G'' are two compound generators, F' and F'' their shunt field windings, R' and R'' their shunt field rheostats, and C' and C'' their series field windings. S' and S'' are two triple-pole switches, A' and A'' are ammeters, s' and s'' are ammeter shunts, and B' and B'' are circuit breakers. The small dotted circles v' and v'' and the dotted lines show the electrical connections of the two voltmeters V' and V''.

189. Alternating-current generator panels. — The conditions which must be met in the operation of alternating-current generators in parallel are discussed in Arts. 133 and 142. Figure 285 shows the complete connections of an alternating-current three-phase generator panel (A.T.G. panel) as arranged for parallel operation, including the connections of the exciter. The generator is represented as a revolving-field machine, the exciting current being delivered to the field windings through collector rings. The field switch is provided with what is called a *discharge resistance* to prevent excessive sparking and possible damage to the insulation of the field windings when the field circuit is opened. This discharge resistance is a moderately high resistance which is connected across the field terminals immediately before the field switch is opened, thus permitting the field current to continue to flow and die away gradually after the open-

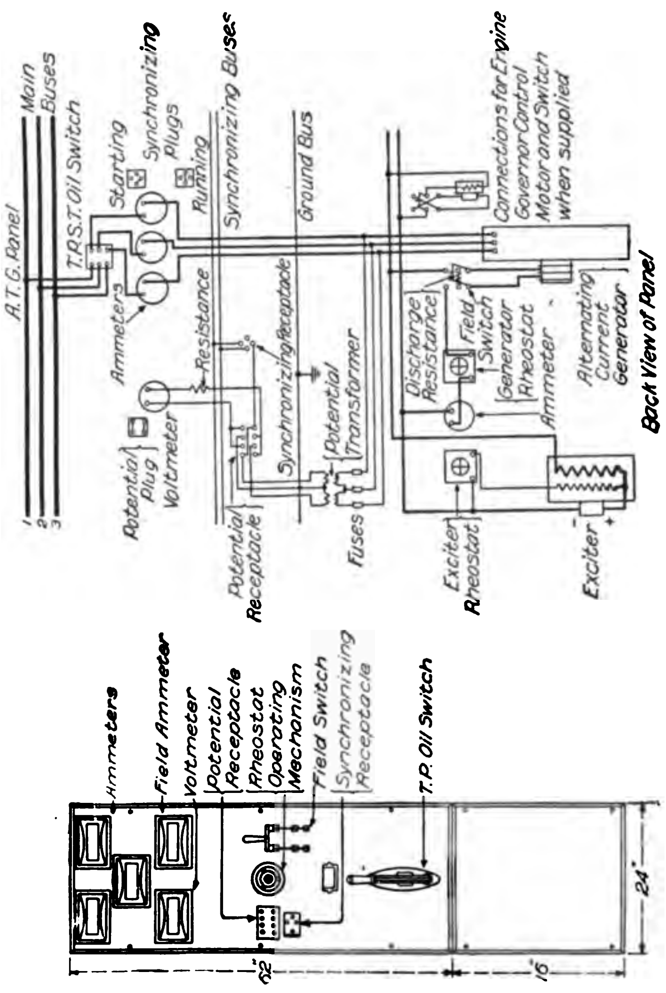

Fig. 285.

ing of the switch. A rheostat is shown in the field circuit of the generator and another rheostat is shown in the field circuit of the exciter in Fig. 285. The armature terminals of the three-phase generator in Fig. 285 are connected through the three ammeters and through a three-phase single-throw oil switch to the main bus-bars.* Two "potential transformers" are shown connected to the three wires which lead from the generator terminals to the oil-switch (the use of two transformers for three-phase transformation is explained in Art. 165); and the low voltage coils of the potential transformers are connected to what are called the "potential receptacle," and the "synchronizing receptacle" as indicated in the figure. The potential receptacle is a plug switch into which the "potential plug" may be inserted to connect the voltmeter so as to measure the voltage of any one of the three phases as may be seen by tracing the connections from the potential transformer through the potential receptacle and potential plug to the voltmeter in Fig. 285. The synchronizing receptacle is a plug switch into which either "synchronizing plug" may be inserted, namely, the "starting plug" or the "running plug."

Other generators being in operation and connected to the main bus-bars, the generator which belongs to the panel shown in Fig. 285 is started as follows: The starting plug is inserted into the synchronizing receptacle (running plugs being in the synchronizing receptacles of all the machines which are already in operation), and the exciter is started and brought up to full speed. The new generator is then started and brought up to approximately full speed, and its field rheostat is adjusted until the voltmeter shows that its voltage is approximately the same as the voltage across the main bus-bars; then the speed of the driving engine is carefully adjusted until the synchronizing device (synchronizing lamps or a dial synchronizer) shows that the two necessary conditions which are explained in Art. 138 are satisfied; and then the three-phase single-throw oil switch is closed so as to connect the alternating-current generator to the bus-bars. The generator is then in full operation and the running plug is put in place of the starting plug in the synchronizing receptacle.

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* In some cases, two separate sets of main bus-bars are installed, and each of the main switches is double-throw so that any generator may be connected to either set of bus-bars.

In many cases the ammeters for indicating the alternating current delivered by each phase of an alternating-current generator

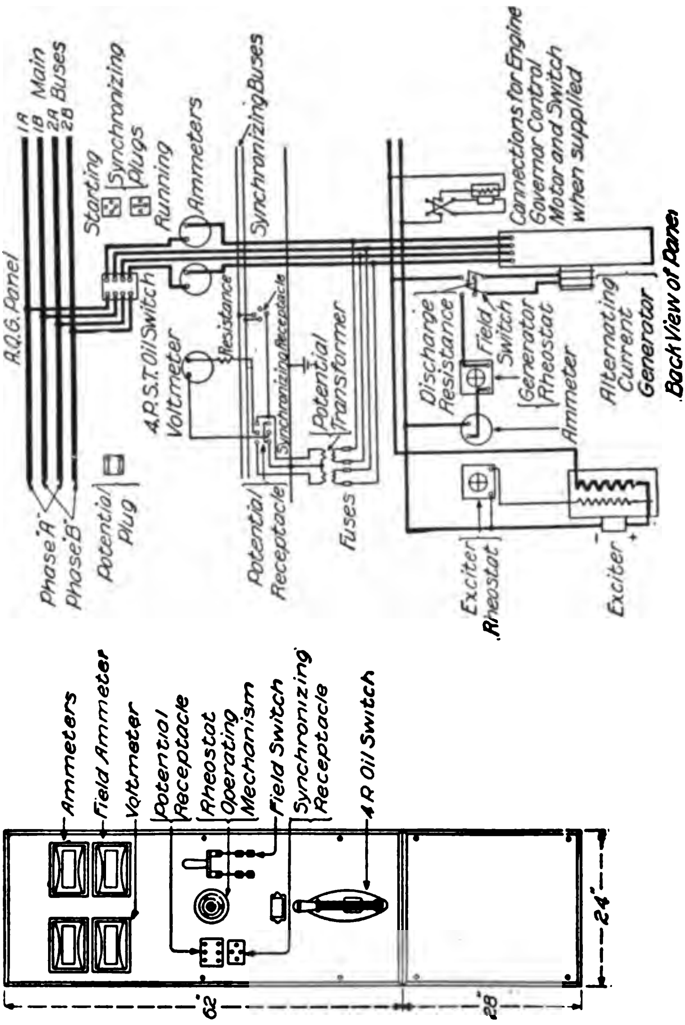

FIG. 286.

are connected through "current transformers." In Fig. 285, however, the ammeters are shown inserted directly in the generator leads.

It is not desirable to use automatic switches (circuit-breakers) on alternating-current generator panels. Thus, the three-phase single-throw oil switch shown in Fig. 285 has no trip coil, and it does not automatically disconnect the machine in case of overload or short-circuit.

Figure 286 shows a quarter-phase[*] alternating-current generator panel (A.Q.G. panel) including exciter connections.

190. Feeder panels. — Figure 287 shows the connections of two pairs of feeders F' and F'' to the main bus-bars in a direct-current generating station; B' and B'' are circuit breakers, s' and s'' are the ammeter shunts, and S' and S'' are the feeder switches. The ammeters which are represented in Fig. 287 are millivoltmeters which may be connected to the terminals of the shunts s' and s'', or they may be connected to the bus-bars through high resistances, as indicated by the dotted lines, and thus serve as voltmeters.

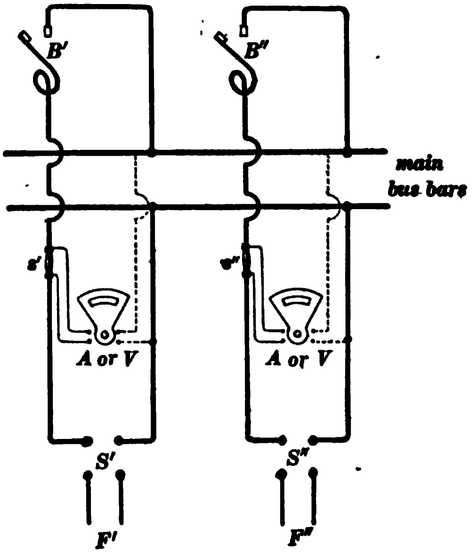

Fig. 287.

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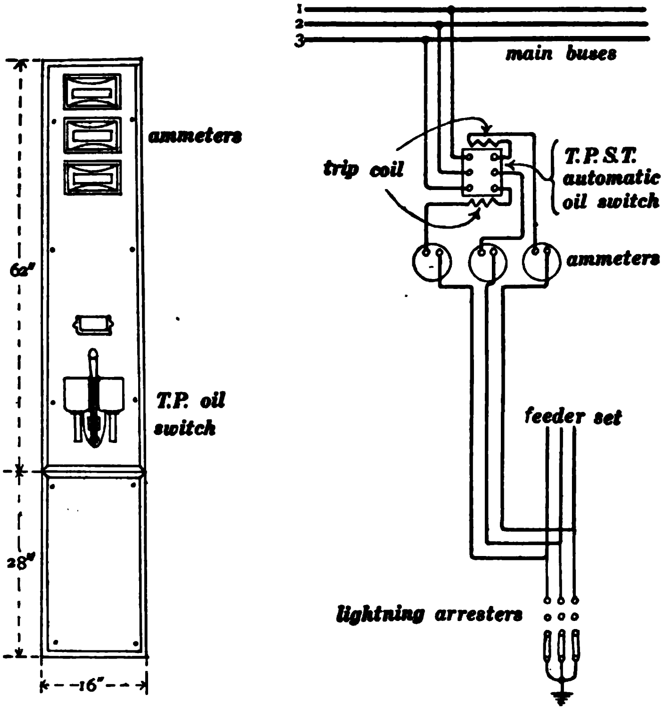
[*] The term quarter-phase is frequently used to designate a two-phase system.

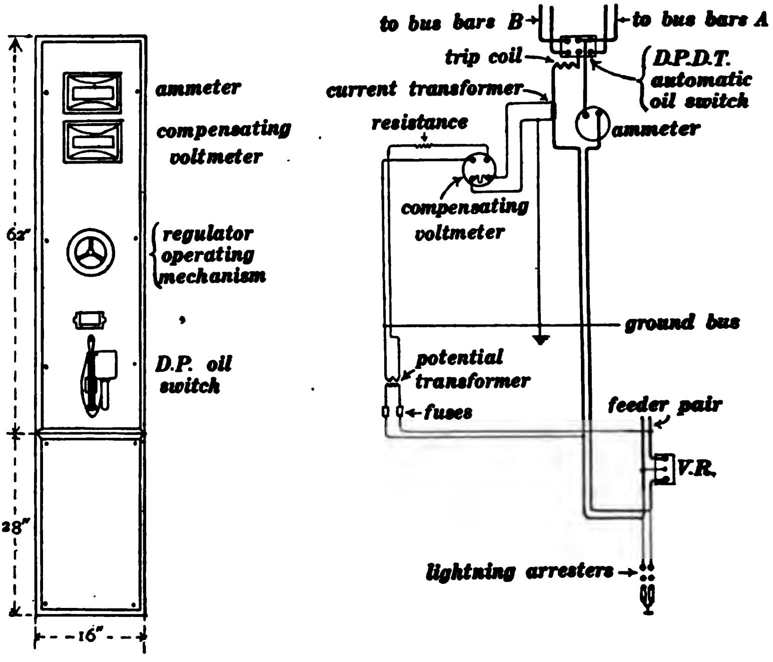
Figure 288 shows an alternating-current three-phase feeder panel (A.T.F. panel) and its diagram of connections. In this case the main switch is a three-phase single-throw automatic oil switch with trip coils in two of the leads so that an excess of current in either phase may trip the switch. Figure 289 shows an alternating-current single-phase feeder panel (A.S.F. panel). This panel is provided with a double-pole double-throw switch so that the single-phase feeder may be connected to bus-bars A or to bus-bars B at will. The voltmeter is represented as being compensated as explained in Art. 197, and the connections of a voltage regulator (see Art. 193) are indicated, the regulator being operated by the hand-wheel shown in the front view.



front view

connections

Fig. 288.



front view *connections*

Fig. 289.

191. Complete wiring diagram of a large alternating-current generating station. — In a very large alternating-current generating station a synchronous motor or an induction motor is sometimes installed instead of a small steam engine for driving the accessory apparatus, and if direct-current is needed for local service, it is usually supplied by a synchronous converter which is driven from the main alternating-current generators. Thus, Fig. 290 is a complete wiring diagram of a power station consisting of a three-phase alternating-current steam-turbine-driven, 2,300-volt, 1,000-kilowatt, generator (A.T.G. 2,300 V. 1,000 K.W.), and a 50-kilowatt exciter driven by an alternating-current, three-phase, 2,300-volt, 75-horse-power, induction motor (A.T.I. 2,300 V. 75 H.P.). The figure shows the complete connections of one alternating-current single-phase feeder panel (A.S.F. 2,300 V.

75 Amp.) and of three alternating-current, three-phase feeder panels (A T.F. 2,300 V. 75 Amp). The figure also shows the connections of a 300-kilowatt three-phase synchronous motor

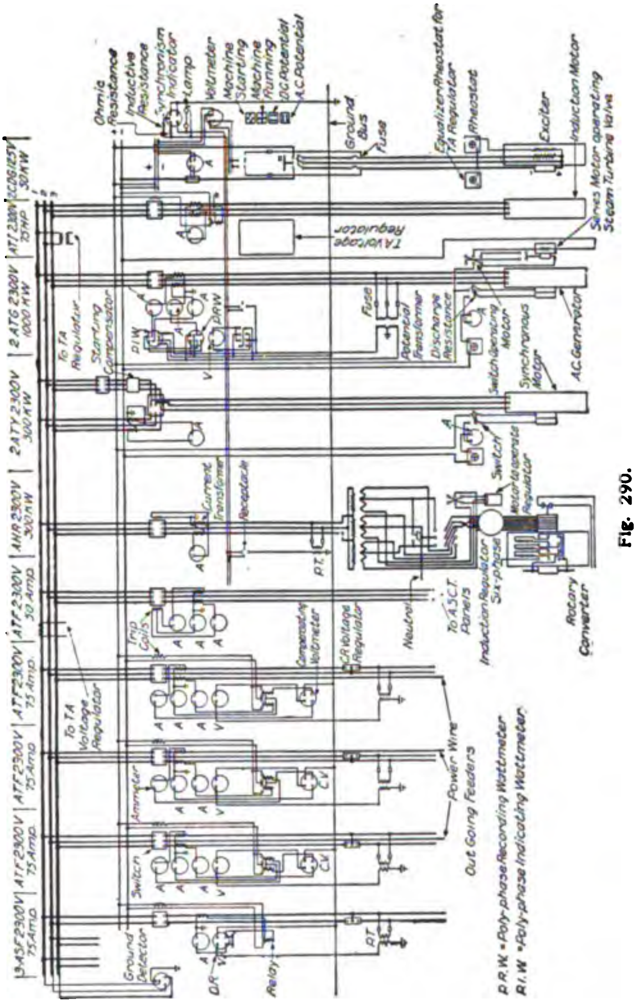


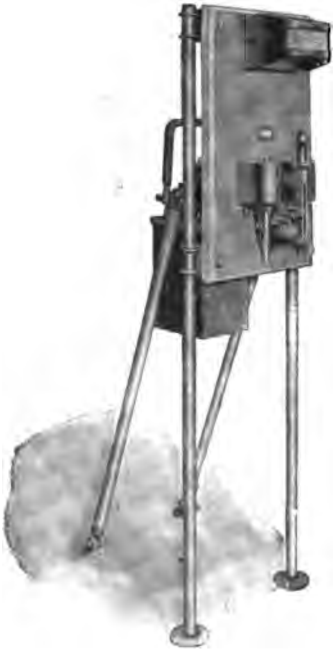
Fig. 290.

(2ATY 2,300 V. 300 K.W.), and the connections of a 300-kilowatt six-phase synchronous converter (AHR 2,300 V. 300 K.W.) with its step-down transformer and synchronizing receptacle.

The "TA" regulator, which is indicated in Fig. 290, is an accessory of the generator (A.T G. 2,300 V. 1,000 K.W.). It is an automatic device for controlling the field excitation of the generator so as to give constant terminal voltage and it is known as the Tirrill regulator.*

The device on each feeder panel marked "relay" in Fig. 290 serves to control the time which elapses between the instant of an overload or short circuit on the pair of feeders and the instant of tripping of the circuit breaker.

The essential features of the ground detector which is shown in Fig. 290 are described in Art. 195. There are several types of synchronism indicators,† the simplest type being the synchronizing lamp which is described in Art. 138.



Induction Motor Panel.
Fig. 291.

192. Switchboards for receiving stations. (*a*) *Induction motor panels.*— Small induction motors are generally provided with a switch for closing the circuit and a fuse for opening the circuit in case of excessive overload or short circuit, and these small accessories are usually mounted on the wall conveniently near to the motor. Large induction motors are usually provided with an autotransformer for delivering current at reduced voltage at starting, as explained in Art. 174, and it is usual to provide an automatic oil switch or circuit breaker and an ammeter. These accessories are usually mounted on a switchboard panel as shown in Fig. 291.

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* See Franklin and Esty's *Elements of Electrical Engineering*, Vol. II, pages 341-345. A full discussion of the Tirrill regulator is given in the Electric Journal, Vol. V, pages 502-509, 1908.

† One of the best known forms of dial synchronizer is the Lincoln synchronizer, which is described on page 337, Franklin and Esty's *Elements of Electrical Engineering*, Vol. II.

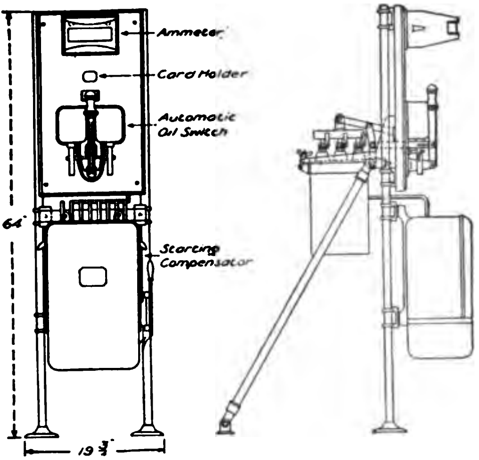

Fig. 292.

Front and side views of this induction motor panel are shown in Fig. 292. Figure 265 is a diagram of connections of a three-phase induction motor and its starting compensator.

(*b*) *Synchronous converter panels.* — The essential connections of two three-phase synchronous converters to the alternating-current supply and to the direct-current bus-bars are shown in Fig. 293, and one mode of connecting a six-phase converter through its step-down transformers to a three-phase alternating-current supply is shown in Fig. 294.*

Figure 295 is a complete diagram of connections of a three-phase synchronous converter arranged to deliver direct current to a street railway. The high-voltage transmission lines come into the receiving station, or substation as it is usually called, and are connected to ground through the lightning arresters as shown in the figure. Just inside of the connections of the lightning arresters choke coils are inserted in the transmission lines but these choke coils are not shown in Fig. 295. The figure shows the complete connections of one synchronous converter. The alternating-current connections and accessories are generally mounted on one panel and the direct-current accessories and connections on another panel. The synchronous converter is arranged to be started by inherent induction motor action as explained in Art. 178. For this purpose the "AC starting switch" connects the low voltage taps of the step-down transformers at starting, and then by throwing the switch over the full running voltage becomes available. The machine is also arranged to be started as a direct-current motor and brought up to synchronism. For this purpose, the "starting switch" on the direct-current side is closed and the

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* See Art. 168.

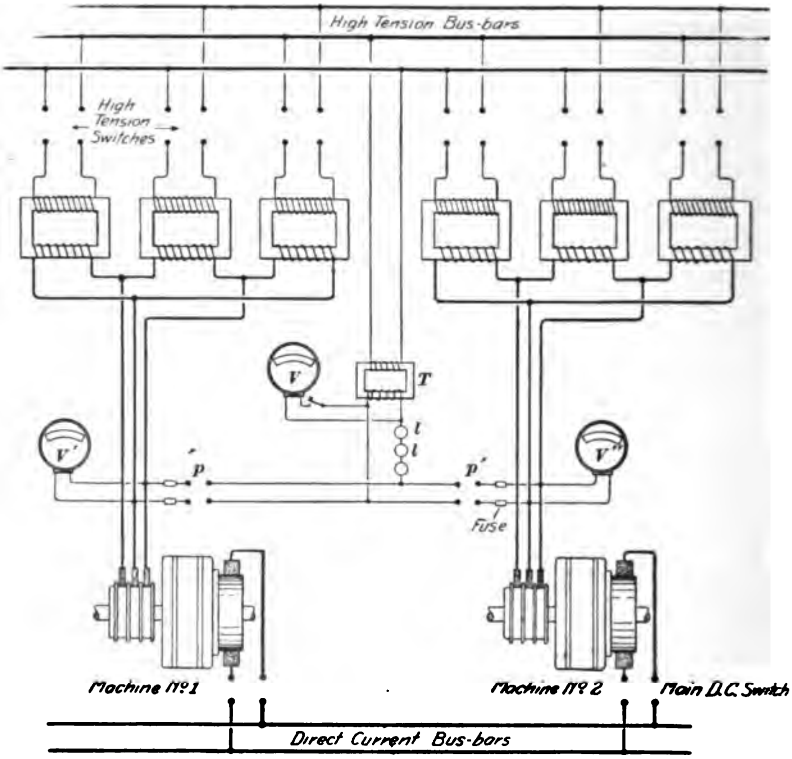

Fig. 293.

"starting rheostat switch" is slowly moved into the closed position, then the field rheostat of the converter is adjusted until the synchronizing lamps show the proper condition for closing the alternating-current supply switch. The direct current for exciting the field of the machine and for starting it as a direct-current motor is taken from the direct-current bus-bars; other converters are supposed to be already in operation.

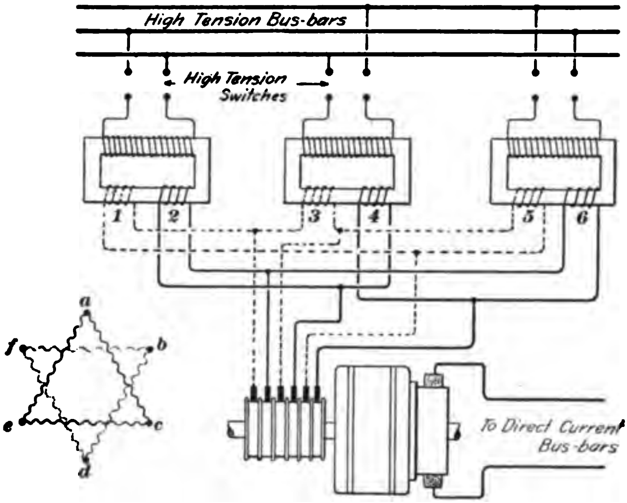

Fig. 294.

(*c*) *Synchronous motor panels.* — A synchronous motor panel is essentially the same thing as an alternating-current panel of a synchronous converter except that a synchronous motor panel usually includes the switches and rheostats of a small direct-current generator, the exciter. Thus, the right-hand side of Fig. 295 shows the essential features of a three-phase synchronous motor panel except that the exciter accessories are absent. Figure 290 shows the connections of a three-phase synchronous motor (2ATY 2,300 V. 300 K W.) with its field current supplied from the same exciter that supplies the field current of the main generator. The synchronous motor, as shown, is arranged to be

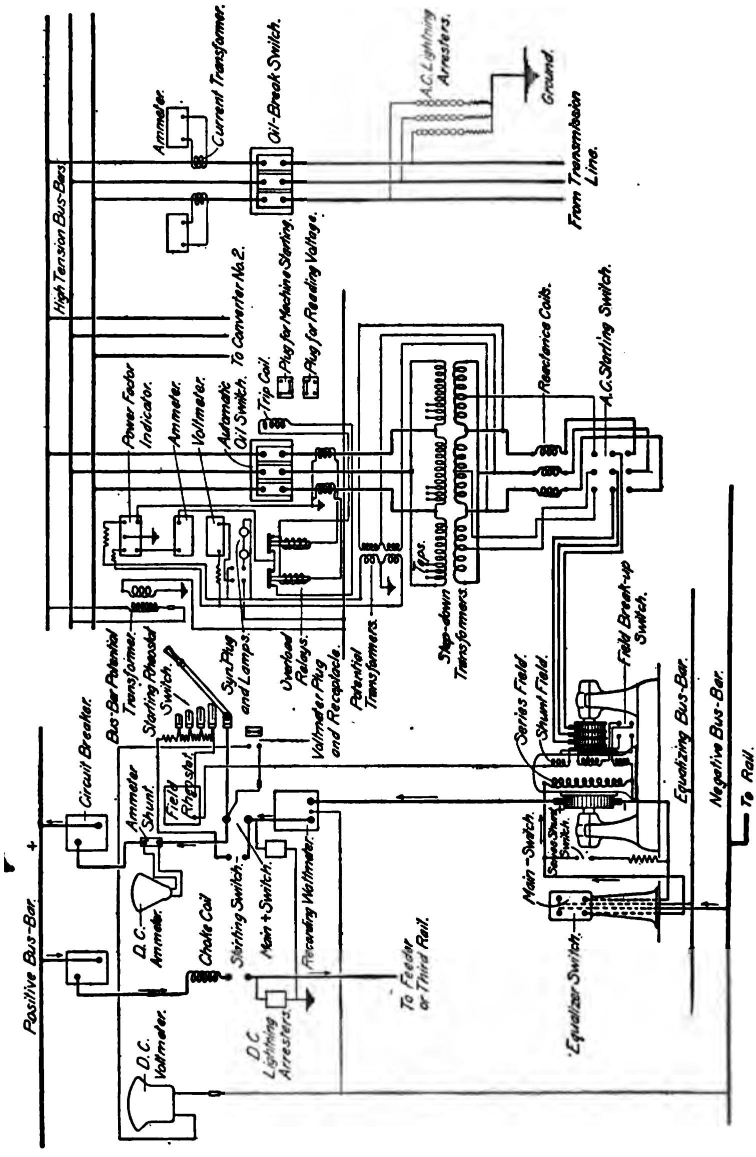

Fig. 205.

started by inherent induction motor action as explained in Art. 178 and in this case no synchronizing device is used.

193. Feeder control. The voltage regulator. — A central station usually delivers current over a number of feeder circuits to a number of subordinate centers from each of which a small district is supplied with current through a network of mains. It is desirable, especially in the distribution of current for incandescent lighting, to maintain an unchanging voltage at every subordinate center of distribution. The importance of this requirement is due to the fact that fluctuation of voltage causes an unpleasant flicker of the lamps, and also to the fact that a slight excess of voltage shortens the life of incandescent lamps.

A constant voltage can be maintained at any *one* of a number of subordinate centers of distribution by proper adjustment of the voltage between the bus-bars at the station; but *to maintain constant voltages at two or more independent subordinate centers of distribution the voltage between each pair of feeders at the station must be adjusted separately.* Therefore the maintenance of a prescribed constant voltage at every subordinate center of distribution depends upon the proper control of the voltage between each pair of feeders at the switchboard. Following are two methods of feeder control:*

The rheostat method. — The voltage between the bus-bars at the station is kept at the highest value required by any of the feeders, a low resistance rheostat is inserted in each feeder circuit at the switchboard, and these rheostats are separately adjusted to give the desired voltages between the respective pairs of feeders. This method of feeder control may be used with direct current or alternating current.

The dial regulator. — The simplest device for controlling the voltage of a pair of feeders in an alternating-current generating station is to bring out a number of taps from the primary (or secondary) coil of the step-up transformer, and arrange a switch so as to vary the number of active turns of wire in the primary (or secondary) coil of the transformer at will, thus changing the ratio of transformation and changing the secondary voltage even though the primary voltage be constant. Thus, Fig. 295 shows three taps brought out from successive turns at one end of the primary coil of each of the three step-down transformers; the figure does not show, however, the mechanical device for connecting the high voltage alternating-current supply mains to one or another of these taps at will. This device is usually in the form of an arm which plays over a circular row of contact points. Such an arrangement is called the *dial regulator*. The details of the dial regulator are shown in Fig. 296. The contact finger

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* Several other methods have been proposed, but are very seldom used.

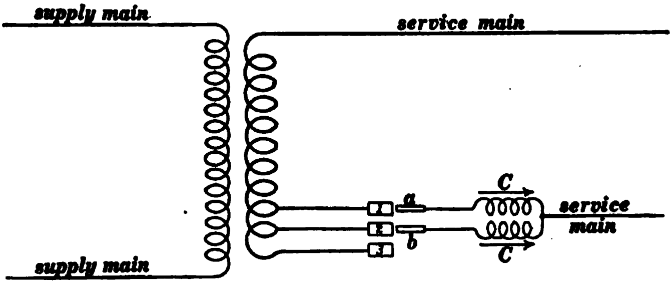


Fig. 296.

which is shifted from one to another of the contact points 1, 2 and 3 consists of two pieces of metal *a* and *b* far enough apart to bridge across from one contact point to the next, as shown. With this arrangement the circuit is never broken as the contact finger is shifted over the contact points, and the short-circuit of one turn of the transformer which takes place when the contact finger touches two contact points includes the two choke coils *CC* which are so wound on an iron core that they permit current to flow inwards or outwards through *both* as indicated by the arrows, but *do not permit* current to flow outwards through one and inwards through the other.

The induction regulator. — The primary of a transformer may be connected across the bus-bars and the secondary of the transformer may be arranged so that any desired number of turns of the secondary coil may be connected in series with the feeder circuit in a direction to assist or oppose the bus-bar voltage, thus giving a decreased or increased voltage between the pair of feeders. In its usual form this type of voltage regulator is made somewhat like an induction motor as follows: An iron carcase similar to the carcase of an induction motor has a single-phase primary winding on its stator and a single-phase secondary winding on its rotor. The primary winding is connected across the supply mains and the secondary winding is connected in series with the feeder circuit; then to turn the secondary member through half a revolution is equivalent to reversing its connections so that the electromotive force generated in the secondary may be added to or subtracted from the supply voltage by adjusting the position of the secondary part member, thus making the feeder voltage greater or less than the bus-bar voltage. This form of voltage regulator is usually called the *induction regulator* because of its similarity to the induction motor. It can be arranged to raise or

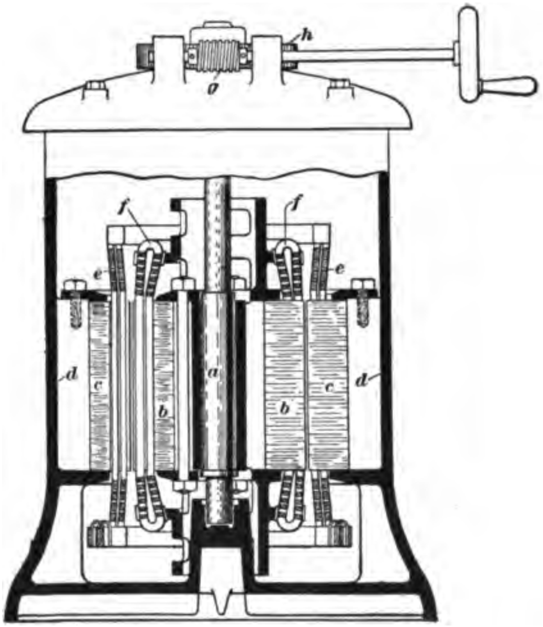


Fig. 297.

lower the voltages of any number of phases simultaneously by providing the primary and secondary members both with polyphase windings. Figure 297 is a sectional view of an induction regulator; *cc* is the laminated iron core of the stator, and *bb* is the laminated iron core of the rotor, which is mounted upon the spindle *a* and turned by means of the hand-wheel and worm gear as shown in the figure; *ee* are the primary windings and *ff* are the secondary windings.

194. Fuses and circuit breakers — A fuse is a link of fusible metal placed in an electric circuit so that when the current exceeds a certain value the link is heated to its fusing point, is melted, and thereby causes the circuit to be broken. Fuse wire is generally made of lead or an alloy of tin and lead, and it is rated at about 80 per cent. of the greatest current it can carry steadily (for a long time). Thus a rated 10-ampere fuse wire is one which can carry no more than 12.5 amperes steadily without melting.

The melting of a fuse link is always followed by an electric arc which, although of short duration, generates a quantity of hot metallic vapor, the sudden expansion of which tends to scatter the melted metal. This *blowing of a fuse*, as it is called, is likely to set fire to nearby inflammable materials, and therefore a fuse link should always be enclosed in a fire-proof receptacle. When two fuse links are mounted side by side, as shown at *ff* in Fig. 83, they should be placed in separate compartments in the fire-proof receptacle so that the arc cannot flash across and short-circuit the supply mains.

A fuse link is more or less sluggish in action because a considerable amount of heat is required to raise the temperature of the wire or link to the fusing point. This is especially true of large fuse links. Furthermore, the amount of current required to fuse a link varies greatly with the mode of mounting. Thus, a fuse link lying flat against the walls of the containing receptacle is cooled thereby and its current-carrying capacity is increased; again, a fuse link which is mounted in a thoroughly ventilated receptacle has a greater current-carrying capacity than it would have if mounted in a closed receptacle.

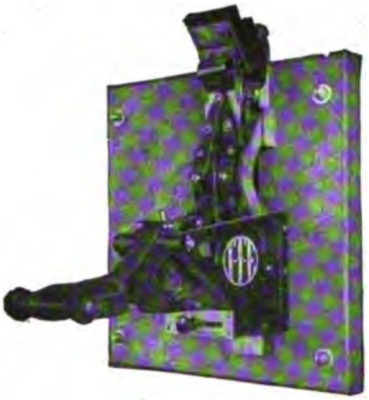
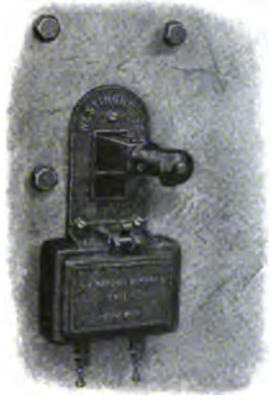


Fig. 298.

The electromagnetic circuit breaker is a switch, single-pole, or double-pole, or triple pole, which is arranged to open automatically when the current is in excess of a certain value. The closing of the switch puts a strong spring in tension, the switch is kept closed by a latch which is controlled by a trigger, and this trigger is released by an electromagnet (through which the main current flows) when the current reaches a certain value. The electromagnetic circuit breaker is very prompt in action and when once adjusted it opens the circuit when the current exceeds a certain definite value.



Type B, Oil Circuit Breaker,
Switchboard Mounting (Front View).
Fig. 299.

Inasmuch as a circuit breaker always opens when an excessive current is flowing, it is evident that an intense arc must always flash across the break. It is necessary therefore to thoroughly protect the metal contacts of the circuit breaker from the destructive action of this arc by having a pair of auxiliary contacts in parallel with the main contacts and so arranged that the main contacts open first without an arc) and the auxiliary contacts open a moment later (with an intense arc). The auxiliary contacts are usually made of carbon blocks. Figure 298 shows a single-pole carbon-block circuit breaker manufactured by the Cutter Company.

Circuit breakers for high voltage alternating-current circuits are usually arranged to open the circuit under oil. Thus, Figs. 299 and 300 show the front and rear views of an automatic oil switch (circuit breaker) of the Westinghouse Electric and Manufacturing Company. Figure 299 shows the tripping magnet which releases the lever of the switch and causes the switch to open, and Fig. 300 shows the essential parts of the switch. This particular switch is a three-phase single-pole switch, that is to say, it is three separate single-pole switches one for each phase of a three-phase system, and each switch breaks under oil. One of the oil vessels is shown removed in Fig. 300. When the

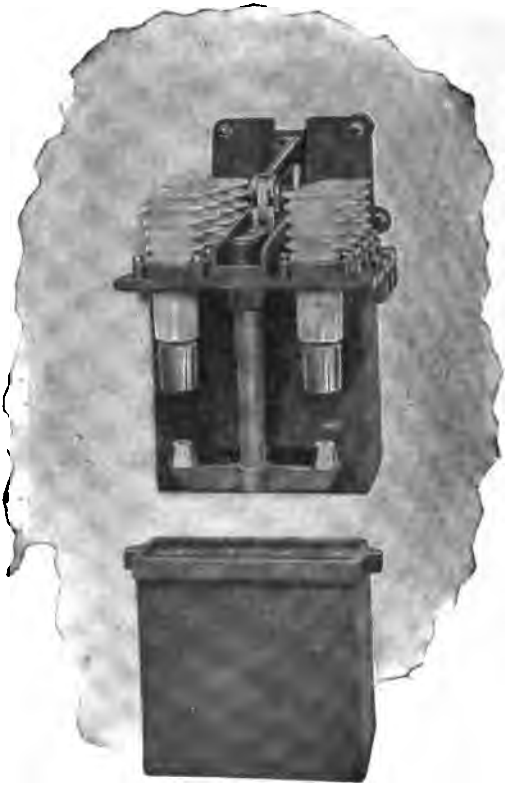


Fig. 300.

switch is closed the metal cross-arm at the lower end of the vertical wooden stud bridges across between the two metal cups, thus making connection.

195. Grounds and ground detectors. — A connection of a wire to ground is called a *ground.* A ground on one of a pair of supply mains is unsafe because an accidental ground on the other wire will complete the circuit. If the second ground is through the human body a personal injury may result; and if either ground connection is through a wire or other conductor, the conductor may become hot enough to start a fire. Since accidental grounds frequently occur on electric mains, a device called a *ground detector* is usually mounted upon each feeder panel in a generating station for indicating the existence of serious grounds on the line wires.

The ground detector which is usually employed on low voltage lines (up to 500 or 600 volts) consists of two incandescent lamp units * connected in series between the mains with the middle point between the units connected to earth. When both line wires have grounds of the same resistance, high or low, both lamp units burn with equal brightness. When one line wire has a fairly low resistance ground and the other a high resistance ground, the lamp units are unequal in brightness, and the main which connects to the dim unit is the main with the low resistance ground. In order to distinguish between the unsafe condition with both mains grounded through nearly equal low resistances, and the safe condition with both mains grounded through very high resistances, the lamp units should be arranged to be disconnected one at a time. If unit A is dim when unit B is disconnected from main B, then main B has a high resistance ground; and if unit A is bright when unit B is disconnected, then main B has a low resistance ground. The reasons for these statements can be easily worked by the reader from a diagram showing the relations of the two ground resistances to the lamp units.

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* Each unit consisting of one or more lamps in series, according to the voltage.

The electrostatic ground detector is almost universally used for moderately high voltage alternating-current feeders. The essential features of the electrostatic ground detector are shown in Fig. 301. Two insulated metal plates *C* and *D* are connected to the two feeders *A* and *B*, and a light metal vane *V* is suspended between *C* and *D* and connected to earth. The grounding of feeder *A* establishes a connection from *A* to *V* so that the voltage between *C* and *V* is zero whereas the voltage between *D* and *V* is large, and the vane is attracted by *D*. The grounding of feeder *B* in the same way causes the vane to be attracted by *C*. In order to distinguish between the safe condition with both feeders grounded through approximately equal high resistances and the unsafe condition with both feeders grounded through approximately equal low resistances, a special switch should be arranged for grounding one or the other of the feeders through a moderately low resistance. The behavior of the ground detector in this case is a matter which can be easily worked out by the reader. Figure 302 shows the electrostatic ground detector of the Westinghouse Electric and Manufacturing Company. In this instrument the moving vane is arranged so that it cannot come into contact with the stationary plates however far it may move, and additional provision is usually made to avoid possible short-circuiting through the ground detector by inserting a con-

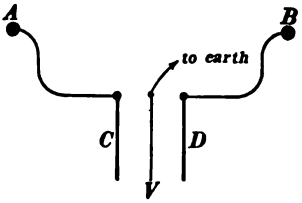

Fig. 301.

Fig. 302.

denser between *A* and *C* and another between *B* and *D* in Fig. 301, which represents the diagram of connections of the Westinghouse ground detector.

196. Lightning and lightning arresters. — When a lightning discharge occurs in the neighborhood of a transmission line, a sudden rush of current takes place over the line due to one or more of the following causes: (*a*) Electric charge accumulated on the line is suddenly released and tends to flow to earth; (*b*) the magnetic action of the lightning discharge induces a sudden pulse of current in the line; (*c*) the line may become the path of the lightning discharge in which case the lightning is said to actually strike the line.

The damage to electrical apparatus by a very intense lightning stroke may be a more or less complete destruction due to the energy of the lightning discharge itself. In nearly every case, however, the damage is done after the lightning discharge has passed; the insulation on a machine or transformer is punctured by the lightning discharge, the machine or transformer is thus short-circuited, and it is burned out by its own current. The protection of electrical apparatus from lightning discharge, therefore, depends upon: (*a*) the prevention of puncturing of insulation, and (*b*) the taking care of the short-circuit which in many cases follows the lightning discharge.

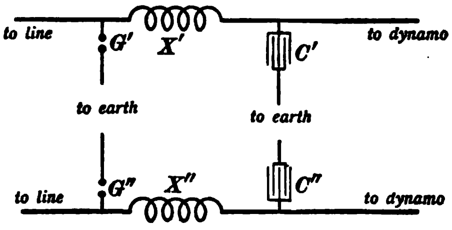

Fig. 303.

Figure 303 is a diagram showing the essential features of a lightning arrester, or rather of two distinct lightning arresters, one for each line wire. Such a pair of lightning arresters when mounted on one base plate is called a *double-pole* arrester. Each of the line wires at the entrance to the station building is connected to a coil of wire called a *choke coil*, or *reactance coil* X' and X'', in Fig. 303, and each line wire has a ground connection in front of its choke coil, this ground connection being broken by a short air gap G. Each line wire should be grounded behind the choke coil through a condenser C with thick insulation.

When a lightning discharge breaks down the two gaps G' and G'' in Fig. 303, the dynamo is short-circuited because the gaps G' and G'' are made conducting by the spark. It is therefore necessary to provide some arrangement for extinguishing the arc which tends to persist across the spark gaps G' and G'' after the lightning discharge has passed. The various arrangements which are used in practice for extinguishing the arc are exemplified in the following descriptions of particular types of lightning arresters.

The action of the choke coils X' and X'' may be explained as follows: The sudden rush of current which enters the station over either line wire is dammed up, as it were, in front of the choke coil because of the very great electrical inertia of the coil, and a sufficient voltage is created to break across the relief gaps G' and G'', thus establishing a conducting path to earth. The function of the condensers C' and C'' may be best explained in terms of a mechanical analogue as follows: A wall is to be shielded from the excessive force of a hammer blow by allowing the hammer to strike against a heavy ball of iron which rests against the ball. If the ball is to take up the momentum of the hammer, it must move to some extent, so that if the ball and wall were both perfectly rigid, the wall would not be protected at all. An elastic cushion may be interposed between the ball and the wall to provide the necessary elasticity. Each condenser in Fig. 303 acts like this elastic cushion, permitting some movement of current through the choke coil without the building up of excessive voltage across the terminals of the dynamo, the circuit of which has an enormously large electrical inertia or inductance. The condenser is not always used with lightning arresters inasmuch as the dynamo and connecting wires usually have sufficient electro-elasticity or capacity. Lightning arresters installed out on a transmission line do not require choke coils, but the choke

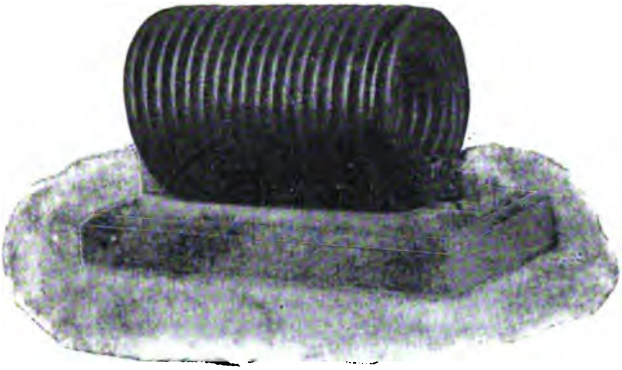

Fig. 304.

coil should always be inserted between the spark gap of a lightning arrester and the generator or motor or other apparatus which is to be protected.

Figure 304 shows a choke coil consisting of two layers of very heavy insulated wire, and Fig. 305 shows an open form of choke

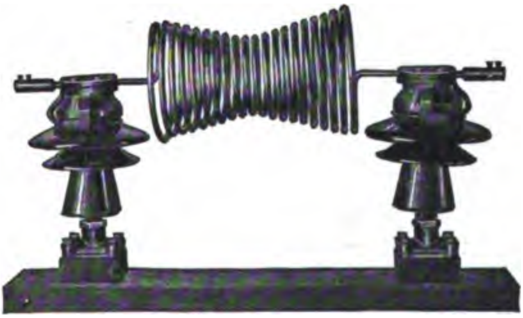

Fig. 305.

coil. Figures 304 and 305 are taken from the publications of the General Electric Company.

The horn arrester consists of a spark gap *G* between two metal rods which curve away from each other like two horns as shown by the lines A and B in Fig. 306. This spark gap is connected between a line wire and the ground with a resistance in series with it. The arc which persists after the lightning discharge is passed is carried upwards by the rising current of hot air and its increasing length finally extinguishes it.

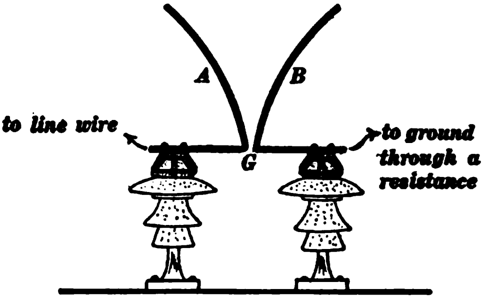

Fig. 306.

The magnetic blow-out arrester.—An electromagnet may be used to extinguish the arc which persists across the spark gap of a lightning arrester. Figure 307 shows a single-pole magnetic blowout direct-current lightning arrester manufactured by the General Electric Company. This particular arrester has two spark gaps in series. The main spark gap is between the two wing-shaped pieces of brass and it lies between the poles of the electromagnet. The auxiliary spark gap is between the two rounded brass points shown at the right in the figure. The winding of the electromagnet forms a shunt around the auxiliary spark gap, and the

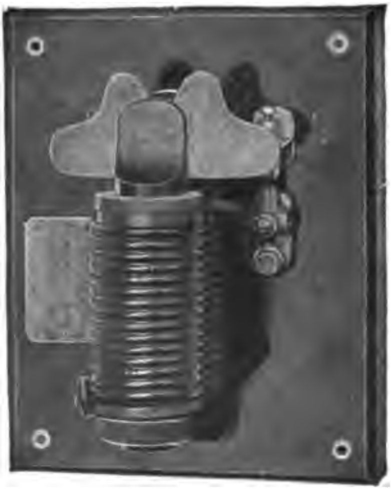

Fig. 307.

current which continues to flow across the two gaps of the arrester after a lightning discharge is passed, quickly establishes itself in the magnet winding, excites the magnet, and blows out the arc in the main gap. The magnetic blow-out arrester is not suitable for alternating-current circuits.

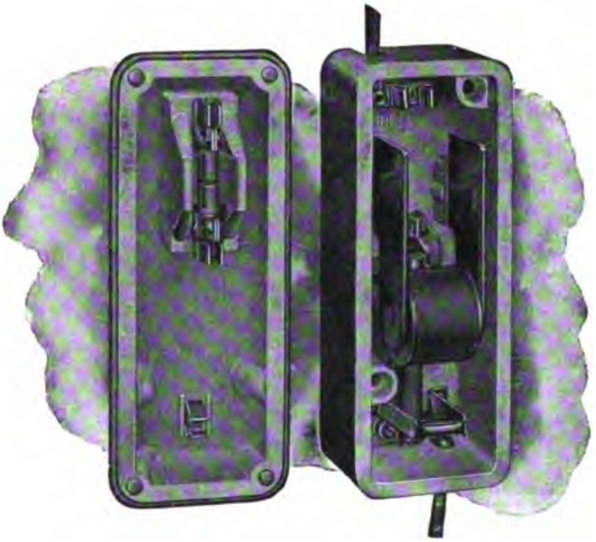
Fig. 308.

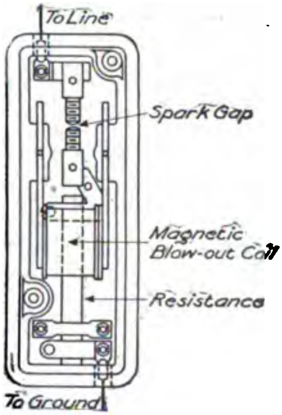

Fig. 309.

Figure 308 shows a compact form of magnetic blow-out lightning arrester with its porcelain box or receptacle. This arrester is made by the General Electric Company. It is of course suitable for direct-current only, and being compact and enclosed, it is specially suitable for protecting the motors on street cars. The spark gap in this arrester is between the ends of two short brass rods, which are shown attached to a porcelain lug on the inside of the cover in Fig. 308. A carbon-rod resistance is connected in series with this spark gap, and the winding of the blow-out magnet is connected as a shunt around the whole or a portion of this carbon resistance. Figure 309 shows the details of the arrester and the manner in which it is connected to the line and to the ground. Figure 310 shows two pairs of these arresters connected to two pairs of feeders, and another pair connected to the generator mains in a station.

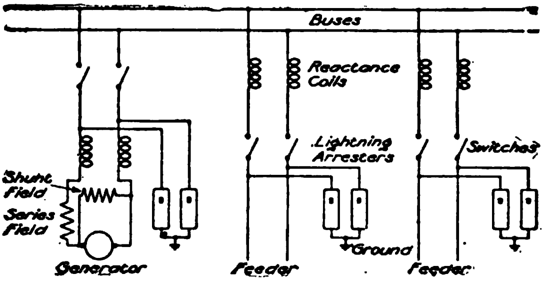

Fig. 310.

The multi-gap non-arcing arrester.—The electric arc seems to depend upon a stream or blast of vapor-like material from the negative electrode of the arc, the so-called *negative blast;* and when this negative blast is once formed the arc can be maintained by a moderately low voltage. To reverse the current in the arc, the negative blast must be established on the opposite electrode, and to do this requires an excessively high voltage if the electrodes are zinc or cadmium, or alloys which contain zinc or cad-

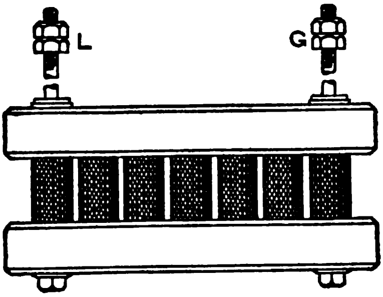

Fig. 311.

mium. Therefore it requires a very high voltage to maintain an alternating-current arc between zinc or cadmium electrodes or between brass electrodes. Thus, an alternating electromotive force of 500 volts (effective value) cannot maintain an arc across a $\frac{3}{64}$-inch gap between blocks of brass. This property of the alloys containing zinc or cadmium was called the *non-arcing property* by its discoverer, Mr. A. J. Wurts, who made use of it in the non-arcing arrester.* Figure 311 shows the essential features of the Wurts alternating-current lightning arrester as manufactured by the Westinghouse Company. It consists of seven brass cylinders held in a porcelain clamp, and it presents six spark gaps in series between the end cylinders which are connected to the terminals *G* and *L*. Figure 312 shows two arresters, like Fig. 311, connected to the two mains of a 2,000-volt alternator. It is always desirable to connect a moderate amount of resistance in series with the spark gaps of a light-

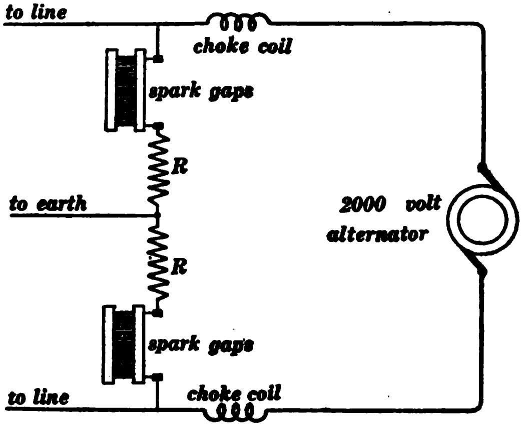

Fig. 312.

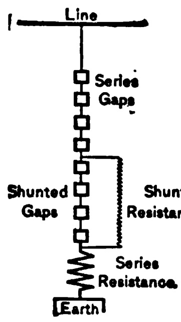

Fig. 313.

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* The contention of Wurts that there is a peculiar non-arcing property of certain metals *not due solely to the cooling action of massive metal electrodes* is justified by the recent researches on the electric arc. See references in Art. 144.

ning arrester as shown in Fig. 312. Figure 313 is a diagram which is taken from the publications of the Westinghouse Electric and Manufacturing Company, showing the arrangement of a lightning arrester for a very high-voltage line. Each small square represents a set of spark gaps like Fig. 311. The "series resistance" and the "shunt resistance" shown in the figure must have considerable carrying capacity so as not to become overheated during the lightning discharge. These resistances are usually made of carbon rods.

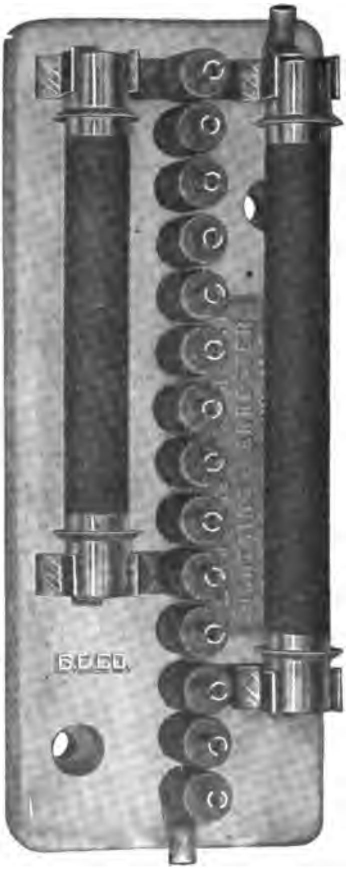

Fig. 314.

Figure 314 shows a single-pole multi-gap lightning arester as manufactured by the General Electric Company, with resistance rods shunted around two groups of spark gaps, the two spark gaps at the bottom of the figure not being shunted. The upper end of the arrester is connected to the line and the lower end is connected to earth.

The aluminum electrolytic lightning arrester. — An electrolytic cell consisting of two aluminum electrodes and an electrolyte of ammonium phosphate, has the remarkable property which is exhibited by the volt-ampere curve in Fig. 315. Up to about 400 volts, an extremely small current flows through the cell, and the current rises to very large values for a slight increase of the voltage above this value. The aluminum electrolytic lightning arrester consists of a number of electrolytic cells like the above connected in series with each other and inserted, in series with a spark gap, between the line wire and the ground. Figure 316 shows a sectional view of the General Electric Company's aluminum arrester. The electrolyte lies between the deep aluminum dishes thus giving a large number of electrolytic cells in series and the stack of cells is submerged in oil which prevents the electrolyte from creeping up over the edges of the dishes and prevents evaporation.

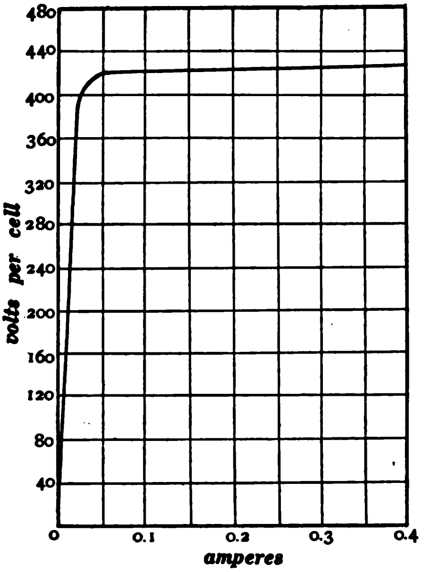

Fig. 315.

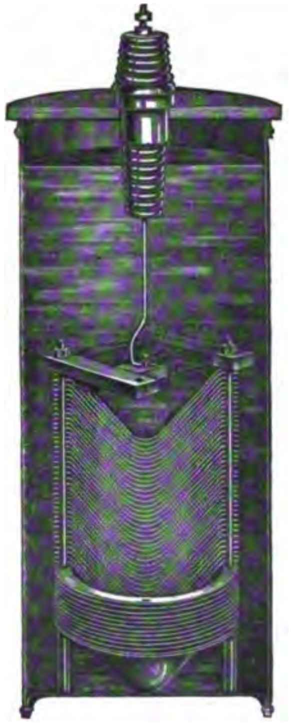

Fig. 316.

197. The compensated voltmeter. — When a voltmeter is connected between the mains at a generating station, either directly or through a "potential transformer," it of course indicates the voltage between the mains at the station, which is generally greater than the voltage between the mains at a distant receiving station. It is frequently desirable to control the voltage between a pair of feeders at a generating station so as to maintain a constant voltage at a distant receiving station which is supplied with current through the given pair of feeders, *in order to do this it is necessary to arrange a voltmeter in such a way that it may indicate the voltage across the mains at the distant receiving station.*

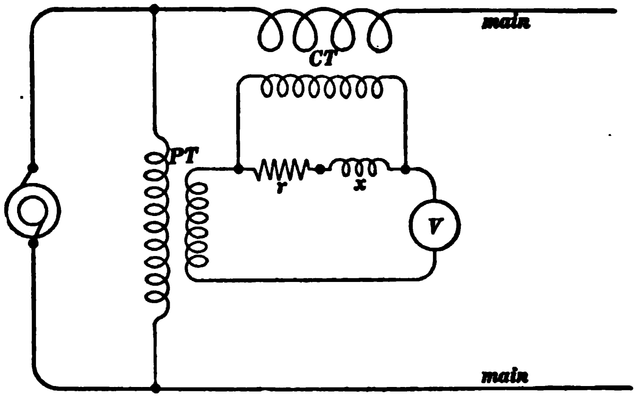

Fig. 317.

A voltmeter so arranged is called a *compensated voltmeter.* The essential features of the compensated voltmeter for alternating-current service are as follows: The voltmeter V is connected to the mains through a potential transformer PT, and in series with an adjustable resistance r and an adjustable reactance x as shown in Fig. 317. A current i proportional to the line current I flows through r and x from a current transformer CT as shown in the figure.

The relationship between the voltage E_0 at the generating station, the voltage E_1 at the receiving station, the line current I, the line resistance R, and the line reactance X is shown in Fig. 318. Let e_0, Fig. 319, represent the secondary voltage of the potential transformer and let ri and xi be the voltage drops across the resistance r and the reactance x of Fig. 317, respectively, so that e_1 is the voltage which acts on the voltmeter. Then, since i bears a fixed ratio to I (the ratio of transformation of the current transformer), it is possible to choose the values of x and r so that the clock diagram, Fig. 319, is exactly similar to the clock diagram, Fig. 318. Then the voltage e_1 which acts on the voltmeter bears a fixed ratio to E_1 at the receiving station, and the voltmeter scale may be numbered so as to indicate the values of E_1 directly.

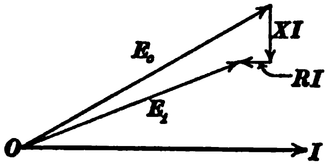

Fig. 318.

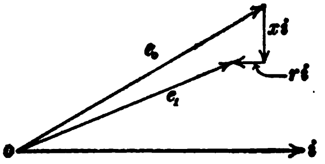

Fig. 319.

CHAPTER XVIII.

PRACTICAL OPERATION OF DYNAMOS.*

198. General instructions for managing generators and motors. — Dynamos are usually provided with an eye-bolt to which ropes or chains for lifting may be attached. In handling a dynamo or in removing an armature for repairs, great care must be exercised to avoid abrasion of the windings. An armature should never be laid upon a hard floor but should always lie upon a bed of sacking.

Bearings should be kept thoroughly lubricated with a fine grade of machine oil. The commutator should be cleaned occasionally with a tightly-woven cloth (free from lint) and a mere trace of oil (preferably vaseline) should be applied to the cloth and thence to the commutator. In any case, oil must be used very sparingly on the commutator because the sparking tends to char it and it collects dust and retains worn off particles of carbon and copper, thus producing a gummy coating which leads to poor electrical contact between the brushes and the commutator.

The tension of the brush springs should be ordinarily adjusted to give a pressure of about 1.5 pounds per square inch of contact surface between the brushes and the commutator. Too little pressure leads to chattering of the brushes and consequent sparking and roughening of the commutator. Excessive brush pressure gives rise to excessive brush friction and undue heating of commutator and brushes.

A dynamo should not be exposed to dust and it should be kept perfectly dry. A water-proof covering should be thrown over the machine when it is not in use.

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* This discussion which is worded to apply explicitly to direct-current machines is in part adapted from the excellent little book *Practical Management of Dynamos and Motors*, by F. B. Crocker and S. S. Wheeler. Everyone who has occasion to handle dynamos should own a copy of this book.

If there is the remotest possibility that a generator is wet or even moist, it should be driven for several hours (without exciting its field) in order to accelerate the drying process by the fan-like action of the rotating armature. The field should then be gradually and cautiously excited by cutting out resistance in the field rheostat and finally the current output should be cautiously increased to its full rated value.

When a motor has been exposed to dampness it should be run as a generator and treated as above described. In the case of an isolated motor, however, it is not feasible to drive it as a generator. If it is very damp, it should be dried by gentle and long-continued heating. In many cases, however, it is allowable to excite the field of the motor, start it and run it for several hours on zero load before putting it into service.

Unless a dynamo is positively known to be dry, it must be assumed to be moist and treated accordingly. A new machine may become damp in transit from the factory.

Switches must be opened and closed with a vigorous and unhesitating movement so as to eliminate as much as possible the destructive action of the arc or spark which is produced between the contact points of the switch.

A belted dynamo, generator or motor, should be provided with a screw device for adjusting the tension of the belt by moving the machine back and forth on slide rails.

199. General directions for starting.*—Before starting a dynamo, generator or motor, see that the machine (especially the commutator and brushes) is clean. Examine the entire machine closely to be sure that all screws and contacts are tight. Fill the oil cups, adjust the oil feed, and see that the oil ducts are clear. In the case of ring-oiling bearings, see that the rings work freely and that they dip into the oil in the reservoir. See that the belt is in place and that it has the proper tension. If it is the first starting of the machine, it should be turned a few times by hand or very slowly by power to make sure that the machine rotates freely and that the belt runs in the center of the pulleys Examine the brushes, see that they press with sufficient pressure against the commutator, and adjust them to the proper position by moving the rocker arm. If the proper position for the brushes is not indicated by a mark this last adjustment must be left until the machine is running and its field excited; the proper position of the brushes is then indicated by minimum sparking, and by maximum voltage in the case of a generator or minimum speed in the case of a motor.

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* Directions for starting direct-current generators are discussed in Arts. 69 and 70. Directions for starting direct-current motors are discussed in Arts. 78, 79 and 80. Directions for starting synchronous motors and for starting alternating-current generators which are to be operated in parallel are given in Arts. 138, 142 and 189. Directions for starting synchronous converters are given in Arts. 147 and 192. Directions for starting induction motors are given in Arts. 174 and 180.

The machine should then be started and brought up to full speed, gradually if possible, all switches in the case of a generator being left open. While starting, the attendant should be in instant readiness to stop the machine if anything should seem to go wrong.

When a dynamo, generator or motor is started for the first time (or after it has been standing unused for a long time), it should be closely watched for an hour or more because an unexpected defect may develop.

200. Diseases of dynamos.—Difficulty in getting a dynamo to operate is most frequently due to a broken connection or a loose contact. Always make sure that the connections are properly made and that the various circuits are free from loose and uncertain contacts. Troubles in the dynamo itself may be classified according to their symptoms as follows: (*a*) Sparking, (*b*) heating and (*c*) unusual noises; and in addition to this, the symptoms peculiar to generators and the symptoms peculiar to motors may be discussed separately.

Sparking and noises are of course readily perceptible. The overheating of a dynamo may be most easily detected by applying the hand to various parts of the machine or by holding the hand in the stream of air that is thrown off by the rotating armature. In general, a temperature that cannot be borne by the hand is considered to be excessive, although carbon brushes and a commutator in which there is no solder may be safely run at a higher temperature than can be borne by the hand. Of course the hand can bear a much higher temperature in a non-conducting substance like wood or varnish than it can in a conducting substance like metal, so that a little experience is necessary to enable one to judge whether the temperature of any part of a dynamo is excessive. An odor of burning varnish is indicative of serious overheating, and a machine which shows this symptom should be stopped at once. Never use water or ice to cool a dynamo.

(*a*) **Sparking.**—A certain amount of sparking at the commutator is inevitable. Excessive sparking may be due to the following causes:

1. Excessive current in the armature on account of a simple overload on a motor or generator, or on account of a short-circuit on the supply mains leading out from a generator.

2. Weak field excitation (see Art. 96). This cause of sparking may be identified in three ways: (i) By the low voltage which is associated with it in a generator or the high speed in a motor (at zero load); (ii) by the weakness of the field-magnet as indicated by the weak attraction of a field-magnet pole for a bit of iron held in the hand (care being taken that the bit of iron is not pulled out of the hand and thrown against the rotating armature), and (iii) by the abnormally large angle of lead of the brushes required to give sparkless running. Weak field excitation is usually due in a shunt or compound dynamo to an excessively high resistance in the shunt field circuit on account of a faulty contact in the field winding itself or in the field rheostat.

3. Improper position of brushes (see Art. 96).

4. Poor contact between brushes and commutator. This may be due to insufficient brush pressure, to the fact that the ends of the brushes are not worn to the exact shape of the commutator, or to the fact that the commutator is foul with grease and dirt. Improper contact between brushes and commutator may be due to jumping of brushes on account of unevenness of commutator or on account of violent vibration due to unbalanced condition of armature, or it may be due to the chattering of the brushes when they are improperly held in the brush holders.

A rough commutator may be greatly improved by the use of fine sandpaper. Never use emery paper or emery cloth on a commutator.

(*b*) **Heating.***—Undue heating of any part of a dynamo may be due to conduction of heat from a hotter part. In attempting to locate the trouble in a machine which heats excessively, this fact must always be duly considered.

1. *Heating of commutator and brushes.* — Carbon brushes always heat very perceptibly and such brushes should not be depended upon to carry much more than about 40 amperes per square inch of contact surface. Undue heating of brushes and commutator may be due to excessive current (over-load) or it may be due to sparking. Short circuits between the commutator segments across the mica insulation sometimes produces excessive heating of the commutator; the remedy is to clean the commutator thoroughly. Imperfect electrical connections between the brushes and the brush holder, which frequently occurs with carbon brushes, may cause the brushes and brush holder to become excessively hot.

2. *Heating of the armature.*—The excessive heating of an armature due to overload is uniform over the entire armature. Short-circuited coils produce local heating. Poor insulation of armature windings due to moisture produces local heating if the

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* This discussion of heating applies equally to direct-current and to alternating-current machines.

moisture is localized and it is indicated by the steaming of the armature.

3. *Heating of field coils.* — This may be due to excessive current in the field produced by excessive voltage or by the short-circuiting of a portion of the field winding. Poor insulation of the field coils due to moisture may produce heating. This heating is local if the moisture is localized and it is indicated by the steaming of the coils.

4. *Heating of bearings.* — Over-heating of bearings may be due to any one of the common mechanical defects; lack of oil, presence of grit, roughness of shaft, too tight a fit between shaft and bearings, crooked shaft, poor alignment of bearings, end thrust of collar or pulley or thrust of shaft against a bearing, or excessive belt tension. The side pull of the field magnet upon the armature when the armature is nearer to one pole piece than to the others may produce excessive friction in the bearings and consequent heating.

(*c*) **Noises.*** — The sound produced by almost any kind of machine is one of the best indications of its condition to an experienced attendant. The legitimate sounds of a dynamo are the humming sounds due chiefly to the armature teeth as they pass the tips of the pole pieces, and to the slight movements of the iron parts especially of the laminations as they are magnetized and demagnetized by pulsating or alternating currents. The abnormal noises are the following:

1. *A fluttering noise,* accompanied by more or less violent vibration, due to unbalanced armature or pulley.

2. *The noise due to the striking of the armature against the pole pieces* or to the striking of the shaft collar or pulley or belt against the bearings. The striking of the armature against the pole pieces may be detected by the scratching noise at low speed, by the abrasion of the armature surface, or by looking through the gap space in front of the various pole pieces.

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* This discussion applies equally to direct-current and to alternating-current machines.

3. *Rattling noises* due to loose parts, such as screws. This noise is most perceptible at moderately low speeds when the loose part is in the rotating element of the machine.

4. *Flapping of belt* or pounding of a laced or cemented belt joint against the pulley. A certain amount of flapping and a certain amount of pounding noise is unavoidable and legitimate, but a whip-like crack as the joint passes over the pulley indicates a loose lacing.

5. *A sharp screeching noise*, like that produced by rubbing the hand over a varnished table, indicates the slipping of the belt due either to overload or to insufficient belt tension.

6. *A squeaking or hissing noise is produced by the brushes.* — When this becomes very prominent a little vaseline is needed on the commutator. The source of this noise may usually be determined by pressing the finger against one brush at a time, or if there is more than one brush in each set, one brush at a time may be lifted from the commutator. The chattering of the brushes is indicated by a noise intermediate between a rattle and a screech.

(*d*) **Symptoms of trouble peculiar to a generator** are those which have to do with its voltage. The generator may not build up at all, it may develop only a weak electromotive force, or, in the case of a direct-current generator, it may become reversed.

1. *Failure to build up.* — This is perhaps the trouble most frequently encountered in the starting of a direct-current generator. The following brief statements of the causes of failure to build up are based upon the full discussion in Art. 61. These statements apply more particularly to the direct-current generator.

(i) Excessive resistance in the field circuit due to a poor contact or an actual break. The brush contacts often have an excessively high resistance when the generator is first started, and the momentary pressure of the fingers on the brush or brushes may enable the machine to build up.

(ii) A short-circuit in the machine or in the external circuit prevents a shunt generator from building up. An excessively high resistance in the external circuit prevents a series generator from building up.

(iii) Absence of residual magnetism prevents a generator from building up. In this case the field winding must be disconnected from the armature and excited for a moment from an outside source.

(iv) Reversed connections of field to armature or reversed direction of driving may prevent a generator from building up (see Art. 61).

(v) Brushes not in proper position. In all of the cuts in this treatise the brushes of direct-current dynamos are represented in their proper positions when they are nearly midway between the field poles. *This is seldom or never the case in actual machines because the connections from the armature conductors to the commutator segments are seldom or never radial.* The proper position of the brushes is that which gives a maximum electromotive force between them, as indicated by a voltmeter.

2. *Excessively low voltage of a shunt generator which is driven at full speed* is due to an excess of resistance in the shunt field circuit.

3. *Reversal of polarity of a direct-current generator.* — When a machine stands idle its residual magnetism may be reversed by proximity to another dynamo or by stray current from an outside source. When the generator is next started it will build up in a direction the reverse of that desired. This reversal of a generator may be corrected by stopping the machine, exciting its field for a moment in the proper direction from some outside source, and then starting it up again. When a number of direct-current generators are to be operated in parallel, the shunt field windings of all of them should be connected to the bus bars; then it is impossible for one of the machines to become reversed with respect to the others.

(*e*) **Symptoms of trouble peculiar to a direct-current motor** are abnormal variations of speed and, inasmuch as the speed of a series motor varies greatly under ordinary conditions of running, the following simple statements apply primarily to the shunt motor:

1. *Failure to start.* — This is most likely due to a faulty connection but it may be due to excessive starting friction or to a combination of moderate starting friction with an under excitation of the field magnet.

2. *Running at low speed.* — This might conceivably be due to a greatly over-excited field, but it is more probably due to an overload or to the combination of a heavy load with an under-excited field

3. *Running at excessive speed.* — This never occurs on very heavy load and when it does occur under light load it is due either to under excitation of field or to displacement of the brushes from their proper positions.

201. Tests for faults in armatures.*— In the four tests described below, the field of the machine is not excited and the armature is stationary.

(*a*) *Tests for broken leads.* — Clean the commutator and substitute for one of the regular brushes a piece of sheet metal which touches one commutator bar at a time. Connect a low voltage low resistance battery through an ammeter to the metal brush and to the carbon brush, and turn the armature very slowly by hand causing the metal brush to make good contact with each commutator bar in succession. Usually there are two leads soldered to each commutator bar. A break in any one of these leads will be indicated by a reduced deflection of the ammeter, and a break in both leads will be indicated by zero deflection of the ammeter when the metal brush touches the commutator segment to which the faulty leads belong.

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* This article applies primarily to the armatures of direct-current machines. Tests (*b*) and (*d*) if slightly modified may be applied to field coils or to armatures of alternating-current machines. See a paper on "Location of Grounds in Armatures, Fields, etc.," by C. E. Gifford, *Transactions American Institute Elec. Engineers*, Vol. XII, pages 260–267.

(*b*) *Test for open circuit.* — If no broken lead is found by test (*a*), a broken circuit in the armature winding itself may be located as follows: Connect the low resistance battery through an ammeter to two metal strips which touch adjacent commutator bars, and turn the armature very slowly by hand. An armature section which is broken, or which has an abnormally high resistance, will be indicated by a reduced deflection of the ammeter. In this test a short-circuited armature section will be indicated by an increased ammeter deflection.

In both of these tests (*a*) and (*b*) the ammeter circuit will be repeatedly broken as the metal brush (or brushes) rides over the mica insulation between the commutator bars, and care must be taken to distinguish these breaks from the partial or complete breaks which may be found while the brushes make good contact with the commutator bars.

(*c*) *Test for short-circuit.* — A short-circuited armature section may perhaps be best located as follows: Pass a current from a battery through the armature, using the regular brushes of the machine. Then take a very low reading voltmeter (or a galvanometer), connect wire leads to it, and touch these wire leads to one pair after another of adjacent commutator bars. A zero or greatly reduced deflection of the voltmeter indicates a short-circuited section.

(*d*) *Tests for ground.* — A ground in the armature winding, that is, a connection between the armature winding and the armature core may be located as follows: Connect a battery to the dynamo brushes. Bring two leads from a very low reading voltmeter (or a galvanometer) and connect one of these leads to the armature shaft and the other to one after another of the commutator bars. A deflection of the voltmeter indicates a ground, and the ground is located near the commutator bar that gives the least deflection.

202. Rules for personal safety. — Personal injury may result from the passage of an electric current through the body, or from severe burns due to a short circuit of the mains by a screw-driver or other metal tool held in the hands. To avoid the first danger the body must not be made a portion of an electric circuit, to avoid the second danger never use a metal tool on live wires as, for example, in the replacing of a burned out fuse.

To make the body a portion of an electric circuit requires, of course, two points of contact with the body. To avoid this it is important to use one hand, only, in manipulating apparatus which is connected with live wires, the other hand being put in the pocket or held behind the back. It must not be forgotten, in this connection, that the greatest risk lies in making a circuit through the hand to the feet when one stands on a moist floor and when there is a ground on one main. A workman should, therefore, always stand on a dry wooden box or platform when working with live wires. Rubber overshoes, if they are whole, insulate a person effectively from the ground, and the use of rubber gloves and of tools with insulated handles is sometimes advisable. It is better, however, not to use such devices on account of the habit of carelessness they develop.

The idea that voltages above a certain minimum, only, are dangerous is wrong. Everything depends upon the nature of the two contacts with the body and their proximity. Voltages as low as 110 volts are dangerous if the contacts with the body are metal plates pressed against the moist skin.

An electrical shock often produces death indirectly by stopping the action of the respiratory organs, and in every case a prompt attempt should be made to restore respiration by the method of artificial breathing employed as a first aid to a person who is taken from the water in a drowning condition.

APPENDIX A.

PROBLEMS.

CHAPTER I. RÉSUMÉ OF ELEMENTS.

1. The anode of an electrolytic cell consists of a copper rod 3 centimeters in diameter, and the cathode consists of a hollow copper cylinder of which the inside diameter is 12 centimeters; 15 centimeters of length of anode and cathode are submerged in the electrolyte, and a current of 25 amperes is passed through the cell. (*a*) Find the current density at the cathode, and (*b*) find the current density at the anode. Ans. (*a*) 0.044 ampere per square centimeter; (*b*) 0.177 ampere per square centimeter.

Note. — The character of the reactions which take place at an electrode in an electrolytic cell depends upon the current density (amperes per unit area of electrode). For example, a large current density deposits both zinc and copper from a mixed solution of zinc and copper sulphates, whereas a small current density deposits only copper.

2. A voltaic cell which is free from local action gives a current of 1.5 amperes for 50 hours. Calculate the number of grams of zinc consumed. Ans. 91.5 grams.

Note. — The zinc consumed in a voltaic cell by voltaic action is equal to the amount of zinc that would be deposited in an electrolytic cell by the current which the cell delivers. The electrochemical equivalent of silver is given on page 3, and the electrochemical equivalent of zinc is 65.4 (2/107.93) as great.

3. A single chromic-acid cell consumes 125 grams of zinc during the time that the current from the cell is depositing 25 grams of copper from a solution of cupric sulphate ($CuSO_4$). What portion of the zinc is consumed by local action? Ans. 79.4 per cent.

Note. — The electrochemical equivalents of zinc (bivalent) and of cupric copper (bivalent) are proportional to their atomic weights, namely, 65.4 and 63.6 respectively.

4. A current of 0.5 ampere flowing through a glow lamp generates 150 calories of heat in 10 seconds. (*a*) Required the resistance of the lamp in ohms. (*b*) What power is expended in the lamp? Express in watts and in horse-power. Ans. (*a*) 252 ohms; (*b*) 63 watts or 0.0844 horse-power.

Note. — One gram calorie is equal to 4.2 joules. One horse-power is equal to 746 watts.

5. A wire having a resistance of 25 ohms is coiled in a vessel containing 2,000 grams of oil of which the specific heat is 0.60. The vessel itself weighs 200 grams and its specific heat is 0.095. A current of 1.5 amperes is passed through the coil of wire. How long will it take to raise the temperature of the oil and the vessel one centigrade degree? Ans. 91.1 seconds.

6. The field coil of a dynamo contains 25 pounds of copper (specific heat 0.094), weight of cotton insulation negligible. The resistance of the coil is 100 ohms. (*a*) At what rate does the temperature of the coil begin to rise when a current of 0.5 ampere is started in the coil? (*b*) How long would it take for the temperature of the coil to rise 20° C. if no heat were given off from the coil by radiation? Ans. (*a*) 0.0056 centigrade degree per second, (*b*) 59 minutes, 31 seconds.

7. What is the resistance at 20° C. of one mile of commercial copper wire 200 mils in diameter. Ans. 1.373 ohms.

Note. — One *mil* is a thousandth of an inch. One *circular mil* is the area of a circle one mil in diameter. The area of a circle d mils in diameter is d^2 circular mils. See table on page 7.

8. A given spool wound full of copper wire 102 mils in diameter has a resistance of 1.5 ohms. An exactly similar spool is wound full of copper wire 204 mils in diameter, what is its resistance? Ans. 0.0938 ohm.

9. Find the resistance at 20° C. of a rectangular copper bar 50 feet long, 4 inches wide, and $\frac{1}{2}$ inch thick. Ans. 0.0002042 ohm.

Note. — Square inches $\times$ 1,000,000 $\times$ $4/\pi$ gives circular mils.

10. What is the resistance at 20° C. of a steel rail 30 feet long weighing 600 lbs.? One cubic inch of steel weighs 0.28 lb. and the specific resistance of the steel is 8 times that of copper. Ans. 0.000343 ohm.

11. What is the resistance at 20° C. of a 20 foot length of wrought iron pipe, 1 inch inside diameter and 1⅜ inches outside diameter? Specific resistance of wrought iron is 7 times that of copper. Ans. 0.001635 ohm.

12. The weight of a cubic inch of copper is 0.32 lb. and the resistance of an inch cube of copper between opposite faces at 20° C. is 0.00000068 ohm. Derive a formula giving the resistance at 20° C. of copper wire in terms of its length in feet and its weight in lbs. Ans. $R = 0.00003133L^2/W$.

13. A sample of commercial copper wire three feet long and 120 mils in diameter is found by test to have the same resistance as 26.2 inches of pure copper wire 100 mils in diameter at the same temperature. Find the conductivity (reciprocal of specific resistance) of the sample expressed in per cent. of the conductivity of pure copper. Ans. 95.42 per cent.

14. The temperature coefficient of a given metal is 0.004 per degree centigrade when expressed in terms of the resistance of the metal at 0° C. Find the temperature coefficient per degree Fahrenheit expressed in terms of the resistance at 0° F. Ans. 0.00239 per degree F.

Note. — Assume a wire of the given metal of which the resistance at 0° C. is one ohm and calculate its resistance R at —17.78° C. (equals 0° F.). The temperature coefficient per degree centigrade expressed in terms of the resistance at —17.78° C. is greater than the temperature coefficient per degree centigrade expressed in terms of the resistance at 0° C. in the ratio of R to unity, and this result must be divided by 1.8 to get the coefficient per degree Fahrenheit in terms of the resistance at 0° F.

15. The field coil of a dynamo (copper wire) has a resistance of 30 ohms after it has been standing for a long time in a room at a temperature of 68° F. After the machine has been running fully loaded for several hours the resistance of the field coil is 35 ohms, what is the temperature of the coil? Ans. 149° F.

Note. — The difference between the temperature coefficient of resistance of a metal expressed as a fraction of its resistance at 0° C. and its temperature coefficient expressed as a fraction of its resistance at any other temperature not greatly different from 0° C. is less than the variation of the temperature coefficient for different samples of the same (commercial) metal, and therefore it is ridiculous to insist on the refined calculations which grow out of the above-mentioned difference. The answers to all temperature-resistance problems in this collection are, however, found by the correct (arithmetically correct!) method. The formula is

$$R_{t'} = R_t \left(\frac{1 + \beta t'}{1 + \beta t} \right)$$

The temperature coefficients of resistance which are given in the table on page 7 are expressed as fractions of the resistance at 0° C.

16. When a certain electric generator is giving out no current it takes 1.75 horse-power to drive it. When the generator delivers a current of 150 amperrs it takes 25 horse-power to drive it. Assuming that the increased power is all used in the maintenance of the 150 amperes of current, find the electromotive force of the generator. Ans. 115.7 volts.

17. A so-called 2,000 candle-power arc lamp has a current of 9.6 amperes flowing through it, and an electromotive force of 47 volts between its terminals. (*a*) Find how many kilowatt-hours and how many horse-power-hours of work are delivered to the lamp during a run of eleven hours. (*b*) If the charge for a run of eleven hours is 27 cents, find the price of power per kilowatt-hour. Ans. (*a*) 4.96 kilowatt-hours or 6.66 horse-power-hours; (*b*) 5.43 cents.

18. Practically all of the energy of the chemical action which takes place in a Daniell cell goes to maintain the current produced by the cell. When one gram of powdered zinc is stirred into a solution of copper sulphate 756 calories of heat are generated. Calculate the electromotive force of the Daniell cell. Ans. 1.07 volts.

Note. — Assume a current of one ampere and find the fraction of a gram z of zinc which would be deposited by this current per second. This is the amount of zinc which is consumed per second by voltaic action. Find the number of calories of heat represented by the reaction of z grams of zinc with copper sulphate, and reduce this result to joules. We thus find the number of joules per second developed by the voltaic action which is produced when one ampere flows through the cell, and this is equal to the desired electromotive force in volts.

19. In the electrolytic refining of copper an electromotive force of 0.3 of a volt suffices to send the current through the electrolytic cell in which the pure copper is deposited. Calculate the number of kilowatt-hours required to deposit a ton of pure copper. Ans. 230 kilowatt-hours.

20. The electromotive force of a battery is 15 volts (measured on open circuit). The battery terminals are connected by a wire, when it is observed that a current of 1.5 amperes is produced and the electromotive force between the battery terminals is 9 volts. Find the resistance of the wire and the apparent resistance of the battery. Ans. 6 ohms and 4 ohms.

Note. — When a voltaic cell is called upon to give current, the terminal voltage of the cell falls off, not only on account of the *ri* drop in the cell, but also on account of what is called polarization. This problem is to be solved on the assumption that the whole of the decrease in terminal voltage is due to *ri* drop, and the value of the resistance as calculated on this assumption is greater than the true resistance of the battery.

21. A coil of which the resistance is to be determined is connected in series with an ammeter across 110-volt mains, and the current is observed to be 26 amperes. What is the resistance of the coil? Ans. 4.23 ohms.

22. A dynamo having an electromotive force of 115 volts between its terminals delivers 200 amperes to a group of glow lamps 1,000 feet distant from the generator. Find: (*a*) the size of copper wire for the mains in order that 95 per cent. of the power output of the generator may be delivered to the lamps; (*b*) the electromotive force between the mains at the lamps. Ans. (*a*) 752,000 circular mils; (*b*) 109.25 volts.

Note. — The resistance of a "mil-foot" of copper wire is taken to be 10.8 ohms in problems 22, 23 and 24. As a matter of fact different samples of copper vary considerably in resistivity.

23. What size of copper wire is required to deliver current at 110 volts to a 10-horse-power motor of 85 per cent. efficiency, the motor being 2,000 feet from the generator, and the electromotive force between the generator terminals being 125 volts. Ans. 230,000 circular mils.

24. A motor receiving 100 kilowatts of power is at a distance of 15 miles from the generator. Line wires 200 mils in diameter are to be used. The line loss is to be 10 per cent. of the generator output. Find: (*a*) the current; (*b*) the voltage at the generator; and (*c*) the voltage at the motor. Ans. (*a*) 16.12 amperes; (*b*) 6,894 volts; (*c*) 6,204 volts.

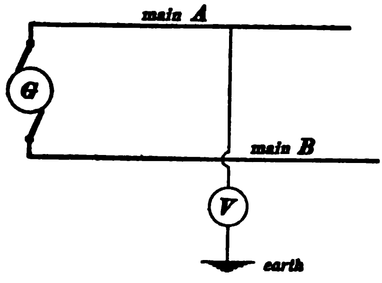

Fig. 25p.

Note. — High voltage direct-current power transmission is not used in American practice.

25. A direct-reading voltmeter *V*, Fig. 25*p*, having 16,000 ohms resistance, is connected from main *A* to earth. The voltmeter gives a reading of 2.6 volts and the electromotive force between the mains is 110 volts. Find the insulation resistance between main *B* and the earth on the assumption that the insulation resistance of main *A* is: (*a*) infinite; (*b*) the same as that of main *B*; (*c*) one tenth of that of main *B*. Ans. (*a*)660,900 ohms; (*b*) 644,900 ohms; (*c*) 500,900 ohms.

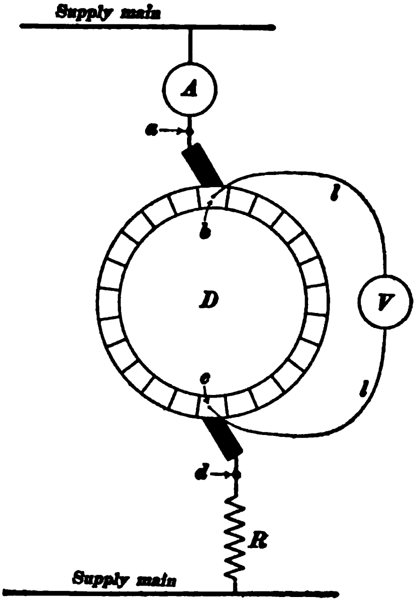

Fig. 26p

26. The armature *D*, Fig. 26*p*, of a dynamo is connected to supply mains in series with an ammeter *A* and a suitable rheostat *R*, and the ammeter reads 50 amperes. The terminal leads *ll* of a low reading voltmeter *V* are connected to the two commutator segments *b* and *c* and the voltmeter reads 4.2 volts. The leads *ll* are then connected to brush lead *a* and segment *b* when the voltmeter reads 1.1 volts; the same reading is obtained when the leads are connected to segment *c* and brush lead *d*. Find: (1) the resistance of the armature between commutator segments *b* and *c*; (2) the resistance of each brush, including contact resistance between brush and commutator; and (3) the total resistance between *a* and *d*. Ans. (1) 0.084 ohm; (2) 0.022 ohm; (3) 0.128 ohm.

27. A direct-reading ammeter has a resistance of 0.05 ohm. The instrument is provided with a shunt so that the total current passing through the instrument and shunt is 10 times the ammeter reading. What is the resistance of the shunt? Would it be practicable to construct such a shunt, measure its resistance by a Wheatstone's bridge, and connect it to the ammeter terminals? If not, how could such a shunt be accurately adjusted? Ans. 0.00556 ohm.

28. A millivoltmeter has a resistance of 15.4 ohms. What resistance must be connected in series with the instrument so that the scale reading may give volts instead of millivolts? Ans. 15,384.6 ohms.

29. The scale of a direct-reading millivoltmeter has 100 divisions, each division corresponding to one millivolt between the terminals of the instrument. This instrument is connected to the terminals of a low resistance shunt and each division of the scale corresponds to 0.25 ampere in the shunt. What is the resistance of the shunt? Ans. 0.004 ohm.

30. The straight metal spoke of a wheel is 40 centimeters long. The plane of the wheel is vertical and magnetic east and west. The horizontal component of the earth's magnetic field is 0.2 gauss. (*a*) How many lines of force does the spoke cut in one revolution? (*b*) How much electromotive force is induced in the spoke when the wheel makes 25 revolutions per second? Ans. (*a*) 1,005.3 maxwells; (*b*) 0.000251 volt.

31. The wheel of problem 30 is replaced by a metal disk 80 centimeters in diameter. What is the electromotive force between the center and the circumference of the disk when its speed is 25 revolutions per second? Ans. 0.000251 volt.

32. The pole face of a dynamo is 30 centimeters long, parallel to the axis of the armature, and the field intensity in the air gap between the pole face and the armature core is 6,000 gausses. The wires on the armature are 12 centimeters from the axis of the armature, and the speed of the armature is 1,800 revolutions per minute. Find the electromotive force in volts induced in each armature wire (30 centimeters long) as it sweeps across the pole face. Ans. 4.072 volts.

33. The core of an induction coil carries 100,000 lines of flux when current is flowing through the primary coil. When the primary circuit is broken the flux through the core drops to 10,000 lines in 0.002 second. How many turns of wire are required in the secondary coil in order that an average electromotive force of 5,000 volts may be induced in it during the 0.002 second? Ans. 11,110 turns.

34. Let it be assumed that the force required to propel a canal boat is proportional to the velocity of the boat, a force of 50 pounds being required to maintain a velocity of 5 feet per second. (*a*) Find the value of the coefficient by which the velocity of the boat must be mutiplied to give the frictional drag and specify the unit in terms of which this coefficient is expressed. (*b*) Find the velocity at which the boat would be propelled by a force of 36 pounds. (*c*) Find the rate at which work is done by a force of 36 pounds in propelling the boat. Ans. (*a*) 10 pounds per (foot per second); (*b*) 3.6 feet per second; (*c*) 129.6 foot-pounds per second.

35a. When a force of 50 pounds is applied to the above canal boat the boat starts from rest, and after some time reaches its full speed of 5 feet per second. At a given instant the velocity of the boat is 3 feet per second. At this instant: (*a*) Find the rate at which work is done on the boat by the propelling force. (*b*) Find the dragging force which is acting on the boat. (*c*) Find the rate at which work is dissipated in overcoming the friction of the water. (*d*) Explain what is becoming of the difference between (*a*) and (*c*). Ans. (*a*) 150 foot-pounds per second; (*b*) 30 pounds; (*c*) 90 foot-pounds per second.

35b. An electromotive force of 50 volts acts on a circuit of which the resistance is 10 ohms. At a certain instant during the time that the current is growing from zero to its full value the current has an actual value of 3 amperes. At this instant: (*a*) Find the rate at which the generator delivers work to the circuit. (*b*) Find the dragging force in volts which is opposing the flow of the current through the circuit. (*c*) Find the rate at which work is dissipated in overcoming the resistance of the circuit. (*d*) Explain what is becoming of the difference between (*a*) and (*c*). Ans. (*a*) 150 watts; (*b*) 30 volts; (*c*) 90 watts.

36. The choke coil of a lightning arrester consists of 50 turns of wire wound in one layer on a cylinder of which the diameter is 15 centimeters and the length is 50 centimeters. (*a*) Calculate the approximate inductance of this coil. (*b*) Calculate the approximate rate of increase of current in the coil at the instant that a lightning discharge jumps across two centimeters of air in preference to going through the coil. Ans. (*a*) 0.000111 henry; (*b*) 360,000,000 amperes per second.

Note. — The inductance of the coil of this problem is to be calculated with the help of equation (18) on page 28. The electromotive force required to strike across 2 centimeters of air is approximately 40,000 volts.

CHAPTER II. THE DYNAMO AND THE TRANSFORMER.

37. (*a*) At what speed must a 40-pole alternator like Fig. 13 be run to give a frequency of 25 cycles per second? (*b*) At what speed would the field magnet shown in Fig. 16 have to be driven to give a frequency of 25 cycles per second? Ans. (*a*) 75 revolutions per minute; (*b*) 750 revolutions per minute.

38. A 6-pole ring-wound armature like Fig. 23, Chapter II, has upon it 1,000 feet of copper wire 325 mils in diameter. What is the resistance at 20° C. of the armature winding from positive to negative brush sets? Ans. 0.889 ohm.

39. The field winding of a shunt generator has a resistance of 40 ohms and the electromotive force between its brushes is 110 volts. (*a*) What is the value of the field current? (*b*) What amount of power is expended in field excitation. (*c*) The full-load current output of the generator being 50 amperes (at 110 volts), express the field current in per cent. of the full-load current output, and express the power consumed in field excitation in per cent. of the full-load power output of the generator. Ans. (*a*) 2.75 amperes; (*b*) 302.5 watts; (*c*) 5.5 per cent.; 5.5 per cent.

40. The field winding of a series generator has a resistance of 0.121 ohm. The full-load current output of the generator is 50 amperes at 110 volts between the generator terminals. (*a*) What electromotive force is lost in the field winding at full-load? (*b*) How much power is consumed in field excitation at full-load? Express the volts lost in the field winding in per cent. of the full-load terminal voltage of the machine, and express the power consumed in field excitation in terms of the full-load power output of the generator. Ans. (*a*) 6.05 volts or 5.5 per cent.; (*b*) 302.5 watts or 5.5 per cent.

CHAPTER III. MEASUREMENTS.

41. An ammeter of negligible resistance is connected in series with 11 ohms (non-inductive) to 110-volt supply mains and a quick-acting switch is arranged to short-circuit 8.25 ohms of this resistance for $\frac{1}{100}$ of a second at a time, 40 times per second. Find the reading of the ammeter (*a*) for the case in which the ammeter is of the D'Arsonval galvanometer type, and (*b*) for the case in which the ammeter is of the electrodynamometer type. Ans. (*a*) 22 amperes; (*b*) 26.46 amperes.

42. Given a direct-reading ammeter (or voltmeter) of the direct-current type of which the scale divisions are of equal length so that an observer can read, say, to $\frac{1}{10}$ of an ampere anywhere on the scale. (*a*) Find the percentage error due to the error of reading when the reading is 1 ampere; (*b*) find ditto when the reading is 10 amperes. Ans. (*a*) 10 per cent.; (*b*) 1 per cent.

43. An electrodynamometer-ammeter (a Siemens instrument) has a scale of equal parts. The position of the pointer can be read to, say, one half of a division anywhere on the scale. (*a*) Find how many times as great the error in amperes (due to a given error in the reading) is when the reading of the pointer is 25 divisions than when the reading of the pointer is 250 divisions; (*b*) find the percentage error in each case. Ans. (*a*) 3.16; (*b*) 1 per cent., $\frac{1}{10}$ per cent.

44. Given a direct-reading ammeter (or voltmeter) of the alternating-current type of which the actual length of a scale division under the pointer is proportional to the reading of the pointer; a scale division being supposed to correspond to, say, 0.1 ampere over the entire scale. The position of the pointer can be read to a certain fraction of a degree of angle, so that the error in amperes due to an error in reading is inversely proportional to the length of the scale division under the pointer, that is, inversely proportional to the reading of the pointer. (*a*) Find how many times as great the error in amperes (due to an error in reading) is when the instrument reads 1 ampere than when the instrument reads 10 amperes. (*b*) Find how many times as great the percentage error is in the first case as in the second case. Ans. (*a*) 10 times as great; (*b*) 100 times as great.

Note. — Ammeters and voltmeters which are suitable for direct-currents only, always give a deflection which is approximately proportional to the current or voltage. Therefore the scales of direct-reading instruments of this type are scales of approximately equal parts.

Ammeters and voltmeters which are suitable for both alternating and direct-current circuits always give a deflection which is nearly proportional to the square of the current or voltage. Let y be the deflection of such an instrument read on a scale of equal parts, or, say, in degrees, let dy be one of the divisions of this scale, and let x be the current or voltage corresponding to the deflection y. Then

$$y = kx^2 \quad \text{and} \quad dy = 2kxdx$$

where k is a constant. Now, in a direct-reading instrument each scale division corresponds to a certain constant fractional part of an ampere or volt which may be represented by dx. Then the length of a scale division of such a direct-reading instrument expressed in degrees is given by the value of dy, which, according to the above equation, is proportional to x, where x is the reading in amperes or volts of the direct-reading instrument.

Problems 42, 43 and 44 touch upon a matter of very great practical importance, and two simple examples covering the entire ground may be worth while. A standardized direct-current ammeter (or voltmeter) which indicates 100 amperes (or volts) with a certain degree of precision will indicate 50 amperes (or volts) with the same actual error or with twice the percentage error. A standardized alternating-current ammeter (or voltmeter) which indicates 100 amperes (or volts) with a certain degree of precision will indicate 50 amperes (or volts) with twice as large an actual error or with four times as large a percentage error.

45. An ammeter indicates 6.2 amperes of alternating current flowing through an arc lamp, a voltmeter indicates 82 volts across the arc, and a wattmeter indicates 440 watts delivered to the arc. What is the power factor of the arc? Ans. 0.865.

46. A coil of wire which is connected across 110-volt alternating-current mains, takes 10 amperes of current. The resistance of the coil is 2 ohms. What is the power factor of the coil? Ans. 0.182.

47. A wattmeter is connected as shown in Fig. 47*p*. The resistance of the coil *B* is 5 ohms. The wattmeter indicates 55 watts when used to measure the power delivered to a single lamp which takes one ampere of current. What is the true power delivered to a lamp? Ans. 50 watts.

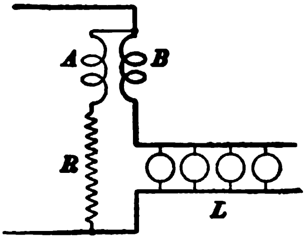

Fig. 47p.

48. An electromotive force of 100 volts acts on a circuit containing resistance only, and the electromotive force is reversed every 0.01 second. The resistance of the circuit is 10 ohms. Plot the electromotive force curve and plot the current curve; and find (*a*) the average current during half a cycle and (*b*) the effective current. Ans. (*a*) Average current 10 amperes; (*b*) effective current 10 amperes.

49. The above repeatedly reversed electromotive force acts on a circuit containing inductance only. The inductance of the circuit is 0.1 henry. Plot the electromotive-force curve and the current curve, and find (*a*) the average current during half a cycle and (*b*) the effective current. Ans. (*a*) 2.5 amperes; (*b*) 2.89 amperes.

Note. — The value 2.89 is found by dividing the half-cycle into 10 equal parts and measuring the middle ordinate of each part.

The current curve which is required in this problem is strictly indeterminate. All that is known from the given data is that the current increases (or decreases) by the amount of 10 amperes during the time that the electromotive force acts in a given direction. Plot the current curve on the assumption that the maximum positive and negative values of the current are equal to each other.

50. An electromotive force which is represented by the curve in Fig. 50*p*, acts on a circuit which contains a condenser only.

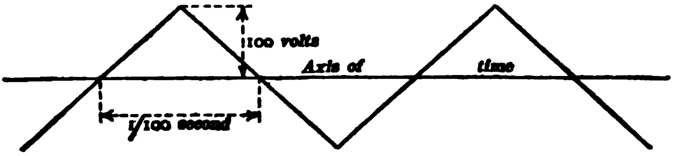

Fig. 50p.

The capacity of the condenser is 10 microfarads. Plot the current curve and find (*a*) the average current during half a cycle and (*b*) the effective current. Ans. (*a*) 0.2 ampere; (*b*) 0.2 ampere.

Note. — The current is in this case at each instant equal to the product of the capacity in farads times the rate of change of the electromotive force.

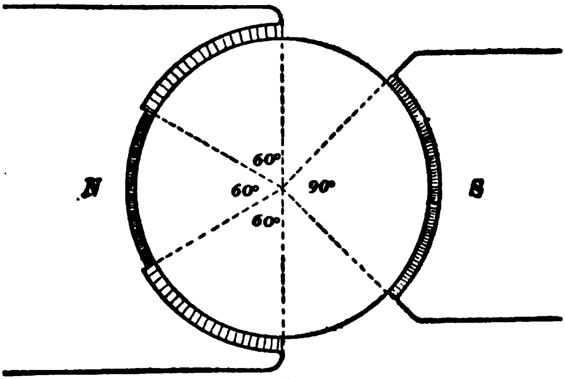

Fig. 51p.

51. A two-pole alternator, having *one conductor* on its armature, has pole pieces shaped as shown in Fig. 51*p*. The total flux emanating from the N pole is 1,000,000 lines, and the armature makes 25 revolutions per second. Plot the electromotive-force curve, neglecting fringes of field at corners of pole pieces, and determine average value and effective value of the electromotive force during each half-cycle. The long and short air gaps under the 180° pole face are as 2 : 1. Ans. Average value 0.5 volt during each half-cycle; effective value is 0.707 volt in the half-cycle corresponding to the 90° pole face, and 0.5303 volt in the half-cycle corresponding to the 180° pole face.

52. The following are instantaneous values, in volts, of the electromotive force of an alternator, taken at equal intervals during an entire cycle: 0, 30, 60, 80, 90, 95, 90, 80, 60, 30, 0, −30, −60, −80, −90, −95, −90, −80, −60, −30 and 0. The corresponding values in amperes of the current are: −65, −45, −25, 0, 25, 45, 65, 75, 78, 75, 65, 45, 25, 0, −25, −45, −65, −75, −78, −75 and −65. Find the instantaneous values of the power, plot the curves of electromotive force, of current and of power, and find the average power. Ans. Average power is 2,138.6 watts.

CHAPTER IV. DIRECT-CURRENT GENERATORS.

53. The ring armature of a bipolar direct-current dynamo has 260 turns of wire upon it, the armature is driven at a speed of 1,200 revolutions per minute, and the magnetic flux from one pole face into the armature core is 3,500,000 lines. Calculate the electromotive force of the dynamo in volts. Ans. 182 volts.

54. The armature specified in problem 53 has 400 feet of 320 mil copper wire wound upon it. What is the resistance of the armature from brush to brush? Ans. 0.01015 ohm at 20° C.

55. The armature specified in problems 53 and 54 is used with a six-pole field magnet with six sets of brushes. (*a*) What is the resistance of the armature between positive and negative brush sets? (*b*) The flux per pole of this six-pole field magnet is 600,000 lines. What is the electromotive force of this six-pole dynamo? Ans. (*a*) 0.00113 ohm; (*b*) 31.2 volts.

56. A given shunt generator gives an induced electromotive force of 110 volts when run at a speed of 1,200 revolutions per minute with a total resistance of 56 ohms in its shunt field circuit. If this machine is driven at a speed of 1,500 revolutions per minute what must be the total resistance in the shunt field circuit to cause the machine to generate an electromotive force of 137.5 volts? Ans. 70 ohms.

57. The electromotive force of a shunt generator decreases from 110 volts to 93 volts when the speed is reduced from 1,000 to 900 revolutions per minute. The armature flux at the higher speed is 1,000,000 lines. (*a*) What is the armature flux at the lower speed? (*b*) What would the electromotive force of the generator be at the lower speed if its armature flux were kept constant at a value of 1,000,000 lines? (*c*) What causes the flux to decrease with the speed? Ans. (*a*) 939,390 lines; (*b*) 99 volts.

58. The resistance of a generator armature, including brushes and brush contacts, is 0.14 ohm. The total electromotive force induced in the armature is 120 volts. Find the value of the electromotive force between the brushes when the current in the armature is 70 amperes. Ans. 110.2 volts.

59. A battery consisting of ten Daniell cells has an electromotive force of 10.8 volts (measured on open circuit) and an internal resistance of 4 ohms. Find: (*a*) The terminal voltage of the battery when it is delivering a current of 1.2 amperes; (*b*) the resistance of the external circuit. Ans. (*a*) 6 volts; (*b*) 5 ohms.

60. The electromotive force of a shunt generator rises from 110 volts at full-load to 130 volts when the receiving circuit is disconnected. What is the percentage regulation of the machine? Ans. 18.2 per cent.

61a. A certain bipolar short-shunt compound generator is rated to deliver a full-load current of 50 amperes at a terminal voltage of 100 volts. To give 100 volts at zero-load a field excitation of 8,100 ampere-turns is required and to give 100 volts terminal voltage at full-load a field excitation of 10,500 ampere-turns is required. Find the number of turns required in the series field coil to give flat-compounding. Ans. 48 turns.

61b. The generator specified in problem 61*a* requires a field excitation of 12,810 ampere-turns to give 110 volts terminal voltage at full-load. Find the number of turns required in the series field coil to give 10 per cent. over-compounding. Ans. 78 turns.

62. The electromotive force of a shunt generator driven at constant speed decreases from 115 volts when the brushes are in the neutral axis to 90 volts when the brushes have 20° forward lead. When the field of this generator is separately excited the electromotive force decreases from 115 volts when the brushes are in the neutral axis to 100 volts when the brushes have 20° forward lead. One million lines of armature flux are required to give 115 volts when the brushes are in the neutral axis. What is the value of the armature flux when the machine is giving, self-excited, an electromotive force of 90 volts with 20° forward brush lead? The armature current in all the above is assumed to be negligibly small. Ans. 900,000 maxwells.

63. A separately excited generator with brushes in the neutral axis gives an electromotive force of 115 volts. The electromotive force between one of the main brushes of this generator and an auxiliary brush 15° from it in the direction of rotation is 8 volts. What would the electromotive force between the main brushes be if they were given a forward lead of 15°? The armature current is assumed to be negligible. Ans. 99 volts.

64a. A series generator has a total internal resistance of 0.2 ohm. When the generator is driven at a speed of 1,000 revolutions per minute it gives a terminal voltage of 99 volts with a current output of 5 amperes. Increasing the current output to 10 amperes increases the useful flux Φ by 50 per cent. Find the terminal voltage of the generator when, driven at a speed of 1,200 revolutions per minute, it delivers a current of 10 amperes. Ans. 178 volts.

64b. The voltage of an over-compounded generator rises from

115 volts at zero-load to 125 volts at full-load. When the series field coil is not used the machine becomes a shunt generator and its voltage falls from 115 volts at zero-load to 90 volts at full-load. What would the voltage of this generator be if it were operated alone at full-load, with its series field coil in parallel with the series field coil of another similar generator which is not in operation. Ans. 107.5 volts.

Note. — The effect, direct and indirect, of the full-load current in the series field winding of this generator is to raise its voltage at full-load from 90 volts to 125 volts, and it may be roughly assumed that one half of the full-load current in the series field winding will raise the full-load voltage half as much.

CHAPTER V. DIRECT-CURRENT MOTORS.

65. A shunt-motor, connected to 110-volt supply mains, takes 2.6 amperes through its shunt field winding, and, when the motor is unloaded, it takes 3.0 amperes through its armature and runs at a speed of 997 revolutions per minute. The rated full-load armature current of this motor is 50 amperes, and the resistance of its armature including brushes and brush contacts is 0.11 ohm. At what speed would this motor run if the friction, eddy current, and hysteresis losses in the armature were zero? Ans. 1,000 revolutions per minute.

Note. — This speed is called the ideal zero-load speed of the motor. It is the speed at which the motor would run if no torque were required to supply the losses due to friction, eddy currents and hysteresis. If these losses were zero, the armature current at zero-load would be zero, the ri drop in the armature would be zero, and the induced electromotive force would be equal to E_x.

66. From the data in problem 65 find: (*a*) The counter electromotive force of the motor at full-load, and (*b*) the full-load speed of the motor (assuming Φ to be constant). Ans. (*a*) 104.5 volts; (*b*) 945.5 revolutions per minute.

67. The actual full-load speed of the motor of problem 65 is observed to be 980 revolutions per minute instead of the speed as calculated in problem 66. Find the value of the ratio Φ/Φ', where Φ is the armature flux due to the field winding alone, and Φ' is the actual value of the armature flux at full-load due to the combined action of field and armature currents. Ans. 1.036.

68. (*a*) Calculate the resistance that must be connected in series with the armature of the motor of problem 65 to give a speed of 500 revolutions per minute under full-load (50 amperes in the armature); (*b*) find the speed of the motor when, with this resistance in the armature circuit, the load is reduced to one half (25 amperes in the armature) and to one quarter (12.5 amperes in the armature) respectively. In this problem assume that Φ remains the same as at zero-load. Ans. (*a*) 0.99 ohm; (*b*) 750 and 875 revolutions per minute, respectively.

69. Find the value of the induction factor $\Phi Z'$ of the motor specified in problem 65; and calculate the torque in pound-inches developed by the motor when its armature current is 50 amperes, assuming Φ to have the same value at zero-load and at full-load. Ans. $\Phi Z' = 6.6$; and torque equals 465.3 pound-inches.

70. A four-pole motor has a four-path armature 30 centimeters in diameter and 35 centimeters long, that is to say, the armature core and pole faces are 35 centimeters long in a direction parallel to the shaft; 0.7 of the periphery of the armature is under the four pole faces, and there are 250 conductors on the surface of the armature. The mean intensity of the magnetic field in the air gap is 4,500 gausses, and the total current flowing through the armature is 100 amperes. (*a*) Calculate the side push in dynes acting on each wire under the pole faces, from this calculate the total torque in dyne-centimeters and reduce the result to pound-inches. (*b*) Calculate the value of Φ and the value of Z' from the above data, and then calculate the torque in pound-inches by equation (31). Ans. (*a*) 393,750 dynes and 915 pound-inches; (*b*) 915 pound-inches.

71. Find the current which would flow through the armature of the motor specified in problem 65 if the armature were to be connected directly to the 110-volt supply mains at starting. Ans. 1,000 amperes.

72. The resistance of the field circuit of the shunt motor specified in problem 65 is doubled by adjusting a field rheostat, the actual zero-load speed of the motor is observed to rise to 1,500 revolutions per minute, and the armature current to 3.3 amperes. In what ratio has the armature flux Φ been changed by the doubling of the resistance of the field circuit? Ans. 1 : 0.664.

73. One terminal of a voltmeter is connected by a lead to the positive brush (which is in the neutral axis) of the motor specified in problem 65, the other voltmeter lead is touched to the commutator at an angular distance of 15° ahead of the positive brush, and the voltmeter indicates 9.2 volts. What would be the ideal zero-load speed of the motor if its main brushes were given a forward lead of 15°? Ans. 1,200.9 revolutions per minute.

Note. — Imagine the machine to be running at its ideal zero-load speed with 110 volts applied to its brushes which are in the neutral axis. Under these conditions the induced electromotive force (counter electromotive force) between the brushes is 110 volts, and the induced electromotive force between two auxiliary brushes 15° ahead of the neutral axis is $(110 - 2e)$. If the main brushes are now shifted 15° forwards, taking the place of the two auxiliary brushes, then the speed must increase sufficiently to make the induced electromotive force equal to the applied voltage. In this problem and the one that follows the armature current is assumed to be negligibly small. It is to be noted that the speed equation (32), page 131, is true only when the brushes are in the neutral axis.

74. The ideal zero-load speed of the motor specified in problem 65 increases to 1,700 revolutions per minute when the brushes are shifted 25° from the neutral axis. Find the electromotive force between: (*a*) An auxiliary pair of brushes in the neutral axis; (*b*) one of the main brushes and an auxiliary brush in the neutral axis. Ans. (*a*) 187 volts; (*b*) 38.5 volts or 148.5 volts.

75. Ignoring the demagnetizing action of the current in the armature, calculate: (*a*) The resistance that must be connected in series with the armature of the motor specified in probem 65 to reduce its speed to 800 revolutions per minute when its load is such as to make the armature intake 50 amperes; (*b*) the watts lost in this resistance expressed in per cent. of the total power intake of the motor. Ans. (*a*) 0.33 ohm; (*b*) 833 watts or 14.4 per cent.

76. The field current of the motor described in problem 65 is increased to 4.8 amperes by cutting out a portion of the field rheostat. This reduces the full-load speed of the motor to 800 revolutions per minute. Calculate the increase of power delivered to the field circuit due to the increase of field current from 2.6 to 4.8 amperes, and express it in per cent. of the power lost in the armature rheosat in problem 75. Ans. 242 watts, or 29.1 per cent.

77. The motor of problem 65 has its field permanently connected to 110-volt mains and its speed is reduced to 200 revolutions per minute at full-load (50 amperes in the armature) by having its armature connected to low-voltage supply mains. Calculate: (*a*) The electromotive force of the low-voltage mains, the full-load speed on the 110-volt mains being 980 revolutions per minute; (*b*) the horse-power developed by the armature at this reduced speed expressed as a fraction of the horse-power developed by the armature when running under full-load on 110-volt mains at a speed of 980 revolutions per minute. Ans. (*a*) 26.8 volts; (*b*) 20.4 per cent.

78. The actual full-load speed of the motor of problem 65 is 980 revolutions per minute (armature current 50 amperes). This machine is driven as a generator at a speed of 980 revolutions per minute, and the field rheostat is adjusted to give the same field current (2.6 amperes). Find: (*a*) The terminal voltage of the generator when the armature current is 50 amperes; (*b*) the total resistance of the field circuit (field coils plus field rheostat) when the machine is operated as a motor and as a generator respectively; (*c*) the resistance which has been cut out of the field rheostat. Ans. (*a*) 99 volts; (*b*) 42.3 and 38.1 ohms respectively; (*c*) 4.2 ohms.

Note.—It is assumed in this problem that the armature flux Φ is the same in value whether the machine operates as a motor or generator. This is approximately true if the backward lead of the brushes for the motor is equal to the forward lead of the brushes for the generator, armature current being the same in each case.

79. A given shunt generator is rated at 10 kilowatts at 100 volts between its terminals when driven at a speed of 1,200 revolutions per minute. The resistance of the shunt field winding is 40 ohms (hot) and the resistance of the armature is 0.08 ohm (hot). Find: (*a*) The armature current of the generator at full load; and (*b*) the speed of the machine when driven as a motor from 100-volt mains with a load such that the armature current is equal to that found under (*a*). Ans. (*a*) 102.5 amperes; (*b*) 1,018 revolutions per minute.

CHAPTER VI. POWER LOSSES AND EFFICIENCY.

80. (*a*) Find the field loss and the stray power loss of the motor specified in problem 65. (*b*) Find the efficiency of this machine when operated as a generator with an armature current of 50 amperes and an electromotive force of 110 volts between its brushes, assuming the stray power loss to be that found under (*a*). (*c*) Find the stray power loss and efficiency of the generator under the conditions specified in (*b*) making a correction for the difference of induced voltage in the armature. Ans. (*a*) 286 watts, and 329 watts; (*b*) 85.4 per cent.; (*c*) 347 watts, 85.42 per cent.

Note. — The difference between the uncorrected and corrected values of the stray power loss, amounting to only 18 watts, is negligible in its effect upon the calculated efficiency of the machine. In the following problems the uncorrected value of the stray power loss is used.

81. Find the efficiency of conversion and the electrical efficiency of the generator specified in problem 80, when its armature current is 50 amperes and its terminal voltage is 110 volts. Ans. 94.6 and 90.3 per cent.

82. Find the true efficiency, the efficiency of conversion, and the electrical efficiency of the generator specified in problem 80, when the armature current is 10 amperes and the terminal voltage is 110 volts. Ans. 56.5, 77.2, 73.1 per cent.

Note. — In this problem neglect the change of field loss due to the adjustment of the field rheostat which must be made to keep the terminal voltage constant.

The stray power loss of a generator at light loads is in fact less than at full-load; this difference depends not only upon the change of induced electromotive force, but also upon the change of distribution of flux in the armature core due to armature reaction. The change of stray power loss due to armature reaction cannot be calculated and is generally ignored in efficiency calculations.

83. (*a*) Calculate the efficiency of the motor specified in problem 65, driven as stated from 110-volt supply mains, when the current intake of its armature has each of the following values: 5, 10, 15, 25, 35 and 50 amperes. (*b*) For what armature current is the efficiency a maximum? (*c*) Calculate the maximum value. Ans. (*a*) 26.1, 54.8, 66.9, 77.5, 82.0 and 84.6 per cent.; (*b*) 74.8 amperes; (*c*) 85.6 per cent.

84. The motor referred to in problem 83 is run ten hours each day as follows: 1 hour with 5 amperes armature intake, 2 hours with 15 amperes armature intake, 4 hours with 35 amperes armature intake, and 1 hour with 50 amperes armature intake. What is the mean all-day efficiency of the motor under these conditions of service? Ans. 78.4 per cent.

Note. — The all-day efficiency of a dynamo is the total output in watt-hours divided by the total input in watt-hours during the day. Thus in this problem find the total input in watt-hours and the total output in watt-hours during the 10-hour run and divide the output by the input.

Evidently a high all-day efficiency, when a motor is run for a large portion of the time at light load, depends upon high efficiency at light loads, and this in turn depends upon small constant losses (field loss plus stray power loss).

85. What is the efficiency of conversion and the mechanical efficiency of the above motor; (*a*) when its armature current intake is 15 amperes, and (*b*) when its armature current intake is 50 amperes? Ans. (*a*) 83.9 and 79.8 per cent.; (*b*) 90.3 and 93.7 per cent.

86. From the results of problem 83 calculate the output of the motor in horse-power corresponding to each value of current intake of the armature, and plot a curve showing efficiencies as ordinates and outputs in horse-power as abscissas.

87. A certain shunt motor is used as a dynamometer to measure the power absorbed by a printing press. The stray power loss corresponding to $E_a = 525$ volts is found by previous test to be 450 watts, and the armature resistance is 0.53 ohm including resistance of brushes and of brush contacts. During the test the current delivered to the armature of the motor is 20.2 amperes, and the voltage across the brushes is 512 volts. Find power delivered to the press. Ans. Stray power during test 430 watts; power to drive press 9,696 watts.

88. A certain shunt motor when connected to 100-volt supply mains takes an armature current of 4 amperes and runs at a speed of 1,063 revolutions per minute at zero load. The armature resistance is 0.1 ohm. Find the stray power loss in watts and the corresponding torque in pound-inches. What opposition to rotation is overcome by this torque? Ans. 398.4 watts; 31.7 pound-inches.

89. An efficiency test of a shunt motor (rated at 5 horsepower, 220 volts, 1,200 revolutions per minute) by the stray power method gave the following results: Volts between brushes 220, and current delivered to armature 2.0 amperes. The machine was then shut down, a current of 10 amperes was passed through the armature, and a voltmeter across brushes read 5.0 volts. Find: (*a*) Stray power loss in watts; (*b*) full-load current in the armature; (*c*) full-load power intake; (*d*) true efficiency at full-load; and (*e*) true efficiency at half-load. The resistance (hot) of the shunt field winding is 733 ohms. Ans. (*a*) 438 watts; (*b*) 19.85 amperes; (*c*) 4,433 watts; (*d*) 84.1 per cent.; (*e*) 76.7 per cent.

Note. — For a given power output P, in watts, the armature current of the motor is determined by the quadratic equation:

$$P = E_x(I_a + I_s) - F - S - I_a^2 R_a$$

in which $F\,(= E_x I_s)$ is the shunt field loss and S is the stray power loss, both of which are assumed to be independent of load.

90. A shunt dynamo when run as a generator at a speed of 800 revolutions per minute delivers 11 kilowatts at a terminal voltage of 110 volts, with a field current of 2 amperes. The resistance of the armature (hot) is 0.05 ohm. When this machine is run as a motor taking 102 amperes from 110-volt mains, it delivers 12.8 horse-power at its pulley. Find: (*a*) Counter electromotive force of the motor; (*b*) speed of the motor; (*c*) induction factor $\Phi Z'$ of the motor; (*d*) stray power loss in the motor; (*e*) full-load torque developed in the generator; and (*f*) full-load torque developed in the motor. Ans. (*a*) 105 volts; (*b*) 729 revolutions per minute; (*c*) 8.63; (*d*) 950 watts; (*e*) 1,240 pound-inches; (*f*) 1,218 pound-inches.

Note. — The armature current is so nearly the same for motor and generator in the specifications of this problem that the armature flux may be assumed to be the same. See note to problem 78.

91. A certain 6.5-kilowatt shunt generator, giving 115 volts between its terminals, has an armature resistance of 0.29 ohm, a field resistance of 57.5 ohms, and a stray power loss of 495 watts. Find the armature current for which the efficiency is a maximum, and find the value of this maximum efficiency. Ans. 50 amperes, 79.2 per cent.

Note. — This problem and the next are based on the condition that maximum efficiency occurs when the variable loss is equal to the constant loss.

92. A certain 5 horse-power 100-volt shunt motor with a speed of 1,200 revolutions per minute has a field loss of 110 watts and a stray power loss of 250 watts, both assumed to be independent of load. The armature resistance is 0.4 ohm. Find the value of the armature current intake for which the efficiency of the motor is a maximum, and find the value of this maximum efficiency. Ans. 30 amperes, 78.9 per cent.

93. A 10-kilowatt 110-volt long-shunt flat-compound generator has a stray power loss of 600 watts, the resistance of its armature including brushes and brush contacts is 0.04 ohm, the resistance of its series field winding is 0.01 ohm, and the resistance of its shunt field winding is 25 ohms. (*a*) Calculate the watts lost in armature winding and series field coil for various armature currents up to 104 amperes and plot a curve showing these losses as ordinates and the corresponding external currents as abscissas. What is the equation to this curve? What is the name of this curve? (*b*) Plot a curve showing total losses as ordinates and values of external current I_x as abscissas; and (*c*) plot a curve showing efficiencies as ordinates and external currents as abscissas.

94. A certain short-shunt compound generator delivers 10 kilowatts of power at a terminal voltage of 100 volts at full-load. The resistance of armature, series field coils and shunt field coils are 0.04 ohm, 0.03 ohm and 20 ohms respectively; and the stray power loss is found by test to be 430 watts. Find at full-load: (*a*) Voltage between brushes; (*b*) armature current; (*c*) total electromotive force induced in the armature windings; (*d*) total electrical power developed in the armature; (*e*) electrical efficiency; (*f*) commercial efficiency. Ans. (*a*) 103 volts; (*b*) 105.15 amperes; (*c*) 107.21 volts; (*d*) 11,300 watts; (*e*) 88.5 per cent.; (*f*) 85.4 per cent.

95. A certain shunt motor is tested for efficiency at full-load by the Prony brake method and the following observations are obtained: Speed, 1,200 revolutions per minute; reading of spring balance, 18.75 pounds; perpendicular distance from center of pulley to the center line of the spring balance, 28 inches; electromotive force applied, 110 volts; total current intake, 83.6 amperes. Find: (*a*) Output in horse-power and in watts; (*b*) torque in pound-feet; (*c*) efficiency (true) of the motor; and (*d*) tangential pull in pounds at the rim of the motor pulley which is 10 inches in diameter. Ans. (*a*) 9.98 horse-power or 7,455 watts; (*b*) 43.75 pound-feet; (*c*) 81.1 per cent.; (*d*) 105 pounds.

96. A certain 6-pole, 250-kilowatt, 550-volt generator runs at a speed of 320 revolutions per minute. The commutator is 37.4 inches in diameter. The machine has six sets of carbon brushes, each set has four brushes, each brush has 1.09 square inches of contact surface with the commutator, the brush pressure is 1.25 pounds per square inch, and the coefficient of friction of the carbon brushes upon the copper commutator is 0.3. Find: (*a*) The watts lost in brush friction; (*b*) the watts lost in contact resistance between the brushes and the commutator at full-load, neglecting the shunt current. Ans. (*a*) 695 watts; (*b*) 911 watts.

Note.— Under ordinary conditions as to peripheral velocity and brush pressure the contact resistance of carbon brushes in ohms per square inch of contact surface is approximately equal to the reciprocal of the current density in amperes per square inch of contact surface. An elaborate discussion of brush contact resistance in its dependence upon peripheral velocity of commutator, brush pressure, quality of carbon and current density is to be found in E. Arnold's *Die Gleichstrommaschine*, Vol. I, pp. 478–484.

CHAPTER VII. RATINGS AND GUARANTEES. ARMATURE REACTION.

97. The drum-wound armature of a certain bipolar dynamo has 384 conductors; the pole faces have an angular breadth of 120°; and the full-load armature current is 200 amperes. Find: (*a*) Per cent. of armature periphery covered by poles; (*b*) the number of demagnetizing ampere-turns on the armature if the angle of lead of the brushes is 30°; (*c*) the cross-magnetizing ampere-turns. Ans. (*a*) 67 per cent.; (*b*) 6,400 ampere-turns per magnetic circuit; (*c*) 12,800 ampere-turns per pole.

98. A certain 6-pole, 250-kilowatt, 550-volt railway generator has a six-path armature winding with 1,200 conductors. The commutator has 600 segments, and at full-load the brushes are shifted forwards through an angle represented by 8 commutator segments. Find: (*a*) The angle of lead of the brushes in degrees; and (*b*) the armature demagnetizing ampere-turns per magnetic circuit at full-load, neglecting the shunt current. Ans. (*a*) 4.8°; (*b*) 2,424 ampere-turns.

99. The pole faces of the generator specified in problem 98 cover 70 per cent. of the periphery of the armature, and the radial length of the air gap between the pole faces and the armature core is 0.527 inch. Find: (*a*) The cross-magnetizing ampere-turns per pole on the armature at full-load; (*b*) the intensity of the magnetic field under the pole tips due to the armature current alone; (*c*) the intensity of the magnetic field under the leading and trailing pole tips due to the combined action of field coils and armature, the field intensity in the air gap due to field coils alone being 6,500 gausses at full-load. Ans. (*a*) 10,640 ampere-turns per pole; (*b*) 5,000 gausses; (*c*) 1,500 gausses and 11,500 gausses.

Note. — The radial length of the air gap in this dynamo is in fact 0.313 inch, but the effect of the high degree of magnetic saturation of the teeth of this armature core is equivalent to a greatly increased radial length of air gap.

100. A bipolar generator has a drum armature with 420 conductors and each pole face subtends an angle of 120°. The distance across the gap space is 0.6 centimeter and the field intensity in the gap space due to the field winding alone is 8,000 gausses. What is the greatest permissible current in the armature of the generator if it is assumed that sparkless running requires the resultant field intensity under the leading pole tips to be at least 25 per cent. of the field intensity due to the field winding alone? Ans. 81.8 amperes.

101. The drum armature specified in problem 100 is arranged so as to be suitable for a four-pole field magnet, the pole faces covering the same percentage of the armature periphery as before, the length of the air gap being the same, and the field intensity in the gap space due to the field winding alone being 8,000 gausses. What is the greatest permissible current in the armature of the machine if it is assumed that sparkless running requires the resultant field intensity under the leading pole tips to be at least 25 per cent. of the field intensity due to the field winding alone? This four-pole drum armature winding has two paths between positive and negative brushes. Ans. 163.6 amperes.

102. The pole faces of the generator specified in problem 101 are widened so as to cover 75 per cent. of the armature periphery and the gap space is increased to 0.7 centimeter. Find the greatest permissible current output of the machine if it is assumed that sparkless running requires the field intensity under the leading pole tips to be at least 25 per cent. of the field intensity of 8,000 gausses which is due to the field winding alone. Ans. 170 amperes.

103. Find the armature demagnetizing ampere-turns per magnetic circuit and the number of turns required in each series field coil to balance the same, in a four-pole 100-kilowatt, 500-volt generator, having a four-path armature winding with 500 conductors; forward lead of the brushes is 10°, and 0.868 of the armature current flows through the series field coils. Ans. 1,389 ampere-turns and 4 turns.

104. The field coil of a dynamo rises 46 centigrade degrees above a room temperature of 35° C., what would be its rise of temperature above a room temperature of 25° C.? Ans. 44.39 centigrade degrees.

Note. — Let t' be the temperature rise above 25° C. and t'' the temperature rise above 35° C. In the first case the actual temperature of the winding is $25 + t'$ and its resistance R' is $R_0[1 + \beta(25 + t')]$. In the second case the actual temperature of the winding is $35 + t''$ and its resistance R'' is $R_0[1 + \beta(35 + t'')]$. The rates at which heat is generated by the given current in the two cases are proportional to R' and R'' respectively, and, since the rise of temperature is proportional (very nearly) to the rates of generation of heat, therefore

$$t' : t'' :: R' : R''$$

105. The armature of a dynamo has a resistance between commutator bars *b* and *c* (see problem 26) of 0.0678 ohm at room temperature of 10° C. After the dynamo has been run for a long time at full-load it is shut down and the resistance of the armature between commutator bars is found to be 0.0806 ohm. (*a*) Find the difference between room temperature and the running temperature of the armature; (*b*) what would this temperature difference be if the room temperature were 25° C.? Ans. (*a*) 49 centigrade degrees; (*b*) 53 centigrade degrees.

106. The armature and all field windings of a given 500 kilowatt, 500-volt generator are connected together (machine not running) and a circuit is made from one main of a 500-volt supply, through a direct-reading voltmeter to one brush of the given generator, whence a very small current passes through the armature and field insulation to the iron frame of the machine which is connected to the other main of the 500-volt supply. The voltmeter indicates 6.2 volts and its resistance is 55,000 ohms. Find the insulation resistance of the given generator as connected. Ans. 4.38 megohms.

Note. — The voltmeter, arranged as specified in this problem, is used as an ammeter, and the current flowing through it is equal to its reading in volts divided by its resistance in ohms.

CHAPTER VIII. HARMONIC ELECTROMOTIVE FORCE AND CURRENT.

107. Two alternators A and B are connected in series. The electromotive force of A is 1,100 volts, and the electromotive force of B is 1,200 volts. The electromotive force of A is 90° ahead of the electromotive force of B in phase. (*a*) What is their combined electromotive force? The two alternators give a current of 125 amperes which lags 30° behind their resultant electromotive force in phase. (*b*) What is the power output of each alternator? Ans. (*a*) 1,628 volts; (*b*) output of A 29,780 watts, output of B 146,470 watts.

108. The electromotive force of alternator A, problem 107, is 135° ahead of the electromotive force of alternator B in phase. A current of 120 amperes flows through both alternators, lagging 30° behind their resultant electromotive force. What is the output of each alternator. Ans. Output of A 30,800 watts negative, output of B 122,770 watts positive.

Note. — Negative output means intake as a motor.

109. Two coils A and B are connected in series across 110-volt alternating-current mains. The voltage across A is 76 volts and the voltage across B is 80 volts. What is the phase difference between the voltages across A and B, assuming all of the voltages to be harmonic? Ans. 90° 21′.

Note. — It is impossible to determine from the data of this problem which of the voltages, A or B, is ahead of the other in phase.

110. A non-inductive rheostat of which the resistance is 60 ohms is connected in series with an alternating-current fan motor to 110-volt alternating supply mains. The voltage across the 60-ohm rheostat is 62½ volts, and the voltage across the fan motor is 64 volts. Find (*a*) the current flowing though the circuit, (*b*) the phase difference between the current and the voltage across the fan motor, and (*c*) the power delivered to the fan motor. Ans. (*a*) 1.04 amperes, (*b*) 59° 10′, (*c*) 34.17 watts.

Note. — It is impossible from the data of this problem to tell whether the voltage across the 60-ohm rheostat is ahead of or behind the voltage across the fan motor in phase. As a matter of fact, the voltage across the 60-ohm rheostat is in phase with the current, and the voltage across the fan motor is ahead of the current in phase.

111. An alternator delivers 200 amperes of current to glow lamps, and 75 amperes to start an induction motor. The power factor of the motor while starting is 0.3. Find the total current. Ans. 233.7 amperes.

112. A harmonic alternating current of which the effective value is 15 amperes has a frequency of 133 cycles per second. (*a*) Find the rate at which the current changes at the instant it passes through zero value, and (*b*) find what value the current would reach if it were to continue changing at this rate for ¼ of a cycle. Ans. (*a*) 17,725 amperes per second, (*b*) 33.3 amperes.

113. A 60-cycle alternator giving a harmonic electromotive force of which the effective value is 110 volts acts upon a circuit of which the inductance is 0.02 henry and of which the resistance is negligible. Find the effective value of the current produced. Ans. 14.6 amperes.

114. (*a*) Find the power component and the wattless component of the current in the fan motor specified in problem 110, these components being taken with reference to the voltage across the terminals of the motor. (*b*) Find the power component and the wattless component of the voltage across the terminals of the fan motor in problem 110. Ans. (*a*) Power component 0.533 ampere, wattless component 0.893 ampere; (*b*) power component 32.8 volts, wattless component 54.95 volts.

CHAPTER IX. FUNDAMENTAL PROBLEMS IN ALTERNATING CURRENTS.

115. A harmonic alternating current, maximum value 100 amperes, frequency 60 cycles per second, flows through a circuit consisting of a non-inductive resistance of 2 ohms, a resistanceless inductance of 0.003 henry, and a condenser of which the capacity is 0.00006 farad, all connected in series. Draw a clock diagram representing the phases of the electromotive forces across the resistance, across the inductance and across the condenser, respectively, and calculate the effective value of each. Ans. 141.4 volts across the resistance, 79.97 volts across the inductance, and 3,126 volts across the condenser.

116. A circuit has an inductance of 0.2 henry and a resistance of 6 ohms. Calculate the effective value of the current produced by an electromotive force of which the effective value is 100 volts and of which the frequency is 60 cycles per second. Calculate the phase difference between electromotive force and current. Ans. Current 1.32 amperes, phase difference 85° 27′.

117. (*a*) Calculate the reactance value of 0.12 henry at 60 cycles per second. (*b*) Calculate the reactance value of a 20-microfarad condenser at 60 cycles per second. (*c*) Calculate the reactance of both in series. (*d*) At what frequency is the reactance value of both in series equal to zero? Ans. (*a*) 45.2 ohms, (*b*) − 132.6 ohms, (*c*) − 87.39 ohms, (*d*) 102.7 cycles per second.

118. A harmonic electromotive force of 1,100 volts (effective) produces a harmonic current of 100 amperes (effective) in a given circuit, the current being 20° behind the electromotive force in phase. Calculate the resistance, reactance, and impedance of the circuit. Ans. Resistance 10.34 ohms, reactance 3.76 ohms, impedance 11 ohms.

119. A harmonic electromotive force of 110 volts produces a current of 15 amperes in a receiving circuit of which the power factor is equal to 0.7. Find the values of resistance, reactance, and impedance. Ans. Resistance 5.13 ohms, reactance ± 5.24 ohms, impedance 7.33 ohms.

Note. — From the data of this problem it is impossible to determine whether the current is ahead of or behind the electromotive force in phase. It is desirable always in specifying the value of a power factor of a receiving circuit to state whether the current leads or lags, thus one would speak of a 70 per cent. leading power factor or a 70 per cent. lagging power factor, as the case may be.

120. Alternating-current mains deliver 100 amperes of current to a non-inductive circuit, to glow lamps for example. An inductive circuit of negligible resistance is then connected to the mains and it takes 10 amperes. What is the total current delivered by the mains? Ans. 100.5 amperes.

121. An inductance of 0.08 henry and a condenser of 25 microfarads capacity are connected in series, the resistance of the circuit being 7 ohms. (*a*) What is the critical frequency of the circuit, that is to say, at what frequency does resonance occur? (*b*) What is the effective value of the electromotive force across the condenser terminals when an electromotive force of 110 volts at the critical frequency acts upon the circuit? Ans. (*a*) 112.5 cycles per second, (*b*) 888.9 volts.

122. An inductance of 0.12 henry and a condenser of which the capacity is 25 microfarads are connected in parallel. The resistance of the inductive circuit is 6 ohms and the resistance of the condenser circuit is 1 ohm. What is the critical frequency at which resonance occurs? Ans. 91.9 cycles per second.

123. An inductance of 0.1 henry and a resistance R are connected in series to 110-volt 60-cycle mains. Calculate the current values when R equals zero, when R equals one ohm, when R equals 5 ohms, when R equals 10 ohms, when R equals 20 ohms, when R equals 50 ohms, and when R equals 100 ohms, and plot a curve of which the abscissas represent R and the ordinates represent corresponding values of the current. Ans. 2.918 amperes, 2.917 amperes, 2.893 amperes, 2.821 amperes, 2.578 amperes, 1.757 amperes, and 1.029 amperes.

124. A non-inductive resistance of 20 ohms, a resistanceless inductance of 0.06 henry, and a condenser of 105 microfarads capacity are connected in series to 110-volt 60-cycle mains. Find the electromotive force, (*a*) between the terminals of the resistance, (*b*) between the terminals of the inductance, and (*c*) between the terminals of the condenser. Ans. (*a*) 109.0 volts, (*b*) 123.3 volts, (*c*) 137.8 volts.

125. A transmission line of which the resistance is 4 ohms and the reactance is 3 ohms delivers 100 amperes to a non-inductive receiving circuit, the electromotive force across the terminals of the receiving circuit being 10,000 volts. (*a*) What is the value of the generator voltage, and (*b*) what is the phase difference between generator voltage and the voltage across the receiving circuit? Ans. (*a*) 10,404.3 volts, (*b*) 1° 39′.

126. A sensibly non-inductive transmission line having a resistance of 4 ohms delivers 100 amperes of current to a receiving circuit. The phase difference between voltage and current in the receiving circuit being sensibly 90°, the voltage across the terminals of the receiving circuit is 10,000 volts. What is the value of the generator voltage? Ans. 10,008 volts.

127. If the transmission line specified in problem 125 were to deliver 100 amperes of current at 10,000 volts to a condenser, what would be the value of the generator voltage? Ans. 9,703.1 volts.

128. The transmission line specified in problem 125 delivers 100 amperes of current at 10,000 volts to a receiving circuit of which the power factor is 0.7. What is the value of the generator voltage? Ans. 10,494.6 volts.

129. Two condensers, each of negligible resistance, have capacities of 0.5 and 0.05 microfarad respectively. The two condensers are connected in series to 1,100-volt alternating-current mains. What is the electromotive force between the terminals of each condenser? Ans. 100 volts, and 1,000 volts respectively.

Note. — **The student should give the physical argument of this problem.**

130. An alternator delivers current to two inductive receiving circuits in parallel. One receiving circuit takes 20 amperes and its power factor is 0.9, and the other receiving circuit takes 25 amperes and its power factor is 0.7. What is the power factor of the combination? Ans. 0.801.

131. An alternator delivers 100 amperes at 1,100 volts and 60 cycles per second to an inductive receiving circuit of which the power factor is 0.85. What capacity condenser would be required to compensate for lagging current? What number of leaves of paraffined paper 22 by 27 centimeters would be required for this condenser, thickness of paraffined paper being 0.08 centimeter, allowing 1 centimeter margin beyond the tinfoil?

Take inductivity of paraffined paper equal to 2. Ans. 127.0 microfarads, 114,800 leaves.

CHAPTER X. POLYPHASE SYSTEMS.

132. The distance from center to center of adjacent poles of an alternator measured on the surface of the armature is 10.4 inches. How far apart must two armature conductors be placed so that there may be a phase difference of 55° in the electromotive forces induced in the respective conductors? Ans. 3.18 inches.

133. A common return wire is used for the two currents of a two-phase system. The system is balanced and each current is equal to 100 amperes. What is the current in the common return wire? Ans. 141.4 amperes.

134. The electromotive force of each phase, problem 133, is 500 volts. What is the electromotive force between the outside wires? Ans. 707 volts.

135. A three-phase alternator is provided with 4 collecting rings and 4 mains as shown in Fig. 164. Three similar receiving circuits are connected as follows: One from main 1 to main 4, one from main 2 to main 4, and one from main 3 to main 4. Each receiving circuit takes 150 amperes. When the armature windings are properly connected to the collector rings the current in main 4 is zero. What is the current in main 4 when one armature winding has its connections reversed? Draw a diagram showing the phase relations between the currents in mains 1, 2, 3 and 4 when the armature winding *A*, Fig. 164, is reversed. In constructing this diagram consider direction outwards from machine as the positive direction in each main. Ans. 300 amperes.

136. Three similar receiving circuits are Δ-connected to 3-phase mains, the electromotive force between each pair of mains being 110 volts. The power delivered to the three circuits is 150 kilowatts and the power factor of each circuit is 0.90. What is the current in each circuit and in each main? Ans. 505 amperes, 875 amperes.

137. Three similar receiving circuits are Y-connected to the 3-phase mains, problem 136; the total power delivered is 150 kilowatts and the power factor of each circuit is 0.90. What is the current in each circuit and in each main, and what is the electromotive force between the terminals of each circuit? Ans. 875 amperes, 875 amperes, 63.5 volts.

138. A given 3-phase alternator is provided with six collector rings so that it may be connected, Δ or Y, for experimental purposes. The electromotive force developed by each winding is 100 volts. Find (*a*) the current in each receiving circuit, (*b*) the current in each main, (*c*) the voltage between mains, and (*d*) the current in each armature winding, when the alternator is Δ-connected to the mains, and the mains are Y-connected to three similar receiving circuits, each one of which has a resistance of 4 ohms and a reactance of 3 ohms. Ans. (*a*) 11.55 amperes, (*b*) same as (*a*), (*c*) 100 volts, (*d*) 6.67 amperes.

139. Find (*a*) the current in each receiving circuit, (*b*) the current in each main, (*c*) the voltage between mains, and (*d*) the current in each armature winding, when the generator specified in problem 138 is Y-connected to the mains and the mains are Δ-connected to the given receiving circuits. Ans. (*a*) 34.64 amperes, (*b*) 60 amperes, (*c*) 173.21 volts, (*d*) 60 amperes.

140. The electromotive force developed in each winding of a 2-phase alternator is 100 volts. What is the electromotive force given by the two windings connected in series? Ans. 141 volts.

141. The electromotive force developed in each winding of a 3-phase alternator is 100 volts. What is the electromotive force developed by the three windings in series? Ans. 200 volts, or zero.

CHAPTER XI. THE ALTERNATOR AS A GENERATOR.

142. Assuming the field flux of an alternator to be harmonically distributed, as explained on page 240, calculate the phase constant of an armature winding in which the conductors lie on the surface of a smooth armature core as bands, the width of each band being $\frac{3}{5}$ of the distance from center to center of adjacent field poles.

Note. — The data of this problem are purposely chosen so that the answer to the problem may not be given in the table on page 243. Assume each band to have an indefinitely large number of conductors.

143. Using the data in Figs. 184 and 185, referring to a given 2,000-kilowatt delta-connected 3-phase alternator, calculate the half-load regulation of the machine for the case in which the receiving circuits have a power factor equal to 0.6; (*a*) by the electromotive force method, and (*b*) by the magnetomotive force method.

Note. — The specifications in this problem are purposely chosen so that the answer to the problem may not be included in the examples given on pages 252-255.

CHAPTER XII. THE SYNCHRONOUS MOTOR.

144. Plot curves showing the values of P', P'' and RI^2 for two synchronous alternators in series, the alternator A having an electromotive force of 1,200 volts, the other machine B an electromotive force of 800 volts, the resistance of the circuit being 1 ohm and the reactance 2 ohms. Result similar to Fig. 194.

145. From the data given in connection with Fig. 195 on page 266 calculate the values of the ratio P''/P' for the following values of P'' (intake of machine B) 25, 50, 75, 100 and 125 kilowatts.

Note. — Solve this problem graphically by using diagrams like Figs. 192 and 193. The result is shown by the curve η in Fig. 195.

146. An alternator A has an electromotive force of 1,100 volts, a resistance of 1 ohm, and a reactance of 0.58 ohm. The machine is driven as a synchronous motor with zero load, from 1,000-volt mains. (*a*) What is the value of the current? (*b*) What is the component of this current which is 90° ahead of the supply electromotive force in phase? (*c*) What capacity of condensers would take the same amount of wattless leading current from the 1,000-volt mains at a frequency of 60 cycles per second? Ans. (*a*) 211.4 amperes, (*b*) 206.6 amperes, (*c*) 548 microfarads.

147. The electromotive force of A is 900 volts, that of B is 800 volts, resistance of circuit is 1 ohm, and reactance of circuit is 1 ohm. What is the maximum intake of machine B as a synchronous motor? Ans. 189.1 kilowatts.

CHAPTER XIII. THE SYNCHRONOUS CONVERTER.

148. A four-pole, two-circuit, single, drum winding has 102 conductors numbered consecutively from 1 to 102. The conductors are connected as follows: 1–26–51–76–101 ... 28–53–78 and back to 1. To what conductors must the three rings A, B and C, of a three-ring converter, be connected? Ans. A to No. 1, B to No. 35, and C to No. 69.

149. Make a diagram of the following winding and show three collecting rings connected to conductors 1, 19 and 37. The winding is a four-pole, two-circuit, single, drum winding with 54 conductors connected up as follows: 1–14–27–40–53 ... 3–16–29–42 and back to 1.

150. At what effective voltage (between adjacent collector rings) must alternating current be supplied to a synchronous converter which is to deliver direct current at 500 volts: (*a*) In the case of a 2-ring converter, (*b*) in the case of a 3-ring converter, and (*c*) in the case of a 4-ring converter? Ans. (*a*) 354 volts, (*b*) 306 volts, and (*c*) 250 volts.

151. A two-ring converter delivers 50 amperes of direct current at 110 volts, the field excitation of the converter being adjusted so that the alternating current which is delivered to the machine is in phase with the alternating supply voltage, that is to say, the power factor of the machine considered as a receiver is unity, and the alternating current which is delivered to the machine is at its maximum value at the instant that the commutator bars to which the collector rings are attached are under the direct-current brushes. Find: (*a*) The effective value of the alternating current delivered to the machine on the assumption that the losses of energy in the machine are zero; (*b*) the value of the alternating current at the instant that the commutator bars to which the collector rings are attached are under the D.C. brushes; (*c*) the actual power delivered to the machine at the instant that the commutator bars to which the collector rings are attached are under the D.C. brushes. Ans. (*a*) 70.7 amperes, (*b*) 100 amperes, (*c*) 11 kilowatts.

152. Consider the 2-ring converter which is specified in problem 151 at the instant when the commutator bars to which the collector rings are attached are 45 electrical degrees from the D.C. brushes. Find at this instant (*a*) the value of the voltage between the collector rings, (*b*) the value of the current entering at the collector rings, (*c*) the value of the power delivered to the machine by the alternating current supply. Ans. (*a*) 77.8 volts, (*b*) 70.7 amperes, (*c*) 5.5 kilowatts.

153. A 50-kilowatt direct-current dynamo is to be used as an n-ring converter. What is its capacity rating when $n = 2$, when $n = 3$, when $n = 4$, and when $n = 6$? Ans. 42.5, 66, 81, and 96 kilowatts.

CHAPTER XIV. THE TRANSFORMER.

154. An electromotive force of 110 volts, from a battery for example, is applied during $\frac{1}{100}$ second intervals in reversed directions to a coil of 100 turns of wire wound on an iron core. Plot the curve representing the above electromotive force and, neglecting the resistance of the coil, plot the curve representing the core flux produced. The iron core is supposed to have a constant magnetic reluctance of 0.00014. Plot the curve representing the magnetizing current.

A secondary coil of 50 turns is wound on the above core. This secondary coil supplies current to a non-inductive receiving circuit having 200 ohms resistance. Plot curves representing (*a*) secondary electromotive force, (*b*) secondary current, and (*c*) total primary current. Resistance of secondary coil is to be neglected.

155. The primary coil of a transformer has 100 turns of wire and its resistance is negligible. The magnetic reluctance of the transformer core is assumed to be constant and its value such that 1,000,000 lines of flux are produced by 200 ampere-turns. The primary coil is connected at a given instant to a 100-volt storage battery. (*a*) Find the current in the primary coil 0.02 second after the given instant, the secondary coil being open, (*b*) find the total current in the primary coil 0.02 second after the given instant, the secondary coil (of 50 turns) being connected to a non-inductive receiving circuit of 5 ohms resistance. Ans. (*a*) 4 amperes, (*b*) 9 amperes.

156. A transformer has 1,300 turns of wire in its primary coil and 130 turns in its secondary coil. The primary coil takes current from 1,100-volt mains and the secondary coil delivers 200 amperes to a receiving circuit of which the power factor is 0.85. What is the equivalent primary resistance and reactance of the secondary receiving circuit? Ans. $r = 46.75$ ohms, $x = 28.98$ ohms.

157. A 10:1 transformer has its primary coil connected to 1,100-volt mains through a non-inductive resistance of 200 ohms and its secondary arranged to deliver current to a non-inductive resistance of 3 ohms. Find: (*a*) The resistance of a simple circuit which is equivalent to the whole arrangement here specified; (*b*) the current delivered to the primary circuit of the transformer; (*c*) the secondary current; (*d*) the secondary terminal voltage; (*e*) primary terminal voltage. Ans. (*a*) 500 ohms, (*b*) 2.2 amperes, (*c*) 22 amperes, (*d*) 66 volts, (*e*) 660 volts.

158. The transformer specified in problem 157 is connected as specified, with the exception that the secondary receiving circuit has a reactance of 3 ohms and a negligible resistance. Find (*a*) the resistance and reactance of the simple circuit which is equivalent to the whole arrangement; (*b*) the primary current; (*c*) the secondary current; (*d*) the secondary terminal voltage; and (*e*) the primary terminal voltage. Draw a clock diagram showing the phase relations of the various electromotive forces and currents. Ans. (*a*) 200 ohms resistance and 300 ohms reactance, (*b*) 3.05 amperes, (*c*) 30.5 amperes, (*d*) 91.5 volts, (*e*) 915 volts.

159. A bundle of iron wires of which the total sectional area is 12.6 square centimeters is to be magnetized by alternating current taken from 133-cycle 110-volt mains so that the maximum flux density in the core may be 4,500 lines per square centimeter. (*a*) How many turns of wire are required? (*b*) How many turns of wire would be required at 60 cycles per second, other things remaining the same? Ans. (*a*) 328 turns, (*b*) 728 turns.

Note. — Resistance of winding is considered negligible.

160. A transformer has 60 turns of wire in its primary coil, and it takes 750 amperes from an alternator at 110 volts and steps up to 20,000 volts. (*a*) How many turns of wire are there in the secondary coil? A usual allowance in transformers is 500 circular mils of sectional area of wire for each ampere of current. (*b*) Find size of primary wire and of secondary wire of the above transformer. Ans. (*a*) 10,909 turns, (*b*) 375,000 circular mils, 2,062 circular mils.

161. The sectional area of the core of the transformer in problem 160 is 161 square centimeters. Find the maximum core flux and maximum flux density at a frequency of 60 cycles per second. Ans. $\Phi = 687{,}700$, $\mathcal{B} = 4{,}272$.

162. A transformer has 800 turns of wire on the primary coil, 40 mils in diameter, and 80 turns on the secondary coil, secondary wire being ten times as large in sectional area. The sectional area of the magnetic circuit of the iron core is 12 square inches. Allowing 500 circular mils of conductor per ampere, and allowing a maximum flux density of 4,000 lines per square centimeter, calculate the electromotive force, current and power ratings of the transformer at 60 cycles per second. Ans. 660.4 volts, 3.2 amperes, 2.113 kilowatts.

163. A given transformer is rated at 5 kilowatts and is designed to take current from 1,100-volt mains at a frequency of 60 cycles per second. Under these conditions P_h, P_e and P_c will be called normal.

(*a*) The transformer gives 6 kilowatts output at the rated electromotive force and frequency. Find P_c in terms of normal.

(*b*) The transformer is used at rated electromotive force but at a frequency of 75 cycles per second. Find P_h and P_e each in terms of normal.

(*c*) The transformer is used at rated frequency but with primary electromotive force of 1,500 volts. Find P_h and P_e each in terms of normal.

(*d*) The transformer is used on a primary electromotive force of 1,500 volts. Find f for which P_h is normal. Show that P_e is independent of frequency with given E'.

(*e*) With primary electromotive force of 1,650 volts what load would give normal P_c?

Ans. (*a*) 1.44 × normal P_c; (*b*) 0.87 × normal P_h, no change in P_e; (*c*) 1.64 × normal P_h, 1.86 × normal P_e; (*d*) 137.2 cycles per second; (*e*) 7.5 kilowatts.

164. Following are the data for a shell type transformer.

A fine wire coil consists of 560 turns of No. 17 B. & S. copper wire. Mean length of turn 29½ inches.

The coarse wire coil consists of 28 turns of two No. 7 wires in parallel. Mean length of turn 29½ inches.

Section of magnetic circuit is $10\frac{1}{2}'' \times 1\frac{13}{16}''$, and $\frac{8}{10}$ of this section is iron. Volume of iron $24\frac{1}{2}$ sq. in. $\times\ 10\frac{1}{2}'' \times \frac{8}{10}$. Thickness of lamination 14 mils.

(*a*) Find the electromotive force and current ratings of this transformer at a frequency of 60 cycles per second, allowing 707 circular mils per ampere, and allowing a maximum flux density of 6,820 lines per square centimeter.

(*b*) Find the copper loss and iron loss at full-load.

(*c*) Find the efficiency of the transformer at full-load.

(*d*) Find the all-day efficiency of the transformer when it is operated at full-load for three hours each day, and at zero-load for the remainder of the time.

Ans. (*a*) 1,000 volts, 2.83 amperes; (*b*) P_c = 112 watts, P_h = 82.6 watts, P_e = 11.4 watts; (*c*) 93.3 per cent.; (*d*) 76.6 per cent.

165. The losses in a direct-current shunt motor when it is running at zero-load are equal to 125 watts, and the losses in the motor when it is running at full-load are equal to 175 watts. The full-load output of the motor is one horse-power. Find the all-day efficiency of the motor when it is operated at full-load for one minute of time every 10 minutes during the day. Ans. 36.5 per cent.

166. A 10-kilowatt 1,100 : 110 volt transformer is connected as an autotransformer to step up from 1,100 volts to 1,210 volts. (*a*) Find the amount of power that can be delivered to a non-inductive receiving circuit at 1,210 volts without exceeding the rating of the transformer; (*b*) find the current in each coil of the transformer; and (*c*) specify the amount of power delivered to or delivered by each coil. Ans. (*a*) 110 kilowatts; (*b*) current in fine wire coil 9.09 amperes, current in coarse wire coil 90.9 amperes; (*c*) 10 kilowatts delivered to fine wire coil, 10 kilowatts delivered by coarse wire coil.

167. A 10-kilowatt 1,100 : 110-volt transformer is connected as an autotransformer to step down from 1,100 volts to 990 volts. Find (*a*) the amount of power that can be delivered to a non-inductive receiving circuit at 990 volts without exceeding the rating of the transformer; (*b*) find the current in each coil of the transformer; and (*c*) specify the amount of power delivered to or by each coil. Ans. (*a*) 90 kilowatts; (*b*) current in fine wire coil 9.09 amperes, current in coarse wire coil 90.9 amperes; (*c*) 10 kilowatts delivered by fine wire coil back to supply mains and 10 kilowatts delivered to coarse wire coil by the current which flows to the service mains.

168. A transformer has two coils *AB* and *CD* arranged as shown in Fig. 168*p*. The coil *AB* is connected to 110-volt mains, point *A* being connected to main number one, and point *B* being connected to main number two. Specify the exact manner in which the coil *CD* must be connected in order that auto-step-up transformation may be accomplished and find the value of the voltage across the service mains.

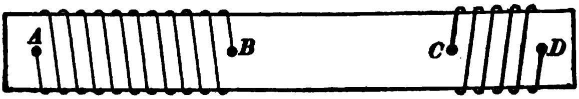

Fig. 168p.

169. Specify the exact manner in which the coil *CD*, Fig. 168*p*, must be connected in order that auto-step-down transformation may be accomplished, with the coil *AB* connected to 110-volt supply mains, and find the value of the voltage across the service mains.

170. The service mains in problem 169 deliver current to a non-inductive receiving circuit of which the resistance is 2 ohms. Find (*a*) the current and power delivered to the receiving circuit; (*b*) the power developed in coil *CD* (specify whether this coil gives out power or receives power); (*c*) the current flowing in coil *AB* and the power developed in coil *AB* (specify whether this coil gives out power or receives power), and (*d*) the total current delivered by the supply mains. Ans. (*a*) 30 amperes, 1,800 watts; (*b*) the coil *CD* receives 1,500 watts; (*c*) 13.64 amperes of current flow in the coil *AB*, this current flows back into the supply mains, that is to say, coil *AB* delivers 1,500 watts to the supply mains; (*d*) of the 30 amperes delivered to the service mains, 13.64 amperes come out of the fine wire coil and the remainder, 16.36 amperes, are delivered by the supply mains.

171. Three similar 1,000 to 100-volt transformers have their 1,000-volt coils delta-connected to three-phase 1,000-volt mains, and the secondaries are Y-connected to service mains. Give a diagram of the connections and find the electromotive force between the pairs of service mains. Ans. 173 volts.

172. The primaries of the transformers in problem 171 are Y-connected to the three-phase supply mains and the secondaries are delta-connected to the service mains. Find the electromotive force between each pair of service mains. Ans. 57.8 volts.

173. The primary coils of two transformers have each 560 turns of wire and they are connected to two-phase mains, the electromotive force of each phase being 800 volts. Calculate the turns of wire in each of two secondary coils (one on each transformer) so that these coils when connected in series may give an electromotive force of 400 volts, 30° ahead of one of the two-phase electromotive forces. Ans. 140 turns, 242.5 turns.

174. A Scott transformer is to transform from 1,000 volts, 60 cycles, two-phase, to 110 volts three-phase. The cross-section of each iron core is 75 square centimeters. Find (*a*) the number of turns of wire in each primary coil, and (*b*) the number of turns of wire in each secondary coil, allowing a maximum flux density of 3,500 lines per square centimeter. Ans. (*a*) $N' = 1{,}429$ turns, (*b*) number of turns in coils *a*, *b* and *c*, Fig. 243 on page 336, are 79, 79, 136, respectively.

175. A three-ring converter is to supply direct current at 500 volts to a street-car line. At what voltage must the alternating currents be delivered to the machine? The step-down transformation is accomplished by three similar transformers with their primaries delta-connected to the high voltage mains and their secondaries Y-connected to the three rings of the converter. What is the ratio of transformation of each transformer, the line voltage being 10,000 volts? Ans. 306 volts. $N''/N' = 0.0177$.

CHAPTER XV. THE INDUCTION MOTOR.

176. The armature of an ordinary direct-current dynamo is short-circuited between the brushes, and the resistance of the armature between brushes is 0.036 ohm. The number of armature conductors is 260, and the flux through the armature is 1,500,000 lines, which is assumed to be invariable. The field magnet is set rotating about the shaft as an axis at a speed of 25 revolutions per second, carrying the direct-current brushes with it. Calculate the torque dragging upon the armature when its speed is 24 revolutions per second and when its speed is 23 revolutions per second. What portion of the power expended in driving the field is available at the armature belt, and what portion is lost in heating the armature, friction losses being ignored?

Ans. (*a*) Torque = 6.86 kg.-meters, 96 per cent. of the power is available at the armature belt and 4 per cent. is lost in heating the armature.

(*b*) Torque 13.73 kg.-meters, 92 per cent. of the power is available at the armature belt and 8 per cent. is lost in heating the armature.

Note.— This problem may be solved by the use of the ordinary equations of the direct-current dynamo, namely:

$$\mathcal{E} = \phi Z' n$$

$$\mathcal{I} = \mathcal{E} / R$$

$$T = 1.41 \Phi Z' \mathcal{I} \text{ pound-inches}$$

where Z' in the present case is $260 \div 10^8$, and n is the relative speed.

177. The synchronous speed n of an induction motor is 100. The motor develops its maximum torque when the rotor speed is 83. How many times must the rotor resistance be increased in order that the motor may develop its maximum torque at standstill? Ans. 5.88 times.

178. Given a 10-horse-power, 4-pole, 60-cycle induction motor which runs under full-load at 1,650 revolutions per minute. The friction losses (mechanical and magnetic) in the rotor are assumed to be negligible, and the copper and core losses in the stator are 0.84 kilowatt. What is the actual efficiency of the motor at full-load. Ans. 85.1 per cent.

179. An ideal three-phase induction motor takes 5 amperes of current into each phase of its delta-connected primary member at 200 volts and 60 cycles per second. The rotor, which has a three-phase winding, delta-connected to collecting rings, supplies 25 amperes of current to each of three similar delta-connected circuits, each having a power factor equal to 0.75. The rotor runs at ⅔ synchronous speed. (*a*) What is the ratio of the stator to the rotor turns? (*b*) What is the rotor terminal voltage? (*c*) What is the total intake of power? (*d*) What is the electrical output of power? (*e*) What is the mechanical output of power? Ans. (*a*) Ratio of turns is 5 : 1. (*b*) Rotor terminal voltage is ⅓ × ⅕ × 200 volts. (*c*) Total intake of power is 2,250 watts. (*d*) Total electrical output is 750 watts. (*e*) Total mechanical output is 1,500 watts.

Note.—In an ideal induction motor the magnetizing current is assumed to be negligibly small or, in other words, the magnetizing action of the current in the stator is assumed to be completely balanced by the magnetizing action of the current in the rotor. Therefore the ratio of rotor to stator turns in each phase is equal to inverse ratio of the currents. The current in each phase of the stator winding being 5 amperes and the current in each phase of the rotor winding being 25 amperes.

The rotor terminal voltage may be found by considering that the rotating stator magnetism cuts the stator windings at synchronous speed and cuts the rotor windings at one third synchronous speed, according to the above specifications.

The phase difference between electromotive force induced in each rotor winding and current delivered by each rotor winding is the same as the phase difference between voltage across each stator winding and current in each stator winding, the magnetic leakage of the motor being ignored.

The remaining details of this problem are fully explained on pages 352–358.

180. An induction motor develops 30 per cent. of its maximum or break-down torque at zero speed when the supply voltages are at their normal rated value. What is the starting torque with supply voltages reduced to ⅓ normal? With supply voltages reduced to ⅔ normal? What would the starting torque be in each case if the rotor resistance were increased so as to give the maximum torque at starting. Ans. 3.3 per cent., 13.3 per cent., 11.1 per cent., and 44.4 per cent. of maximum torque, respectively.

181. A 4-pole 60-cycle induction motor runs at a speed of 1,650 revolutions per minute at full-load. The rotor resistance is to be increased so that the motor may run at 800 revolutions per minute and develop its full-load torque. In what ratio must the rotor resistance be increased? Ans. 1 : 6⅔.

CHAPTER XVI. THE SINGLE-PHASE SERIES MOTOR.

182. Given a 2-pole, 25-cycle, single-phase, series motor of which the full-load current rating is 25 amperes effective. The resistance of the field winding alone is 0.15 ohm, and the resistance of armature and neutralizing windings together is 0.25 ohm. With the armature stationary the full-load current is produced through the motor by 70 volts effective at rated frequency. With the field winding open the full-load current is produced through the armature and neutralizing windings by 20.73 volts effective at rated frequency. What is the total reactance of the motor, what is the reactance of the field winding alone, and what is the reactance of the armature and neutralizing windings together? Ans. (*a*) 2.77 ohms; (*b*) 1.979 ohms; (*c*) 0.791 ohm.

183. The motor of problem 182 is driven by an auxiliary motor at a speed of 1,500 revolutions per minute with its field excited by 20 amperes of direct current, and the steady electromotive force across the armature terminals is observed to be 60 volts.

The machine is to be driven as a single-phase motor from a 25-cycle source at a speed of 1,500 revolutions per minute taking full-load current. Find: (*a*) The terminal voltage effective that is required, and (*b*) the power factor of the motor. Ans. (*a*) 109.6 volts; (*b*) 77.5 per cent.

Note.— In this and the following problems assume that the reluctance of the magnetic circuit of the motor is invariable so that the useful flux may be considered to be at each instant proportional to the field current.

184. The motor of problems 182 and 183 has 240 conductors upon its armature. Find the maximum value of the alternating armature flux when the motor is running as specified in problem 183, and find the number of field turns. Ans. (*a*) 1,770,000 lines, (*b*) 55 turns.

Note.— In this problem assume the field flux and armature flux to be equal, or, in other words, assume magnetic leakage to be zero.

The maximum value of the alternating armature flux corresponds to the maximum value of the alternating current in the field winding as explained in the note to problem 183. The reactance voltage $X_f I$ is induced in the field winding by the alternating field flux exactly as in the coil of a transformer so that equation (63) of Art. 157 gives the relation between maximum field flux, field turns, induced voltage and frequency.

185. The motor of problems 182–184 is driven from a 15-cycle supply at a speed of 1,200 revolutions per minute taking 25 amperes effective of current. What is the terminal voltage of the motor and what is its power factor? Ans. (*a*) terminal voltage 81.4, (*b*) $\cos \theta = 0.86$.

186. The motor of problems 182–184 is driven from a 60-cycle supply at a speed of 1,800 revolutions per minute taking 25 amperes effective of current. What is the terminal voltage of the motor and what is its power factor? Ans. (*a*) terminal voltage 192.5, (*b*) $\cos \theta = 0.52$.

187. The motor of problems 182–186 has 60 commutator bars, that is, a section of the armature winding consists of 4 conductors (two turns). The motor operates at full-load current at 25 cycles per second frequency. What is the effective value of the voltage which is induced in the sections of the armature winding as they are successively short-circuited by a brush? Ans. 3.93 volts.

188. With the motor operating as specified in problem 187, calculate the resistance required in each resistance lead to give a short-circuit current of 62½ amperes effective; each armature section consists of 36 inches of 102 mil copper wire, and the brush tip introduces a resistance into the short circuit such that the *RI* drop through the brush is 2.0 volts. Ans. 0.014 ohm.

Note. — The answer to this problem is calculated on the assumption that the armature section is non-inductive. This assumption is not exactly true. The tendency of the current in the armature section to produce flux through the main magnetic circuit of the motor does not give to the armature section a reactance which has to be overcome by the electromotive force induced by the actual given flux through the magnetic circuit of the motor. The only thing which does give a reactance to the armature section in this problem is the leakage flux which circles around the bundles of wires in the armature slots.

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